Massachusetts Institute of Technology
1.070 Introduction to Hydrology
Problem Set 4

Assigned October 7th 2004
Due October 28th, 2004

Problem 1

(Use Matlab, and remember to include your matlab script)

The objective of this problem is to investigate how different processes affect the transport of a pulse of contaminant injected into an aquifer with unidirectional flow. The most general solution to the advection-dispersion equation for a so-called "slug" or "pulse" input is as follows:

$$C(x,t) = \frac{M}{\sqrt{4\pi \frac{D}{R} t}} \exp\left\{ -\frac{(x - v\frac{R}{t})^2}{4\frac{D}{R}t} \right\} \tag{1}$$

This accounts for advection, dispersion and retardation due to adsorption. D is the hydrodynamic dispersion coefficient, which includes both molecular diffusion and mechanical dispersion. So:

$$D = D^* + \alpha v \tag{2}$$

Let's assume that the slug input of mass was 15kg m^{-2}. We want to look at the distribution of the plume along the direction of flow at t=20 days, 50 days and 100 days, to see how it's distribution changes with time.

(a) Start with the fluid at rest, so assume that there is no advection occuring. Furthermore, there will be no mechanical dispersion because the fluid is not moving. This means that v=0, R=1 and $D = D^* = 1 \times 10^{-7}$ m^2 s^{-1}. Write the simplified version of equation 1 under these conditions. Plot C(x,t) for the three time steps on a single graph (**Figure 1**). *C*omment on the location of the maximum and the change in the concentration with time.

(b) Include advection. Now the chemical is being carried by advection in addition to diffusion/dispersion. Use a pore velocity of 1 m day^{-1}, $D^* = 1\times10^{-7}$ m^2 s^{-1}, and assume a longitudinal dispersivity of 10m. Plot C(x,t) for the three time steps on a single graph (**Figure 2**). Comment on the location of the maximum and the change in the concentration with time.

(c) Include adsorption, where some of the chemical adsorbs to soil grains. Use a retardation factor R=2.5. Now, equation 1 is used. Plot C(x,t) for the three time steps on a single graph (**Figure 3**). Comment on the location of the maximum and the change in the concentration with time. In particular, compare it to your results from Figure 2. How does retardation affect both the mean distance travelled by the pulse and the spread in concentration?

Problem 2
(Use Matlab, and remember to include your matlab script)

An industrial accident at a pharmaceutical plant spills a harmful chemical into the underlying aquifer. The instantaneous mass loading was 4kg m^{-2}.The chemical can only be detected if its concentration exceeds 1mg/L. However, it is harmful to humans if ingested at concentrations greater than 15mg/L. The aquifer has porosity n=0.3, and the specific discharge is 0.95 m day^{-1}. Assume a molecular diffusion coefficient of 1 $\times 10^{-7} m^2 s^{-1}$.

(a) Plot the breakthrough curve at a monitoring well 300m downgradient of the spill location, and answer the following:
(i) How many days was it before the chemical was detectable at the monitoring well?
(ii) How long was the chemical detectable at the monitoring well?
(iii) For how many of those days was the chemical at a concentration considered unsafe for humans?

(b) An engineer from the plant calls to tell you that the situation may not be as bad as first feared. The chemical has a decay rate of 0.8 day^{-1}.
Plot the new breakthrough curve at the monitoring well. (You can superimpose it on the first graph using "**hold on**" in matlab). Taking the decay rate into account, for how many days will the chemical concentration at the monitoring well be higher than 15mg/L?

Problem 3
A column test is performed to determine the longitudinal dispersivity [cm] and the retardation factor [.] of a conservative tracer in a soil of porosity n=0.35. The cylindrical column has a diameter of 10cm and is 100cm in length. You maintain a constant flow of $0.45 cm^3 s^{-1}$ through the column and a constant concentration of C_0 at the upgradient end. After 2.5 hours, you find C/C_0 at the outlet to be 0.19. After 3 hours, you find C/C_0 to be 0.5. Determine the longitudinal dispersivity and the retardation factor. The tabulated error function **erf(**β**)** is attached.

Problem 4
(Use Matlab, and remember to include your matlab script)
In this problem, you will use the over-relaxation method to find the steady-state head field in an unconfined aquifer.The aquifer is 10km wide, and is bound by a lake on one side, and a river on the other side. The hydraulic head is 12m at the lake, and 6m at the river. The aquifer may be considered infinitely long compared to its width, but you are asked to model just a 20km strip which contains two wells. This may be approximated using a 200 by 100 grid, so that the spatial resolution is 100m. The wells are located in grid pixels (x,y)=(70,50) and (x,y)=(50,120), and are pumping at 3000 m^3 day^{-1} and 2000 m^3 day^{-1} respectively. Recharge is uniform at 0.5 $\times 10^{-3}$ m day^{-1}. Starting with an initial guess of 13m, and use over-relaxation to find the steady state head field.

Some hints to get you started:
1) Use $\phi=h^2$. Solve for ϕ in your over-relaxation scheme.
2) The pumping rate Q must be expressed in the same units as the recharge, so normalize it by the area of the grid cell.
3) The source/sink term is then given by:

$$S = \frac{(Rp + Qp)\Delta s^2}{4K} \tag{3}$$

where $\Delta s = \Delta x = \Delta y = 100m$, Qp is the normalized pumping rate (m per day) and Rp is the recharge rate (m per day) You will note that this ensures that S has units of $[L^2]$, just like ϕ!
4) In general, iterating involves obtaining the old value of ϕ at the four surrounding nodes and getting the new ϕ from:

$$\phi = \frac{1}{4}(\phi_{up} + \phi_{down} + \phi_{right} + \phi_{left}) - S \tag{4}$$

5) At the constant head boundaries, ϕ is found from the constant head, e.g:

$$\phi_{right}(Nx, :) = \frac{1}{2} h_{lake}^2 \tag{5}$$

6) At the other edges there is no flow gradient across the boundary, so we generally write:

$$\phi_{up}(2 : Ny, :) = \phi_{old}(1 : Ny - 1, :) \tag{6}$$

this becomes:

$$\phi_{up}(1, :) = \phi_{old}(1 + 1, :) \tag{7}$$

at the top boundary.

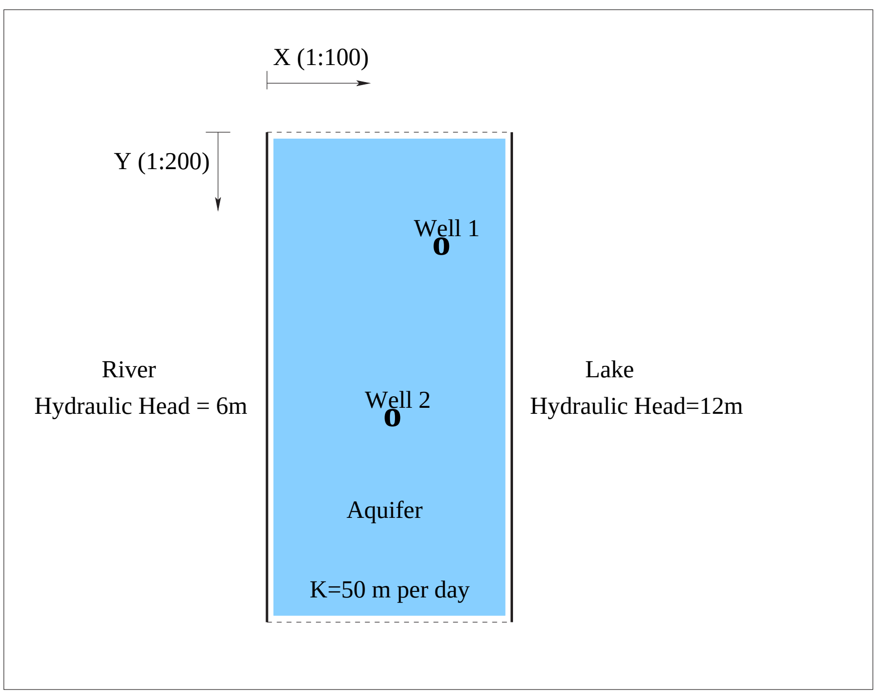

Figure 1: Problem 4:Model Domain