8.02 with Kai   Spring 2003

Problem Set 2 Solution

Problem 1: 24.7
A point charge q is located at the center of a uniform ring having linear charge density λ and radius a, as shown in Figure P24.7. Determine the total electric flux through a sphere centered at the point charge and giving radius R, where $R<a$.

Solution:

The only charge that contributes to the total flux are the charge that situates inside radius R. Therefore

$$\Phi_E = 4\pi k_e q \qquad\qquad (1.1)$$

Problem 2: 24.14
Calculate the total electric flux through the paraboloidal surface due to a constant electric field of magnitude E_0 in the direction shown in Figure P24.14.

Solution:
1. Since there is no charge inside the closed surface, so the net flux going through the surface is zero.
2. Thus, flux going inside the closed surface equals the flux going out from the surface.
3. Since the flux comes in through the flat surface and goes out through the paraboloidal surface (the electric field is perpendicular to the flat surface),
4. so the flux going through the flat surface is the same as the flux going through the paraboloidal surface.

Thus, the flux through the paraboloidal surface is

$$\boxed{\Phi_E = E_0\left(\pi r^2\right)}$$
(2.1)

Problem 3: 24.21
Consider an infinitely long line charge giving uniform charge per unit length λ, Determine the total electric flux through a closed right circular cylinder of length L and radius R that is parallel to the line charge, if the distance between the axis of the cylinder and the line charge is d. (Hint: Consider both cases: when $R < d$, and when $R > d$.)

Solution:
Case I: When $R < d$,
In this case a part of the line of charge would be inside the closed cylinder. Since the line of charge has a uniform charge density,

$$\frac{dQ}{dl} = \lambda \tag{3.1}$$

Therefore the charge enclosed by the surface would be

$$q_{enc} = \lambda L \tag{3.2}$$

Applying Gauss's Law,

$$\oint \vec{E} \cdot d\vec{a} = 4\pi k_e q_{enc} \tag{3.3}$$

and since the flux through the flat surface would be zero, we get

$$E(2\pi R L) = 4\pi k_e (\lambda L) \tag{3.4}$$

which gives

$$\boxed{E = \frac{2k_e \lambda}{R}} \tag{3.5}$$

which has a direction of radially outward.

Case II: When $R > d$,

In this case, the line of charge would not go through the closed cylinder surface. Thus, the charge enclosed by the cylinder would be zero. By Gauss's Law,

$$\boxed{E = 0} \tag{3.6}$$

In conclusion, we have

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{2k_e\lambda}{R} & , R < d \\ 0 & , R > d \end{cases}$$

(3.7)

Problem 4: 24.28

A cylindrical shell of radius 7.00 cm and length 240 cm has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is 36.0 kN/C. Use approximate relationships to find
    (a) the net charge on the shell and
    (b) the electric field at point 4.00 cm from the axis, measured radially outward from the midpoint of the shell.

Solution:
(a) Assume that the line is long enough that the electric field at the point 19.0 cm away from the line of charge is still uniform. Then, as we know that for an infinite line of charge

$$E_r = \frac{2k_e\lambda}{r} \tag{4.1}$$

substituting, then we get

$$\left(3.60\times10^4\right) = \frac{2\left(8.99\times10^9\right)\left(\dfrac{Q}{2.40}\right)}{0.190} \tag{4.2}$$

which gives

$$\boxed{Q = 9.13\times10^{-7}\,\text{C}} \tag{4.3}$$

(b) Since the point that we are evaluating at is inside the shell, thus there is no charge enclosed if we construct a cylindrical Gaussian surface with the radius 4.00 cm, therefore, by Gauss's Law,

$$\boxed{E = 0} \tag{4.4}$$

Problem 5: 24.42
A hollow conducting sphere is surrounded by a larger concentric, spherical, conducting shell. The inner sphere has a charge $-Q$, and the outer sphere has a charge $3Q$. The charges are in electrostatic equilibrium. Using Gauss's law, find the charges and the electric fields everywhere.

Solution:
Bearing in mind that the two spheres are conductors, thus the electric field inside a conductor would always be zero.
So there are 5 cases:

- Case I: Inside the inner sphere

Since it is inside a conductor,

$$\boxed{\vec{E} = 0} \tag{5.1}$$

- Case II: Between the inner and the outer sphere

If we draw any gaussian spherical surface that is concentric with the inner sphere with radius r away from the center, we get

$$E\left(4\pi r^2\right) = 4\pi k_e\left(-Q\right) \tag{5.2}$$

which gives

$$\boxed{E = \frac{-k_e Q}{r^2}} \tag{5.3}$$

- Case III: Inside the outside spherical shell

Since it is inside a conductor, we get

$$\boxed{\vec{E} = 0} \tag{5.4}$$

- Case IV: Outside the outer spherical shell

The charge enclosed by any gaussian surface would be $2Q + \left(-Q\right) = Q$. Thus, the electric field would be

$$\boxed{E = \frac{k_e Q}{r^2}} \tag{5.5}$$

Problem 6: 24.49
A solid conducting sphere of radius 2.00 cm has a charge $8.00\ \mu E$. A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge $-4.00\ \mu C$. Find the electric field at

    (a) $r = 1.00$cm
    (b) $r = 3.00$ cm
    (c) $r = 4.50$ cm
    (d) $r = 7.00$ cm

from the center of this charge configuration.

Solution:
This problem is similar to the last question, and it is not difficult to show that the electric field at part (a) and part (c) is zero. Part (b) and part (d) would be left to you as another exercise.

Answer: (b) $E = 79.9$ MN/C

          (d) $E = 7.34$ MN/C

Problem 7: 24.57

An infinitely long cylindrical insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ $\left(C/m^3\right)$. A line of charge density $\lambda\left(C/m\right)$ is placed along the axis of the shell. Determine the electric field intensity everywhere.

Solution:

By symmetry, the electric field everywhere is perpendicular to the surface of the cylindrical shell, and point from the centerline of charge to the outside. So we break this down into three cases, and build the gaussian surface to be a cylindrical face with its curved surface parallel to the cylindrical shell, radius r and length L.

- Case I: $0 < r < a$,

In this case, the charge enclosed would be just due to the line of charge. Therefore, applying Gauss's law, we have

$$E\left(2\pi rL\right) = 4\pi k_e\left(\lambda L\right) \tag{7.1}$$

which gives

$$E = \frac{2k_e\lambda}{r} \tag{7.2}$$

- Case II: $a < r < b$

Charge enclosed would include the one from the line of charge and the proportion of the charge from the cylindrical shell. The volume of the charge from the cylindrical shell that would be enclosed by the Gaussian surface is

$$V = \pi r^2 L - \pi a^2 L = \pi L\left(r^2 - a^2\right) \tag{7.3}$$

Therefore, applying the Gauss's Law,

$$E\left(2\pi rL\right) = 4\pi k_e\left(\lambda L + \rho\pi L\left(r^2 - a^2\right)\right) \tag{7.4}$$

which gives

$$E = \frac{2k_e\left(\lambda + \rho\pi\left(r^2 - a^2\right)\right)}{r} \tag{7.5}$$

- Case III: $r > b$

Similar to case II, we have

$$E(2\pi rL) = 4\pi k_e \left(\lambda L + \rho\pi L \left(b^2 - a^2 \right) \right) \tag{7.6}$$

which gives

$$E = \frac{2k_e \left(\lambda + \rho\pi \left(b^2 - a^2 \right) \right)}{r} \tag{7.7}$$

In conclusion,

$$E = \begin{cases} \dfrac{2k_e \lambda}{r} & , 0 < r < a \\[4mm] \dfrac{2k_e \left(\lambda + \rho\pi \left(r^2 - a^2 \right) \right)}{r} & , a < r < b \\[4mm] \dfrac{2k_e \left(\lambda + \rho\pi \left(b^2 - a^2 \right) \right)}{r} & , r > b \end{cases} \tag{7.8}$$

Problem 8: 24.58
Two infinite, non-conducting sheets of charge are parallel to each other, as shown in Figure P24.58. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the value of the electric field at points
   (a) to the left of,
   (b) in between, and
   (c) to the right of
the two sheets. (Hint: See Example 24.8)

Solution:

Each sheet would give an electric field of magnitude $\dfrac{\sigma}{2\varepsilon_0}$ to the whole space. (Why is it not distance related? Why is it the same for the whole space no matter how far it is from?) However, the positive sheet would give a field that is pointing away from the sheet while the negative sheet would produce a field which is pointing towards itself. Therefore, on the left of the positive and on the right of the negative, the two fields would just cancel out while the field in the middle of the sheets would sum up together. Therefore,
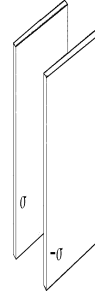
$$E = \begin{cases} 0 & \text{on the left} \\ \dfrac{\sigma}{2\varepsilon_0} + \dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0} & \text{between the two plates} \\ 0 & \text{on the right} \end{cases} \qquad (8.1)$$

$\mathbf{E}=0 \quad \mathbf{E}=\dfrac{\sigma}{\epsilon_0} \quad \mathbf{E}=0$