Disagreement and Information Collection

Eric Van den Steen*

April 10,2002

Abstract

This note shows that disagreement, in the sense of differing priors, may increase the incentives to collect information when two agents work on a joint project. The reason is that each agent believes that new data will confirm his own beliefs and thus 'convince' the other agents to do what the focal agent thinks is right.

1 Introduction

Disagreement is important, especially when people have to undertake a joint project. This paper models disagreement as differing priors about the right course of action or true state of the world. Assuming that a group of agents' payoff is influenced by each of their actions or decisions, it shows that heterogeneity in beliefs may increase their incentives to collect information.

The paper considers a situation where all collected information becomes immediately public. There are thus no issues of communication as in Milgrom (1981), Crawford and Sobel (1982), or Milgrom and Roberts (1986). Each agent's payoff is a function of how his own and his colleagues' actions fit the (unknown) optimal course of action. The conclusion is that heterogeneity in beliefs generally gives extra incentives to collect information. This is caused by the fact that each agent thinks that new information will confirm his own beliefs and disprove those of the people who disagree with him. This typically induces the latter to undertake actions that are more 'correct' from the focal agent's perspective. Collecting information has thus a 'convincing effect' over and above the regular 'uncertainty reduction (value of information)' effect. This convincing effect increases in the level of disagreement. Note that this effect is subjective in that it raises the expected utility of the focal agent using his own beliefs, but not necessarily from an outsider's perspective.

The general question of the value of information goes back at least to Blackwell (1951). More recent contributions include Athey and Levin (2001). Incentives for collecting information were also at the heart of the paper by Aghion and Tirole (1997) on delegation. The contribution of the current paper is the insight that the *subjective* value of information increases when agents undertake a joint project and have differing priors about the right course of action.

The next section sets out the basic model. Section 3 studies the incentives to collect information when such action and its outcomes are publicly observable, while section 4 concludes.

2 The Basic Model

Consider a situation with two agents, A and B, who undertake a joint project. In undertaking the project, each agent i must choose an action $x_i \in \mathbb{R}$. The payoff of their joint project depends

*MIT-Sloan School of Management (evds@mit.edu). I thank David Scharfstein for the very valuable input.

on how well each of the agents's actions fit with the true state of the world, x. Assume that the true state x is unknown, but both agents hold a subjective belief about the distribution of x. In particular, agent i believes that x is distributed according to some distribution F_i with differentiable density f_i. Let $f_i(x) = f_j(x + \overline{x}_j - \overline{x}_i)$, with $\overline{x}_i$ and $\overline{x}_j$ denoting the means of f_i and f_j respectively. This says that the agents have identical beliefs up to their mean. To simplify later notations, define f by $f_i(x) = f(x - \overline{x}_i)$. The F_i are common knowledge, so that agents have differing priors.

Let agent i's payoff from the project, when the actions are x_1 and x_2 and the true state is x, be

$$u_i = g_{i,1}(x - x_1) + g_{i,2}(x - x_2)$$

with $g_{i,j}$ symmetric around zero and strictly quasi-concave. This specification implies an agent's payoff increases as his or his partner's action corresponds more closely to the state of the world x. A special case is that where $g_{i,j} = g_{j,j}$ so that $u_i = u_j$. In that case, all agents get the same payoff, as in team theory.

Agent i's expected payoff is thus

$$E[u_i] = \int [g_{i,1}(y - x_1) + g_{i,2}(y - x_2)]\, f_i(y) dy$$

which, using notation $D(x - z) = \int g(y - x) f(y - z) dy$, becomes

$$E[u_i] = D_{i,1}(x_1 - \overline{x}_i) + D_{i,2}(x_2 - \overline{x}_i) \tag{1}$$

Let $\hat{x}_i$ denote i's optimal action when his utility is given by equation (1) and $\overline{x}_i = E_i[x]$ the expected value of x according to i. The following are two properties that will be useful in the upcoming analysis.

Lemma 1 *If f_i is symmetric and strictly quasi-concave, then $\hat{x}_i = \overline{x}_i$.*

Lemma 2 *$D(x - z)$ is symmetric around zero. If g is concave, then $D(x - z)$ is concave (in $x - z$).*

The proofs are in appendix. Combining the lemmas implies that at the optimum $E[u_1] = D_{1,1}(0) + D_{1,2}(\overline{x}_1 - \overline{x}_2)$.

3 Public Information Collection

Assume that agent 1 has the opportunity to collect a piece of information at a cost c, with $c > 0$. The information consists of an observation v with distribution $x + \epsilon$ with ϵ being distributed according to h.

Assumption 1 *Let $E_i[x \mid v = \hat{v}]$ be differentiable in $\hat{v}$ with $0 < \frac{dE_i[x|v=\hat{v}]}{d\hat{v}} < 1$.*

Note that the expectation will be weakly increasing in $\hat{v}$ when $h(v \mid x)$ satisfies MLRP, since MLRP implies that v and x are associated, so that, by Milgrom and Weber, $E[x \mid v = \hat{v}]$ increases in $\hat{v}$. Assumption 1 is thus a bit stronger than MLRP. It will be satisfied, among others, when the distributions are normal.

If the agents had common priors, i's expected payoff would be $2D_{1,1}(0)$. Write therefore

$$E[u_1] = 2D_{1,1}(0) - (D_{1,1}(0) - D_{1,1}(\overline{x}_1 - \overline{x}_2))$$

Let $f_1(\cdot \mid v)$ denote the belief of agent 1 after observing v and

$$\tilde{D}_{1,1}(x - z) = \int g_{1,1}(y - x) f_1(y - z \mid v) dy$$

Let furthermore $\overline{x}_{1|v}$ and $\overline{x}_{2|v}$ denote the means of $f_1(\cdot \mid v)$ and $f_2(\cdot \mid v)$.

Lemma 3 *According to agent 1,* $[D_{1,1}(0) - D_{1,1}(\overline{x}_1 - \overline{x}_2)] - E_v\left[\tilde{D}_{1,1}(0) - \tilde{D}_{1,1}(\overline{x}_{1|v} - \overline{x}_{2|v})\right] > 0$ *and increases in* $|\overline{x}_2 - \overline{x}_1|$ *when* g *is concave.*

Proof : Note that $E_v[f_1(\hat{u} \mid v)] = f_1(\hat{u})$. It thus follows that

$$\begin{aligned} E_v[\tilde{D}_{1,1}(x-z)] &= E_v\left[\int g_{1,1}(y-x) f_1(y-z \mid v) dy\right] \\ &= \int g_{1,1}(y-x) E_v\left[f_1(y-z \mid v)\right] dy \\ &= \int g_{1,1}(y-x) f_1(y-z) dy \\ &= D_{1,1}(x-z) \end{aligned}$$

or

$$D_{1,1}(0) - D_{1,1}(\overline{x}_1 - \overline{x}_2) = E_v[\tilde{D}_{1,1}(0) - \tilde{D}_{1,1}(\overline{x}_1 - \overline{x}_2)]$$

So now we're left to prove that

$$E_v[\tilde{D}_{1,1}(0) - \tilde{D}_{1,1}(\overline{x}_1 - \overline{x}_2)] > E_v[\tilde{D}_{1,1}(0) - \tilde{D}_{1,1}(\overline{x}_{1|v} - \overline{x}_{2|v})]$$

or

$$E_v[\tilde{D}_{1,1}(\overline{x}_1 - \overline{x}_2)] < E_v[\tilde{D}_{1,1}(\overline{x}_{1|v} - \overline{x}_{2|v})]$$

for which it is sufficient that for any realization of v, $|\overline{x}_2 - \overline{x}_1| > \left|\overline{x}_{2|v} - \overline{x}_{1|v}\right|$.
Note now that, for $\overline{x}_2 > \overline{x}_1$,

$$\begin{aligned} \overline{x}_{1|v} &= E_1[x \mid v] = E_2[x \mid v + (\overline{x}_2 - \overline{x}_1)] - (\overline{x}_2 - \overline{x}_1) \\ &> E_2[x \mid v] - (\overline{x}_2 - \overline{x}_1) = \overline{x}_{2|v} - (\overline{x}_2 - \overline{x}_1) \end{aligned}$$

(with the inequality running the other way when $\overline{x}_1 > \overline{x}_2$), which proves the first part of the proposition.
For the second part of the proposition, it suffices to show that $\tilde{D}_{1,1}(\overline{x}_{1|v} - \overline{x}_{2|v}) - \tilde{D}_{1,1}(\overline{x}_1 - \overline{x}_2)$ increases in $|\overline{x}_2 - \overline{x}_1|$. Assume again $\overline{x}_1 < \overline{x}_2$. Fix $\overline{x}_1$, let $\overline{x}_2$ increase to $\overline{x}_{2'} = \overline{x}_2 + \delta$, and denote the updated value by $\overline{x}_{2'|v}$. Since $\overline{x}_1$ and thus $\overline{x}_{1|v}$ remain unchanged and since g and thus D are concave, we just have to show that $\overline{x}_{2'} - \overline{x}_2 > \overline{x}_{2'|v} - \overline{x}_{2|v}$. Now

$$\begin{aligned} \overline{x}_{2'|v} &= \int x f(x - \overline{x}_{2'} \mid v)\, dx = \int x f(x - \overline{x}_2 \mid v + \delta)\, dx \\ &< \delta + \int x f(x - \overline{x}_2 \mid v)\, dx = \delta + \overline{x}_{2'} \end{aligned}$$

where the inequality follows from assumption 1. ■

Proposition 1 *From agent 1's perspective, the expected benefit from an extra piece of information*

- *is larger when* $\overline{x}_1 \neq \overline{x}_2$ *than when they are equal.*
- *increases in* $|\overline{x}_1 - \overline{x}_2|$ *when* g *is concave.*

Proof : The expected benefit is

$$\begin{aligned} &2D_{1,1}(0) - (D_{1,1}(0) - D_{1,1}(\overline{x}_1 - \overline{x}_2)) \\ &- \left[2\tilde{D}_{1,1}(0) - \left(\tilde{D}_{1,1}(0) - \tilde{D}_{1,1}(\overline{x}_1 - \overline{x}_2)\right)\right] \end{aligned}$$

Since $D_{1,1}$ and $\tilde{D}_{1,1}$ do not depend directly on the $\overline{x}$'s, it is sufficient to look at

$$D_{1,1}(\overline{x}_1 - \overline{x}_2) - \tilde{D}_{1,1}(\overline{x}_1 - \overline{x}_2)$$

but then the proposition follows directly from the above lemma.

■

Monotone comparative statics then imply the following corollary.

Corollary 1 *The amount of information collected is larger when there is disagreement than when there is none. Moreover, when g is concave then the amount of information increases in the level of disagreement.*

Proof : This follows from applying monotone comparative statics on the above at each step of the information collection process. ■

This result essentially confirms the intuition that disagreement gives people incentives to collect information.

4 Conclusion

This paper showed how differing beliefs about the right course of action may increase the incentives to collect information. The reason is that new information not only reduces uncertainty but also allows an agent to 'convince' his colleagues.

A more complete study should first consider how this generalizes to other payoff functions and then show that the result does not hold when agents just have different information or preferences.

A Proofs

Lemma 1 *If f_i is symmetric and strictly quasi-concave, then $\hat{x}_i = \overline{x}_i$.*

Proof : Let us prove the lemma for, say, agent 1. Agent 1 chooses x_1 to maximize

$$E[u_1] = \int g_{1,1}(y - x_1) f(y - \overline{x}_1) dy + \int g_{1,2}(y - x_2) f(y - \overline{x}_1) dy$$

Since the second term is independent of x_1, it suffices to maximize $\int g_{1,1}(y - x_1) f(y - \overline{x}_1) dy$. A change of variable allows to rewrite this as $\int g_{1,1}(u) f(u + x_1 - \overline{x}_1) du$. The first derivative is

$$\begin{aligned} \frac{dE[u_1]}{dx_1} &= \int g_{1,1}(u) f'(u + x_1 - \overline{x}_1) du \\ &= \int_{-\infty}^{0} \left(g_{1,1}(v + \overline{x}_1 - x_1) - g_{1,1}(-v + \overline{x}_1 - x_1) \right) f'(v) dv \end{aligned}$$

When $\overline{x}_1 - x_1 > 0$ then (since $v < 0$)

$$\begin{aligned} & g_{1,1}(v + \overline{x}_1 - x_1) - g_{1,1}(-v + \overline{x}_1 - x_1) \\ & \quad > \; g_{1,1}(v - (\overline{x}_1 - x_1)) - g_{1,1}(-v + \overline{x}_1 - x_1) = 0 \end{aligned}$$

while with $\overline{x}_1 - x_1 < 0$

$$\begin{aligned} & g_{1,1}(v + \overline{x}_1 - x_1) - g_{1,1}(-v + \overline{x}_1 - x_1) \\ & \quad < \; g_{1,1}(v - (\overline{x}_1 - x_1)) - g_{1,1}(-v + \overline{x}_1 - x_1) = 0 \end{aligned}$$

It thus follows (since $f' > 0$ for $v < 0$) that the derivative is positive when $x_1 < \overline{x}_1$, negative when $x_1 > \overline{x}_1$, and zero when $x_1 = \overline{x}_1$. It follows that $x_1 = \overline{x}_1$ is the unique optimum. ■

Let now $D(x - z) = \int g(y - x) f(y - z) dy$.

Lemma 2 *$D(x - z)$ is symmetric around zero. If g is concave, then $D(x - z)$ is concave (in $x - z$).*

Proof : For the first part,

$$\begin{aligned} D(x - z) &= \int g(y - x) f(y - z) dy \\ &= \int g(x - y) f(y - z) dy \\ &= \int g(u - z) f(x - u)(-du) \\ &= \int g(u - z) f(u - x) du \\ &= D(z - x) \end{aligned}$$

where we did a change of variable $x - y = u - z$.
Note further that $D(x - z) = \int g(y - x) f(y - z) dy = \int g(u - (x - z)) f(u) du$ so that we can write $D(\alpha) = \int g(u - \alpha) f(u) du$ and $D''(\alpha) = \int g''(u - \alpha) f(u) du$. If g is concave, then so is D. ■

References

AGHION, P., AND J. TIROLE (1997): "Formal and real authority in organizations," *Journal of Political Economy*, 105(1), 1– 29.

ATHEY, S., AND J. LEVIN (2001): "The Value of Information in Monotone Decision Problems," Working Paper, MIT and Stanford University.

BLACKWELL, D. (1951): "Comparisons of Experiments," *Proceedings of the Second Berkeley Symposium on Mathematical Statistics*, pp. 93– 102.

CRAWFORD, V., AND J. SOBEL (1982): "Strategic Information Transmission," *Econometrica*, 50, 1431– 1452.

MILGROM, P. (1981): "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12, 380– 391.

MILGROM, P., AND J. ROBERTS (1986): "Relying on the Information of Interested Parties," *Rand Journal of Economics*, 17(1), 18– 32.