THE MECHANICAL SEAL

by

James Kerr Roberge

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
BACHELOR OF SCIENCE

at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
May, 1960

Signature of Author.....James K. Roberge.........
Department of Electrical Engineering

Certified by.....Leonard A. Gould.........
Thesis Supervisor

Accepted by..
Chairman, Departmental Senior Thesis Committee

ACKNOWLEDGMENTS

I wish to express my most sincere gratitude to my thesis adviser, Professor Leonard A. Gould. Professor Gould was responsible for the original suggestion which started me thinking about a broom balancer as well as guidance throughout the course of the work. Without his excellent technical advice and encouragement the device could not have been completed.

I would also like to extend my thanks to Mr. Walter Boynton of the instrument room in the Electronic Systems Laboratory. In work of this type a large quantity of equipment is necessary. Mr. Boynton always seemed able to locate any required component in a minimum amount of time. On numerous occasions he allowed me the use of his own personal equipment.

J.K.R.

CONTENTS

ABSTRACT

A servomechanism has been designed and built which balances a broom. This thesis presents the analytic techniques used to determine the compensation needed to maintain a broom in an inverted position. The construction of the components used to realize the indicated compensation is also described.

I. INTRODUCTION

The problem of stabilization of a set of fixed elements which are dynamically unstable is an inherently interesting one to the servo designer. This condition is not commonly met with in practice. Usually any instabilities in the feedback control system are a direct consequence of feedback. This type of instability can often be eliminated by reducing loop gain if the resulting decrease in performance can be tolerated.

An excellent example of an inherently unstable device is the inverted pendulum or broom. This system has a transfer function with a pair of poles on the positive and negative real axis. If a servomechanism is used to maintain a broom in an inverted position, any closed loop instabilities cannot be corrected simply by reducing loop gain since, if the gain is decreased below a certain limit, the broom falls over.

A device has been designed and constructed which accomplishes the task of maintaining a broom in a vertical position when constrained to fall in one plane only. This control mechanism is currently in operation and has been demonstrated at the recent M.I.T. Parents' Weekend. It is capable of sustained operation for long periods of time. It generally functions correctly until someone (usually the designer) attempts to demonstrate the ability of the mechanism to correct for large disturbances and pushes the broom too far off its null position.

In its present version, there seems to be little practical application for a device of this type. However, the equations apply almost directly to a rocket at take-off. The rocket at low speeds behaves exactly like an inverted pendulum.

The basic method of stabilization outlined here should have modifications which apply to a wide variety of control systems in which fixed elements have one or more poles in the right-half plane.

II. PRELIMINARY INVESTIGATION OF STABILITY

Figure 2.1 shows the idealized physical situation from which the equations of motion of the broom can be determined. The broom is assumed to be pivoted at the base on frictionless bearings, such that it can fall only in the plane of the paper. The supporting rod is assumed to be massless and inflexible. X_1 is the variable which can be controlled to maintain stability. The only information available is the magnitude of the angle θ.

Summing forces at the broom head,

$$\text{Force} = M\ddot{X}_2 = Mg \sin \theta - B\dot{X}_2 \qquad (2.1)$$

$$\sin \theta \approx \theta = \frac{X_2 - X_1}{L} \qquad (2.2)$$

From this point on in the preliminary analysis, the damping, B, will be assumed to be zero. Later in the development it will be included primarily as a method of limiting the maximum required values of $\dot{X}_1$ and $\ddot{X}_1$. Its

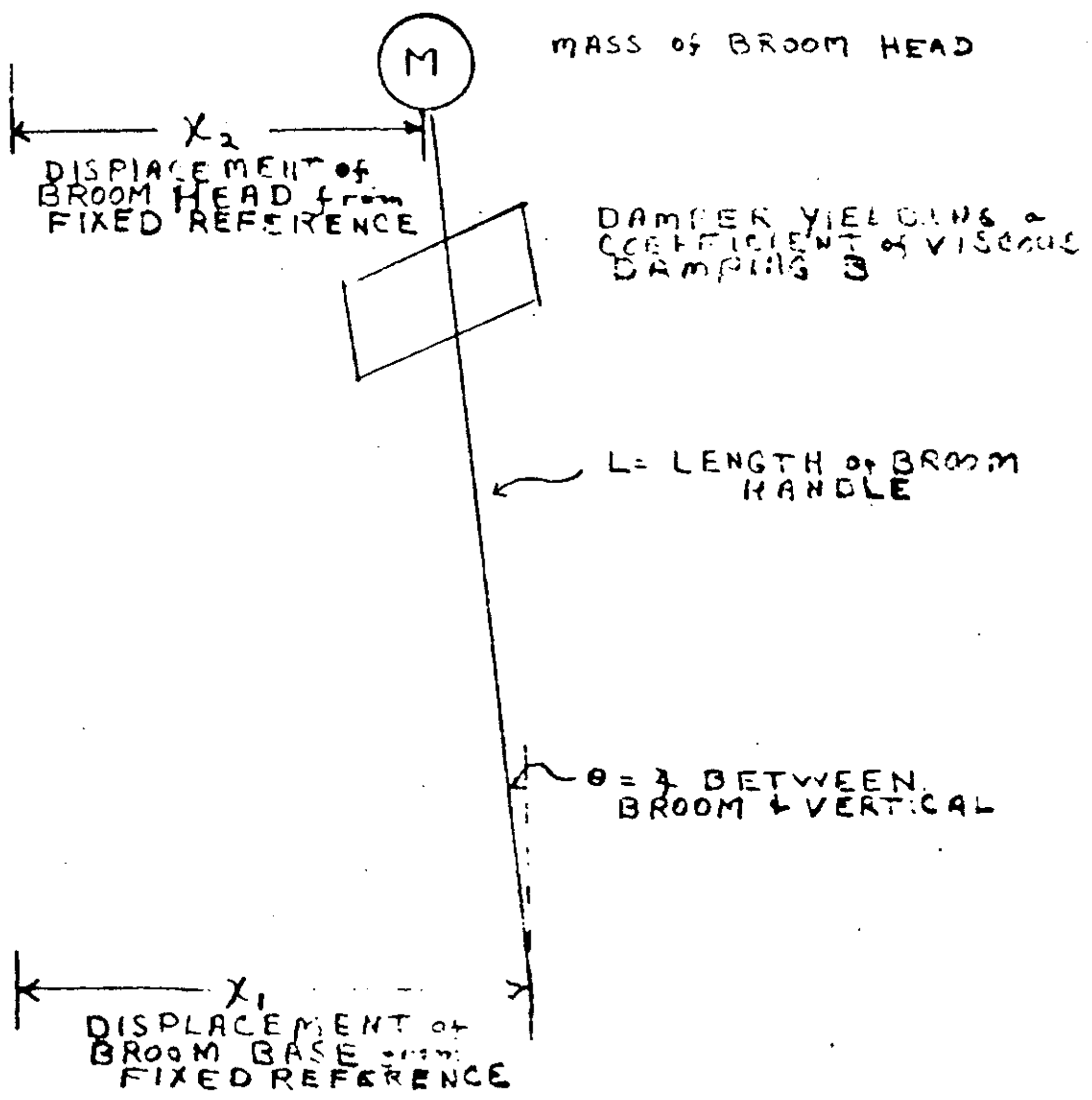

FIGURE 2.1 BROOM DYNAMICS

inclusion does not seriously alter the stability considerations and thus for the present it can be ignored without drastically changing the form of compensation necessary for stability.

The equations of motion can be used to form the block diagram shown in Figure 2.2. $G_c(s)$ is the compensation required to yield a stabilizing X_1 as a function of θ.

To eliminate the necessity of handling a multiloop system, the compensation loop is opened at the indicated point and the loop containing the broom dynamics is collapsed. The broom loop can be replaced by a block having the transfer function

$$\frac{1/L}{1 - g/s^2 L} = \frac{s^2/g}{(as+1)(as-1)}, \quad a=(L/g)^{1/2} \tag{2.3}$$

As an attempt at stabilization, let $G_c(s) = K/s^2$. This produces an open-loop transfer function

$$\frac{K}{g}\,\frac{1}{(as+1)(as-1)} \tag{2.4}$$

The corresponding closed-loop transfer function is

$$\frac{\frac{K}{g}\,\frac{1}{(as+1)(as-1)}}{1 + \frac{K}{g}\,\frac{1}{(as+1)(as-1)}} = \frac{K}{g(as+1)(as-1) + K} = \frac{K}{ga^2s^2 - g + K} \tag{2.5}$$

From Routh's stability criterion we notice that this form of compensation has removed the pole from the right-half plane providing $K > g$, but has left a pair of poles on the imaginary axis. Notice that the term $1/s^2$ must be a true double integration rather than of the form $b^2/(bs+1)^2$.

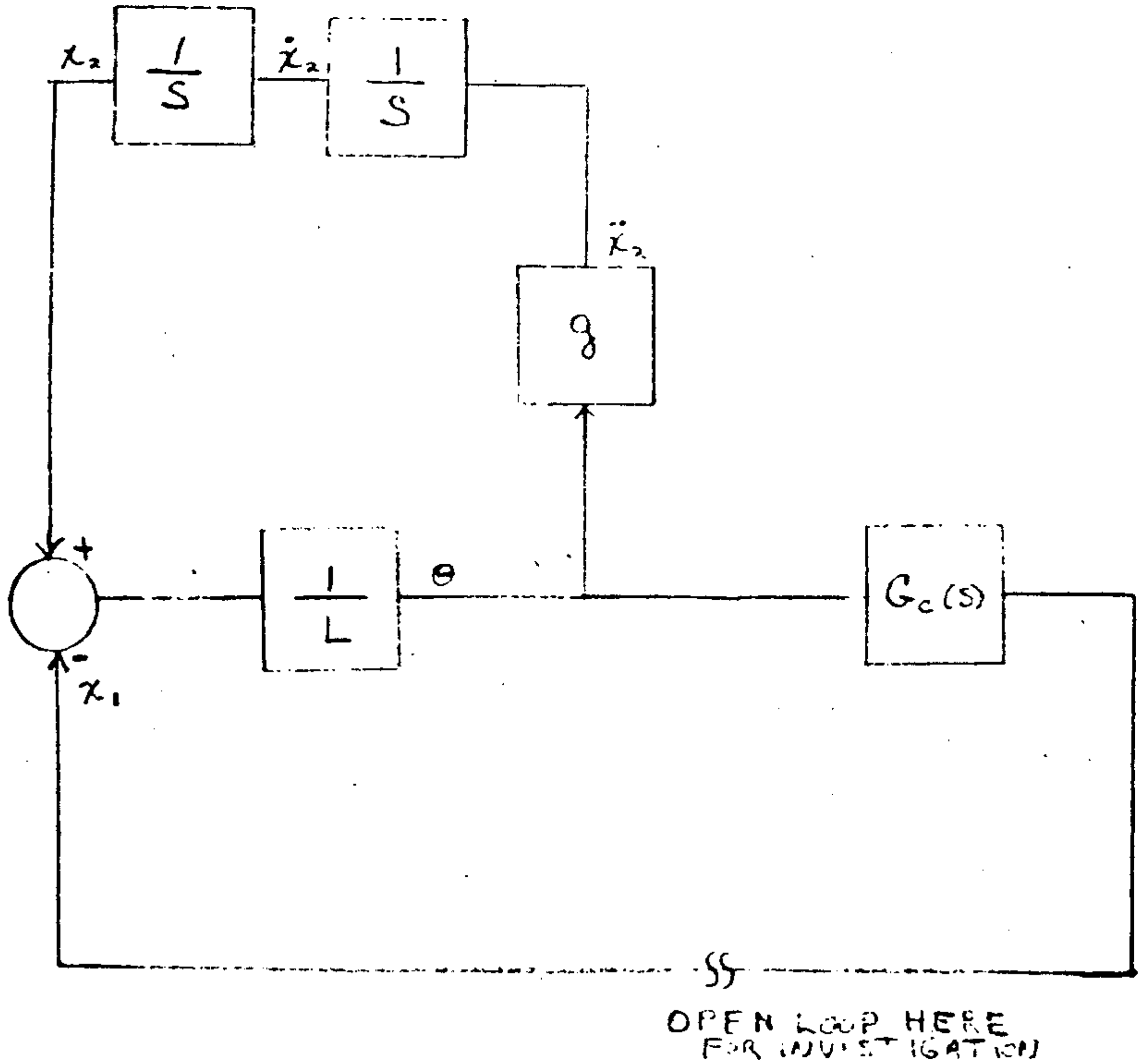

FIGURE 2.2 PRELIMINARY BLOCK DIAGRAM

If we assume $1/s^2$ to be approximated by $b^2/(bs+1)^2$, the resulting open-loop transfer function is

$$\frac{K}{g}\,\frac{s^2b^2}{(as+1)(as-1)(bs+1)^2} \qquad (2.6)$$

The corresponding characteristic equation of the closed-loop system is

$$g(as+1)(as-1)(bs+1)^2 + Kb^2s^2 =$$
$$g((a^2b^2s^4+2a^2bs^3+(a^2-b^2)s^2-2bs-1)) + Kb^2s^2 = 0 \qquad (2.7)$$

We notice that there is no way to eliminate the negative coefficient of s^1 and s^0 in this equation. Thus at least one pole remains in the right-half plane. A similar result is obtained if an approximation to $1/s^2$ is made by $b/s(bs+1)$.

It is concluded that, if damping in the broom is neglected, at least a true double integration is necessary. The only practical way to obtain this seems to be the use of electro-mechanical integrators such as motor-tach units. It is assumed that such integrators will have some lag associated with them. Therefore, a reasonable expression for the double integration would be $1/s^2(cs+1)^2$.

To offset the lag associated with the integrators, and to eliminate the oscillation which would be present if $G_c(s)$ were K/s^2, a lead network is employed. The resultant $G_c(s)$ is of the form

$$\frac{K}{s^2}\,\frac{1}{(cs+1)^2}\,\frac{(\alpha ds+1)}{(ds+1)}\ , \alpha > 1 \qquad (2.8)$$

When the dynamics of the broom are included, the overall open-loop transfer expression becomes:

$$\frac{K}{g}\,\frac{1}{(as+1)(as-1)}\;\frac{1}{(cs+1)^2}\;\frac{\alpha ds+1}{ds+1} \qquad (2.9)$$

The term $1/(as+1)(as-1)$ is a non-minimum phase expression. It has magnitude characteristics that are the same as the magnitude of $1/(as+1)^2$ but has a phase shift of -180° at all frequencies.

A Nyquist plot of Equation (2.9) is shown in Figure 2.3. It is assumed in this plot that the integrator time constants, c, can be made shorter than a by proper choice of integrators and the length of the broom. It is also assumed that α and d can be found such that over some range of frequencies the net phase of the expression is greater than -180°.

We see from the Nyquist plot that stability can be attained. From the plot, there is -1 net encirclement of the -g/K point. Since there is one open-loop pole in the right-half plane the system is stable. The diagram further shows that it is possible to use a Bode plot for the determination of gain and time constants of the lead network providing the D.C. gain of the expression is made greater than 0 dg.

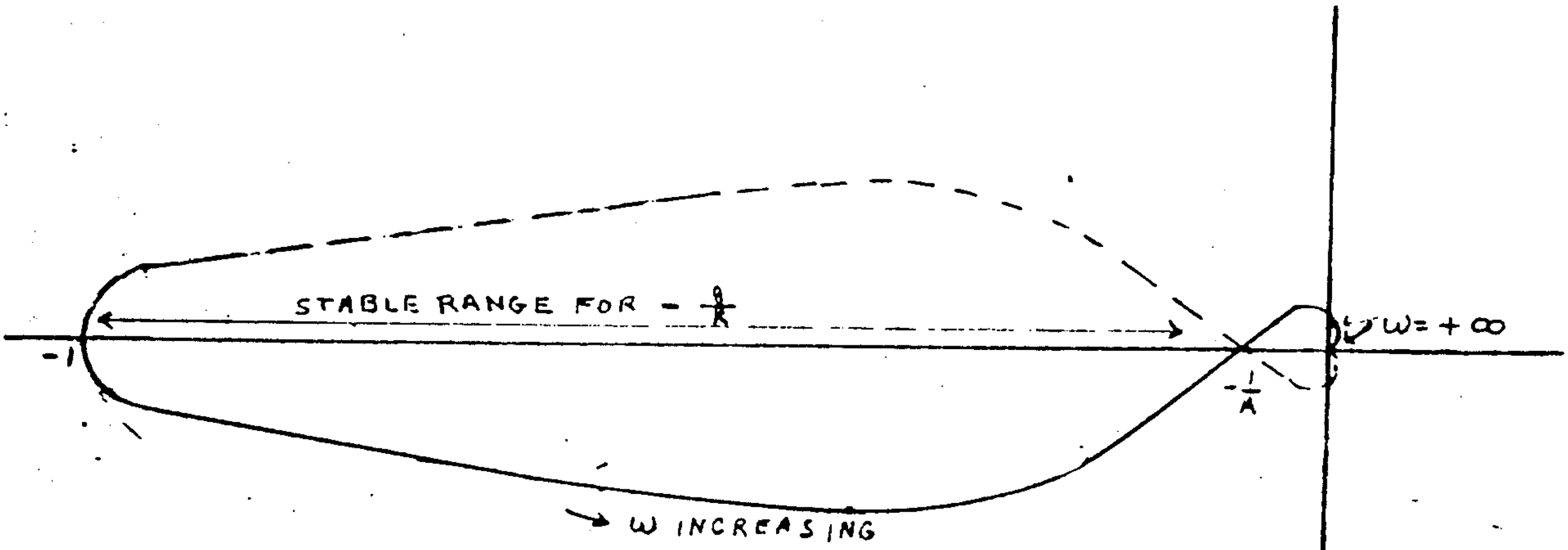

FIGURE 2.3 NYQUIST PLOT OF EQUATION (2.9)

III. DESCRIPTION OF COMPONENTS

Once the basic type of compensation necessary to stabilize a broom was determined, the individual components of the loop were assembled. the data obtained from measurements on these elements enabled selection of the final compensation.

The platform used for movement of the handle of the broom was originally a Z(X,Y) generator constructed by the M.I.T. Dynamic Analysis And Control Laboratory. It permits movement of a small output fixture along two perpendicular axes. The drive is from two-phase servomotors via gearpasses, pulleys, and steel belts. At the start of the work, it was intended to give the broom freedom to fall in any direction. This would have made use of both axes of the Z(X,Y) generator. However, construction of the second axis was not carried out due to lack of time. Accordingly, only the x axis of the generator is being used. The only work done on the generator was mechanical maintenance and modification of the input adapter on the x axis to accept a new servomotor. The generator had been out of use for a long period of time prior to the start of this project and some of the components were badly rusted. Various adjustments such as gearpass backlash and belt tension were incorrect. When these conditions were corrected, the table functioned excellently. The x axis was originally driven by a Bendix servomotor. A Diehl unit of higher output power was substituted to permit higher acceleration. The maximum travel in the X direction on the table is .55 meters. The gear ratio is such as to limit the maximum slewing speed to about .45 meters/second.

The "broom" was constructed as follows. A piece of steel conduit 1.55 meters long was cut. A mass was connected to one end. A hole was drilled through the other end to accept a shaft. The shaft is held in place with gears pinned to the shaft and bolted to the conduit. The ends of the shaft are supported on ball bearings. This arrangement insures that the broom falls in one plane only. Since the conduit is not massless, the effective length and mass of the broom were determined by measurement. A natural frequency of 2.65 radians/second was observed. This corresponds to an effective length of 1.4 meters. The mass was measured as 1.6 kilograms. The angle between the broom and vertical is measured by a Clifton Precision Products type CT-11-B-2 synchro. This is driven from the shaft at the base of the broom through a seven:one stepup gearpass. An attempt to reduce backlash in this drive was made by employing a rubber band to keep the small gear always wound up against the large gear in the same direction. Figure 3.1 shows the construction of the support and synchro assembly.

The synchro is intended for operation at 400 cps with 26 volts on the rotor. For system considerations it is necessary to operate the synchro at 60 cps. To ac-

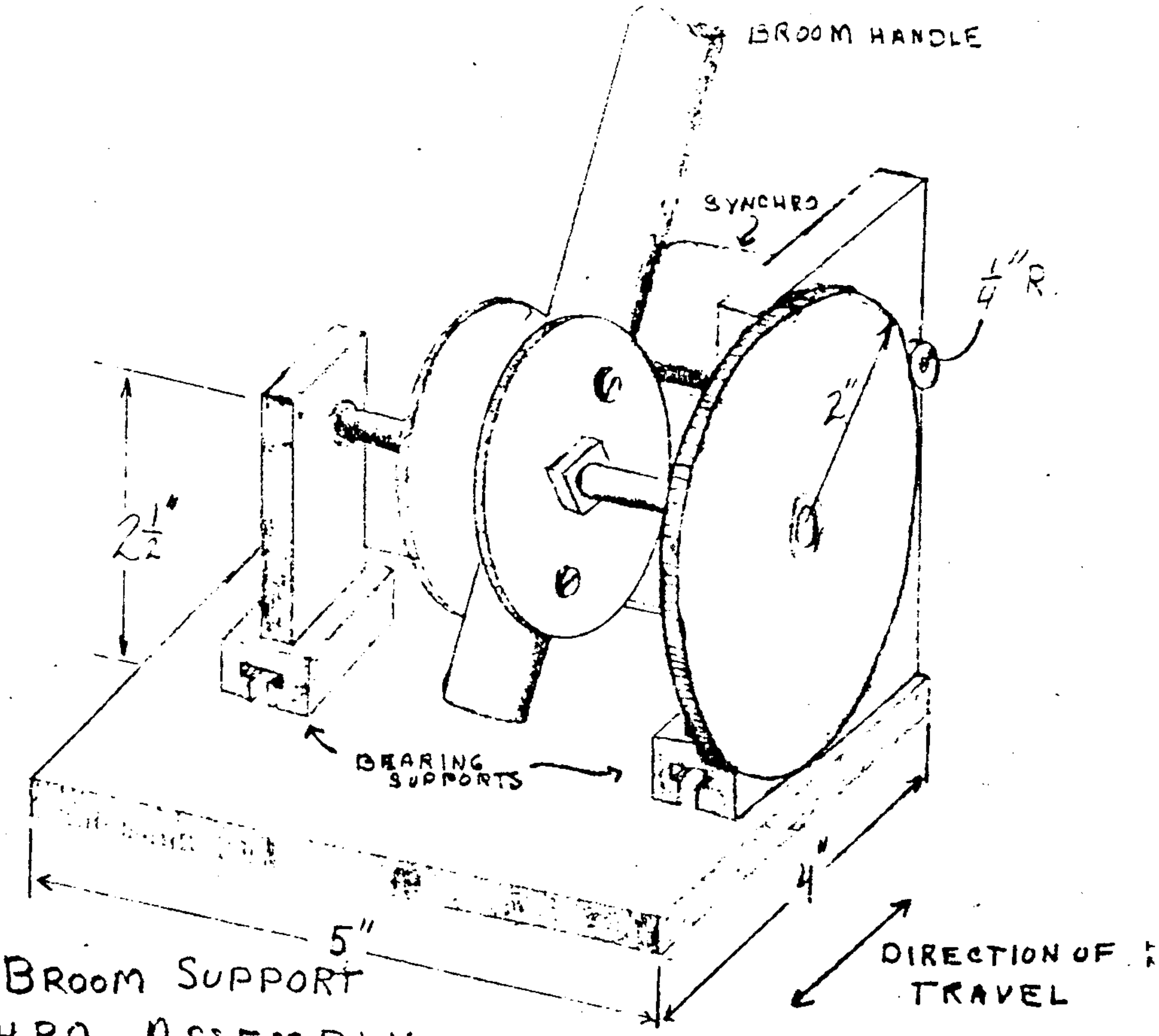

FIGURE 3.1 BROOM SUPPORT AND SYNCHRO ASSEMBLY

complish this, the rotor voltage was decreased to 6.5 volts which is obtained from the line through a Variac driving an 8 volt transformer. The maximum output of the synchro is 2.3 volts rms. Thus, with the gearpass, the sensitivity is 16 volts/radian for regions where the small angle approximation is valid (2^o-3^o at the broom).

D.C. power for the electronics in the system is obtained from two regulated power supplies, one delivering 300 volts at 250 ma. , the other 750 volts at 250 ma. . 90 volts D.C. is obtained from batteries for fixed bias on the output amplifier and as a supply for a transistor amplifier serving as an active compensation network.

The first integrator (Figure 3.2) consists of a 5 watt Diehl servomotor driving a 20K pot through a 10:1 gear reduction. Tachometric feedback is applied around the motor to improve the frequency response of the integrator. The fixed phase of the motor is connected to the line: phase shift for the control signal is obtained by the network shown as part of Figure 3.1. This network provides more than 90° of lag at 60 cps. However, the voltage at the control terminals of the motor is only about 80° lagging with respect to the line, indicating some positive phase shift through the amplifier. The amplifier is a 15 watt servo amplifier obtained from the instrument room in the Electronic Systems Laboratory. The gain from the tachometer output to the control winding of the motor is set at 15.

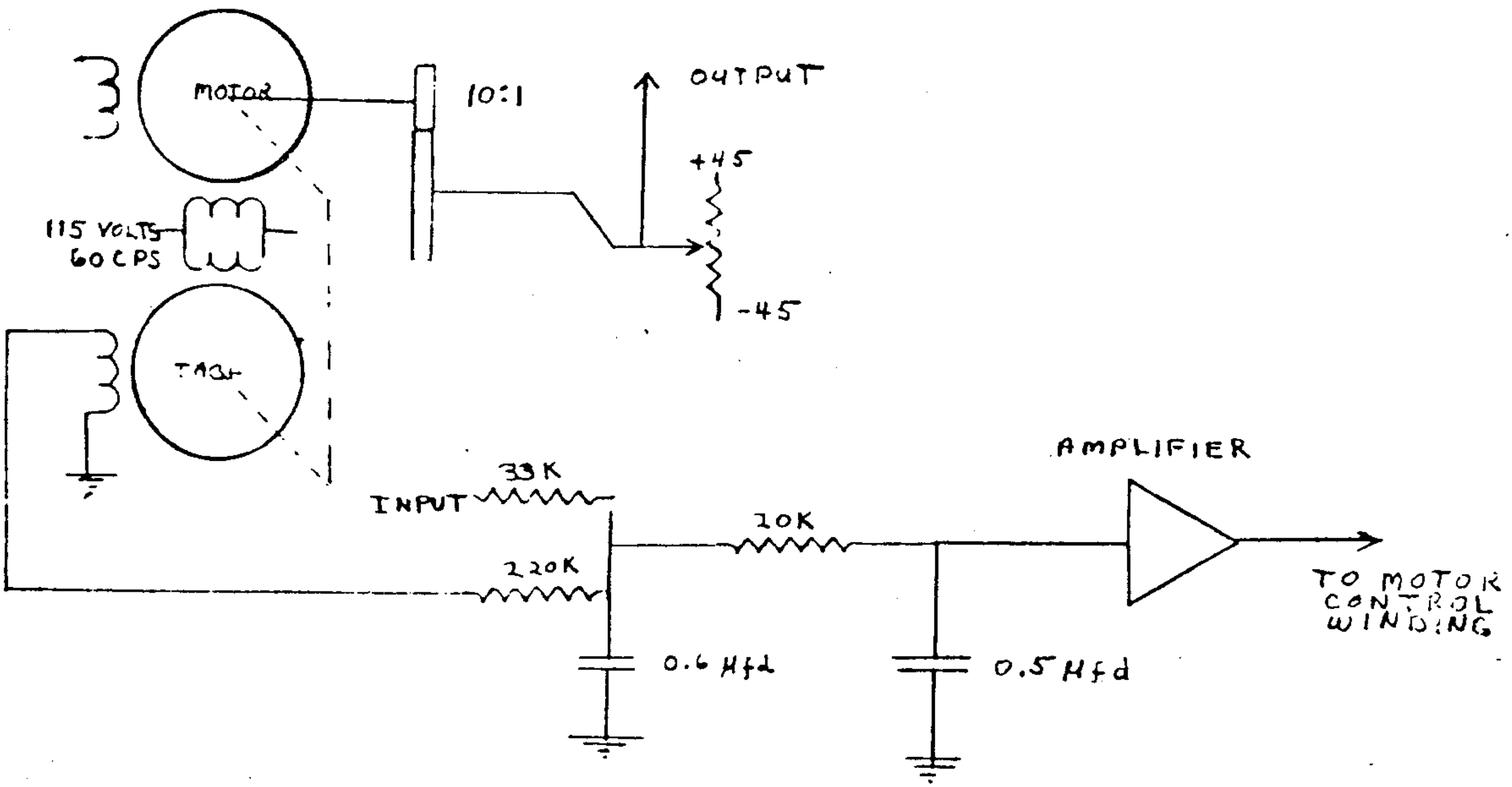

FIGURE 3.2 FIRST INTEGRATOR

From measurements made on the first integrator, the gain with 90 volts across the pot is 140 volts/second D.C. output per volt rms in. The log magnitude plot falls at 10 dg/decade until 50 radians/second where the slope becomes -20 dg/decade. The transfer function for the first integrator is then 140/s(.02s+1). Since the carrier frequency for this system is 60 cps, a bandwidth of 50 radians/second seemed reasonable and no attempt was made to improve it.

The first integrator produces a D.C. output proportional to the integral with respect to time of the envelope of a 60 cps input signal from the synchro. Because the output is D.C., this seemed a good point in the loop to insert compensation. From considerations which will be discussed in more detail later on compensation of the form K(.2s+1)/(.015s+1) was desired. The D.C. attenuation inherent in a purely passive network could not be tolerated so a transisterized active network was designed. The drift problem is not too severe in the design of this circuit since it is preceded by an integrator. Any shift in the D.C. output can be compensated for by a shift of the output of the first integrator and will not be detrimental to closed-loop operation, assuming the drift is not too rapid.

The schematic for the network is shown in Figure 3.3. The operation is as follows. A signal from the first integrator is attenuated by the 100K pot. This serves as

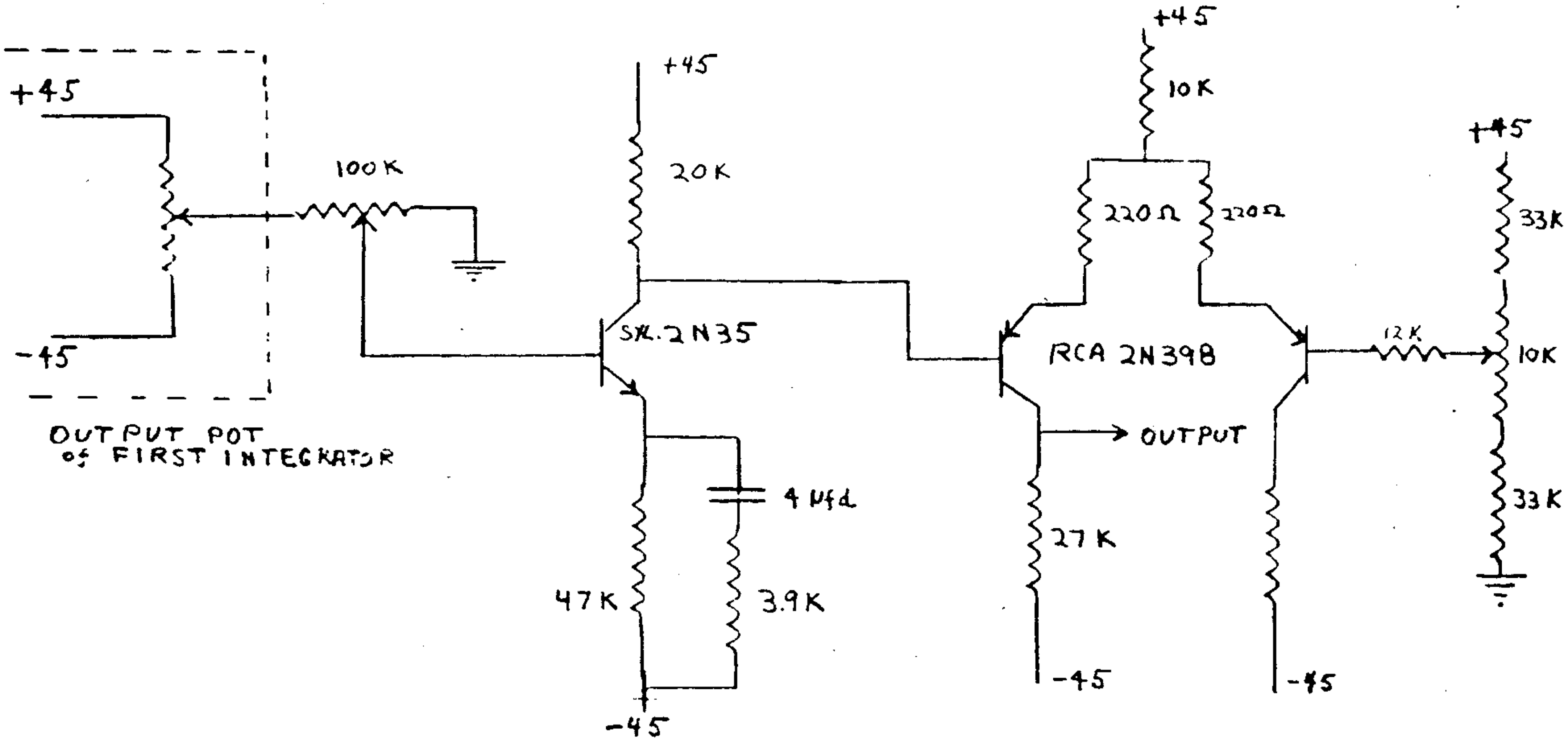

FIGURE 3.3 ACTIVE NETWORK

a gain control for the network and subsequently for the entire loop. Because the 2N35 has a β of 75, the input impedance is greater than 250K for any frequency and thus the gain pot is not seriously loaded. The first stage behaves basically as an emitter follower to the emitter of the 2N35. Thus on an incremental basis the gain to the output of the first stage is $-\alpha Z_c/Z_e$ where Z_c is the collector load and Z_e is the emitter load. Substituting values this becomes

$$\frac{20x10^3(47x10^3+3.9x10^3+1/4x10^{-6}s)}{47x10^3(3.9x10^3+1/4x10^{-6}s)} = \qquad (3.1)$$

$$.4\,\frac{(.204s+1)}{(.016s+1)}$$

In the first stage the lead is inserted with a D.C. attenuation of only 2.5, rather than an attenuation of 15 which would be associated with a passive network. The D.C. operating point of this stage is about +24 volts with no signal in providing a safe margin on the V_{ce} of the 2N35. Because the gain of this stage depends on α rather than β, drift and variations in gain are minimized.

The output stage consists of a pair of 2N398s connected as a difference amplifier. A fairly large dynamic output swing was required. Therefore a transistor with a high maximum V_{ce} was necessary. The 2N398 is normally intended for use as a high voltage switch and has a V_{ce} of -105 volts. Except for this, the transistor is not too good a choice. The two units were checked on a curve tracer. One β was 50; the othwr 30- as opposed to a design center of 60. Fortunately β was not too strongly temperature dependent. I_{co} was quite high and was strongly temperature dependent. However, some of this effect will cancel out due to the use of the transistors as a difference amplifier. In spite of these limitations the 2N398s were used because they are inexpensive.

If the two halves of the difference amplifier are assumed identical and r_e and r_b for the transistors are neglected the gain of this stage will be

$$\frac{1}{2}\,\frac{Z_i}{Z_o+Z_i}\quad\frac{27}{.220} \tag{3.2}$$

where Z_o is the output impedance of the previous stage and Z_i is the input impedance of the difference amplifier = β(220) ohms. Substituting values the gain of the difference amplifier becomes 15. The 10K pot is used for balance of the output. The overall transfer function of this amplifier becomes $K_a(.204s+1)/(.016s+1)$, where K_a is adjustable between 0 and 6 depending on the gain pot setting. The output impedance of the unit is 27K but this is not a hindrance since the output integrator which this network drives has an input impedance of about 100K. A collector load of 27K had to be used to limit the power dissipation in the transistors.

The circuit was assembled and tested. The maximum gain was near 5 and the frequency dependence was as predicted. Maximum swing at the output was ± 20 volts, sufficient to drive the following unit. Drift was on the order of 2 volts/minute after warmup which can easily be absorbed by the preceding integrator.

To complete the major loop, an output integrator was designed and constructed. (Figure 3.4). The alternative to this arrangement would be a separate second integrator driving a positional servo. The velocity servo was chosen because of the better frequency response obtainable without compensation and to reduce the complexity of the system.

The operation of the output integrator is as follows. The D.C. input signal from the preceding network is chopped to 60 cps by a Brown Converter. This unit is normally operated as a full-wave chopper; however adjustments were made to permit spst operation with 180° of dwell. The feedback signal from the tachometer is filtered and phase shifted by a RC network. A 15K pot is used to adjust the relative phases of the chopped input and tachometer signals. In this integrator as in the previous one the fixed windings of the motor and tach are supplied from the line and the control signal is phase shifted before amplification. One advantage of this method of obtaining

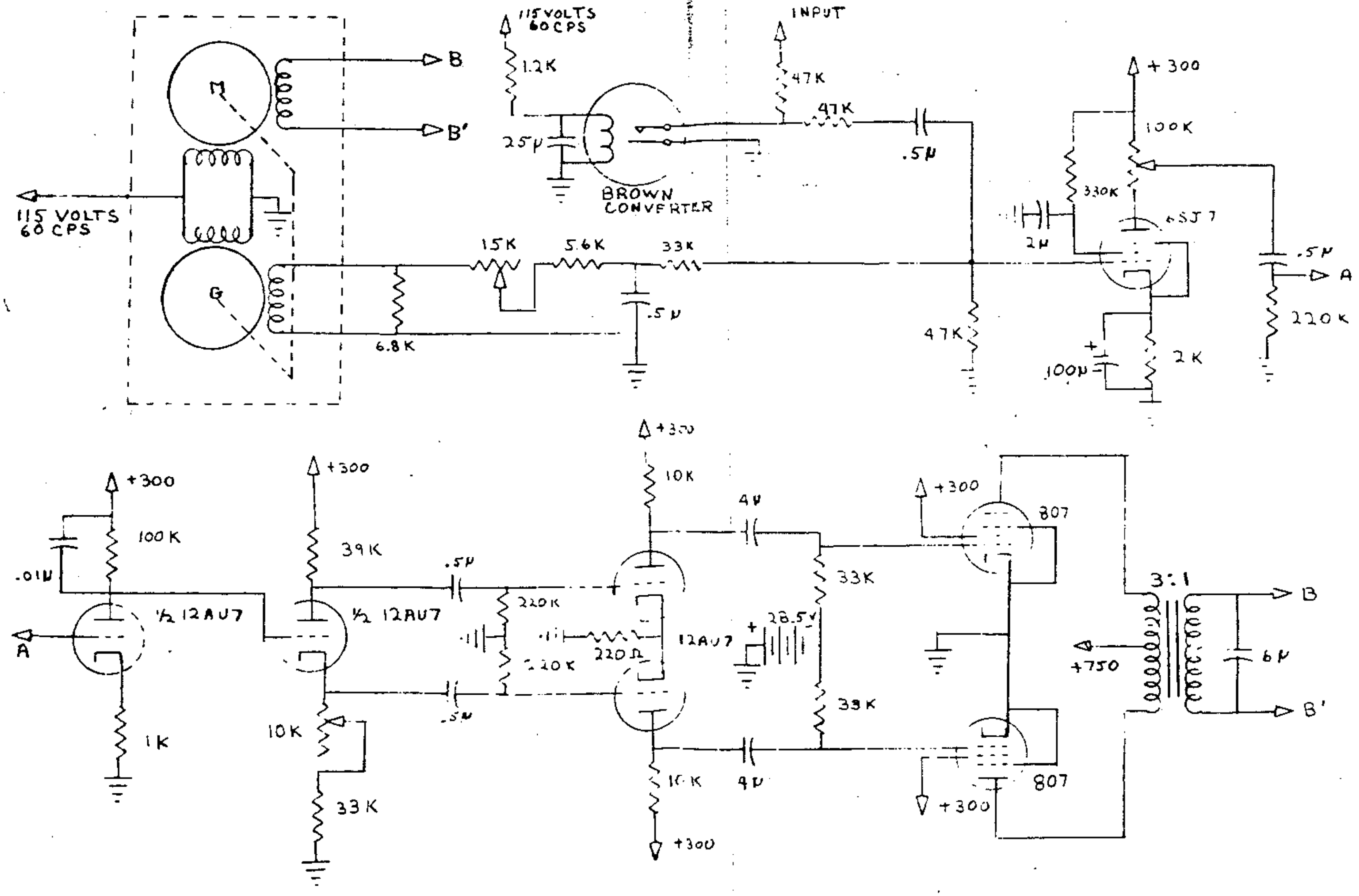

FIGURE 3.4 OUTPUT INTEGRATOR

phase shift is that the full output voltage of the power amplifier is applied to the motor control windings, which is not the case if a series capacitor in the output circuit is used to shift phase. Also, the output of the tachometer is filtered. This largely removes undesired harmonics. Any hum picked up in the circuit is likely to be in phase with the signal on the fixed windings and thus does not affect operation. The lag network in the foward gain path of the integrator loop amounts to carrier compensation and contributes some phase shift at the data frequency. Over the frequency range of interest this effect is negligible. The chopped D.C. and tachemetric feedback signals are summed across a 4.7K resistor in the grid circuit of the 6SJ7 used as a high gain preamplifier. Loop gain can be controlled by the 100K pot which is the plate load for the 6SJ7. One-half of a 12AU7 functions as an inverter driver. Proper choice of components in this circuit permits direct coupling to the grid of the inverter. The .01mfd capacitor was included to elimimate an oscillation at about 50kc which occurred in the completed circuit. This was possibly due to feedback through the power supply from the screens of the output tube to the plate of the 6SJ7. A 10K pot in the cathode circuit of the inverter accomplishes A.C. balance. Both halves of a 12AU7 are used as pull-pull drivers. This combination has an output impedance slightly greater than 5K. The output stage is a pair of 807s operating class AB_2

with 750 volts on the plates. Better performance of the circuit would have been possible if transformer coupling had been used between the driver and output stages. An increase of about 20% in the maximum output power could have been realized. A driver transformer was not used primarily because none was available at the time of construction. Also, the present power output is sufficient to saturate the drive motor so that an increase in amplifier capability would not improve performance. The 807s are transformer coupled to the control windings of a Diehl type FPE49-38-1 servomotor. This is a 10 watt output unit with an integrally mounted tach.

The amplifier was assembled and tested. A 100 watt light bulb was used as a load for these tests. The amplifier could deliver 60 watts with very low distortion, and a maximum of 100 watts with some peaking due to the capacitive driver coupling. This power is double the rated input to the servomotor. The servomotor was then included in the loop and gain adjusted to 9 from the tachometer output to the motor control winding. The phase of the control signal was 75° lagging with respect to the line. The log magnitude plot of the output integrator behaves as 1/s until about 20 radians/second where a first order lag occurs. Sensitivity was measured as .033 meters/second/volt. Thus the transfer function of the output integrator is $.033/s(.05s+1)$.

During the frequency response tests a severe mechanical resonance was noticed in the Z(X,Y) generator at about 30 radians/second. To eliminate this a 2"x4" beam was bolted to the structure as a cross-member.

Since the major loop as developed thus far has a double integrator in the foward gain path and no position feedback, drift becomes a problem. Drift could cause the platform to reach the limits of travel of the Z(X,Y) generator and thus control would be lost. Even if no drift is assumed in the loop an initial synchro misalignment with respect to vertical of only one second of arc (certainly much smaller than can be achieved in practice) would cause the broom to reach the limits of travel in about 100 seconds. To eliminate this problem position feedback was employed as shown in Figure 3.5. The slide-wire was already available on the Z(X,Y) generator. The position signal is summed with the synchro signal to form the input of the first integrator. Polarity is chosen to cause positive feedback- if the base of the broom moves to the right the synchro null is effectively shifted towards the center of the table, thus causing the broom base to move slightly further to the right, and the broom handle tips inward. The net result is to force the broom to fall back towards the center of the table. This positive feedback introduces an additional open-loop pole in the right-half plane. The effects of this will be discussed later. Referring to Figure 3.5, the 100K pot is used to adjust the feedback ratio. In operation it is

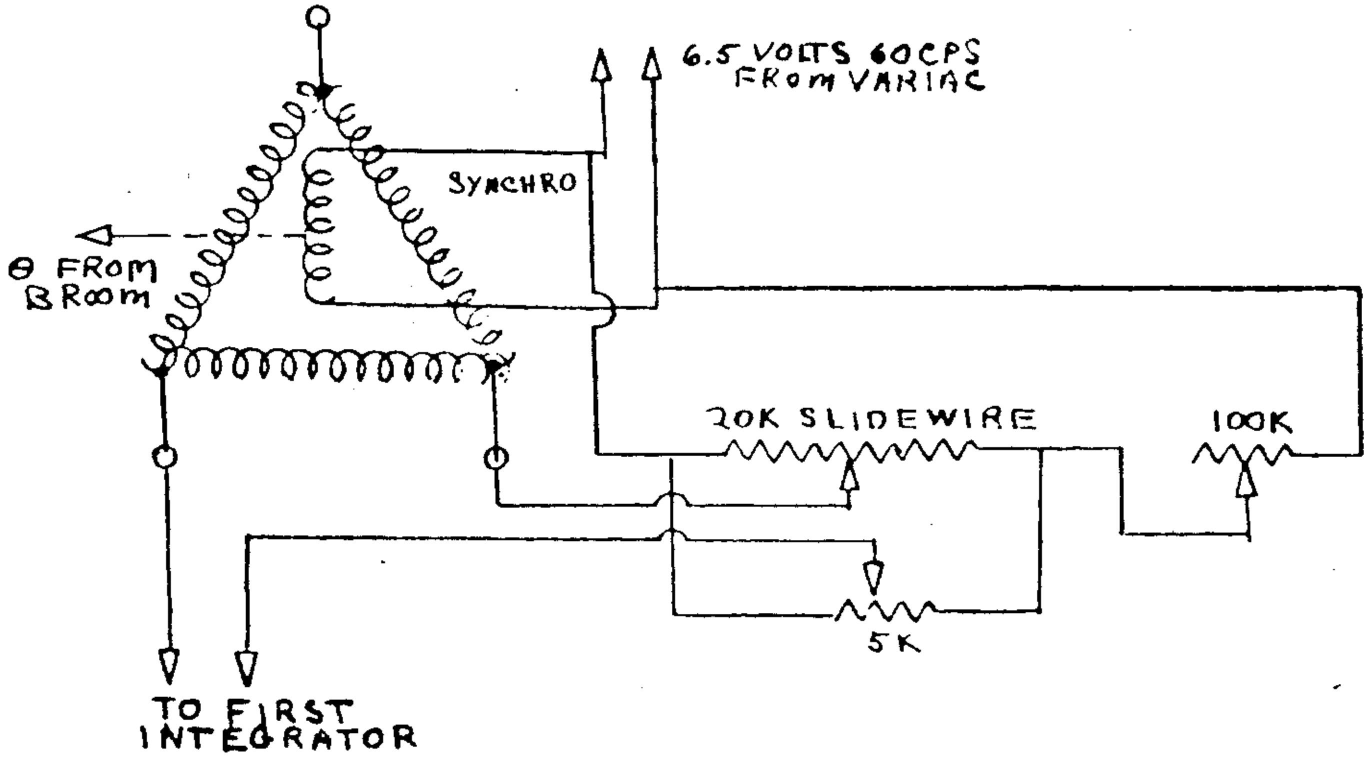

FIGURE 3.5 POSITIVE FEEDBACK ARRANGEMENT

set at about .75 volts/meter. The 5K pot is used to locate the null point of the synchro and slidewire.

Since the output power of the motor is relatively low, some method was necessary to limit the maximum acceleration and prevent motor saturation. Also, negative phase shift is required in the loop at low frequencies for stability considerations. To accomplish these objectives a piece of cardboard with an area of .2 square meters was attached to the top of the broom to introduce damping. The damping force produced by air drag is proportional to the square of the velocity. To linearize this damping as well as to approximate friction losses in the bearings supporting the broom, tests of damping for the broom were conducted by suspending it in a head down position and measuring the decay of oscillations with time. The results indicated that for the normally encountered velocity ranges the coefficient of viscous damping, B, is approximately .18 Newtons/meter/second.

It was also noted from the operation of the system that at times the shaft of the broom would start to flex at about 50 radians/second. With high values of overall loop gain this oscillation would be maintained causing a jitter of about .25 centimeters amplitude in the output position. To eliminate this problem a small piece of cardboard was attached to the center of the broom shaft. This damped the fundamental mode of the shaft sufficiently to prevent oscillation.

IV. FINAL STABILITY CONSIDERATIONS

The completed loop including the values developed in the preceding section is shown in Figure 4.1. The symbols and values of constants used are repeated here for convenience.

B = approximated viscous damping coefficient = .18 Newtons/meter/second

L = effective length of broom = 1.4 meters

M = effective mass at top of broom = 1.6 Kilograms

θ = angle of broom with respect to vertical (radians)

g = acceleration of gravity = 9.8 meters/second^2

X_2= displacement of top of broom from reference (meters)

X_1= displacement of base of broom from reference (meters)

K_f= positional feedback sensitivity (volts/meter), adjustable with pot

K_p= synchro sensitivity = 16 volts/radian

First integrator transfer function = 140/s(.02s+1) (volts/second)/volt

Active network transfer function =

$$K_a\frac{(.204s+1)}{(.016s+1)}$$ volts/volt, K_a is adjustable with pot

Second integrator transfer function =

$$\frac{.033}{s(.05s+1)}$$ (meters/second)/volt

The null set voltage is summed with the synchro voltage to null the synchro as explained previously.

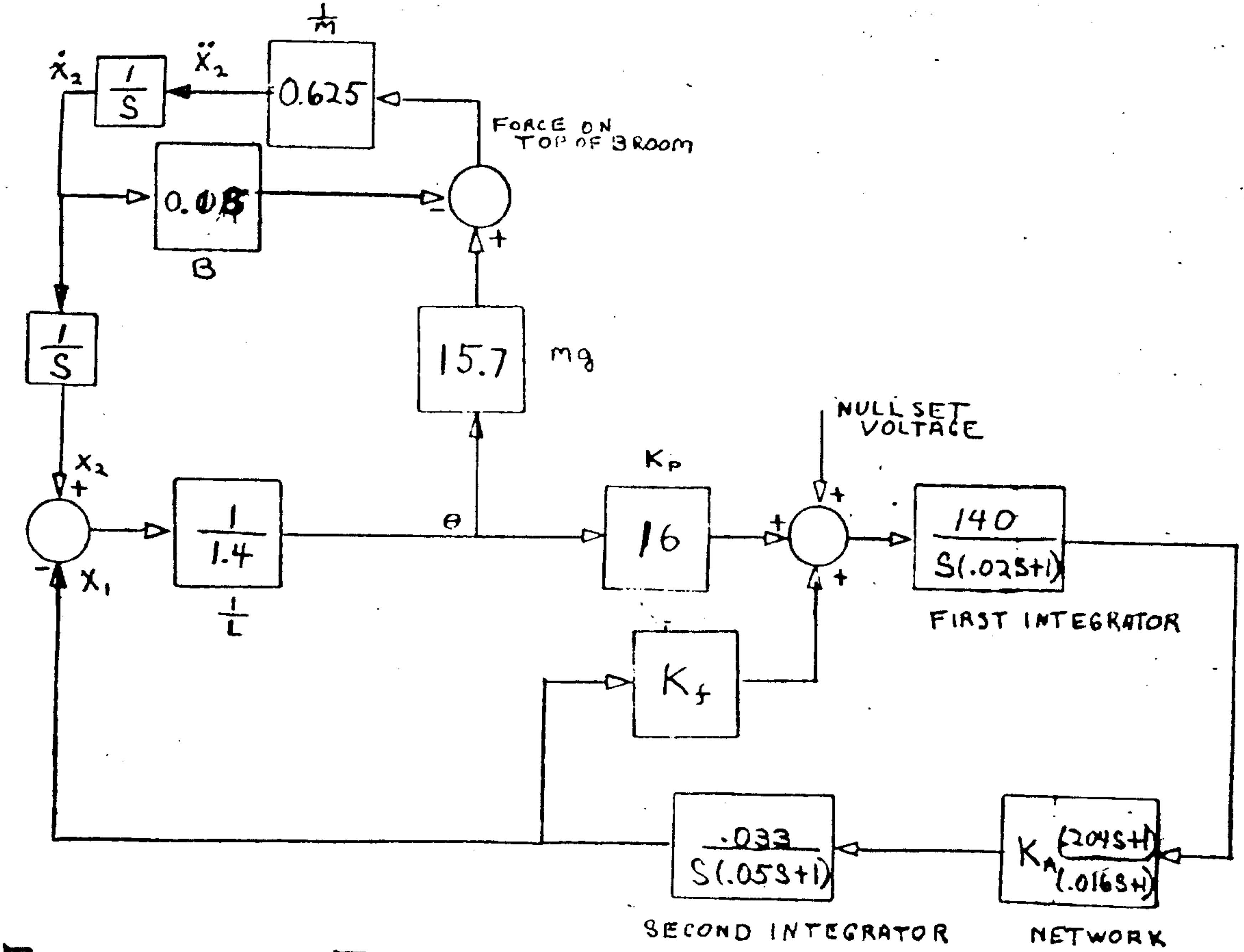

FIGURE 4.1 FINAL BLOCK DIAGRAM

To set the loop gain as a first approximation K_f is ignored. Under this condition a Bode plot can be used. The minor loop containing the broom is collapsed, yielding a minor loop transfer function of

$$\frac{s(s+.115)}{1.4(s+2.7)(s-2.6)} \tag{4.1}$$

The net result of damping is to change the numerator from s^2 to $s(s+.115)$, and to shift the pair of poles on the real axis slightly to the left. The magnitude difference between $1/(s+2.7)(s-2.6)$ and $1/(s+2.65)(s-2.65)$ is insignificant, and the phase difference is less than 1° at any frequency. Therefore it is reasonable to replace Equation (4.1)

$$\frac{s(s+.115)}{1.4(s+2.65)(s-2.65)} = \frac{s(s+.115)}{9.8(.378s+1)(.378s-1)} \tag{4.2}$$

With this approximation, the resulting open major loop transfer function is

$$(7.55K_a)\frac{(s+.115)(.204s+1)}{s(.378s+1)(.378s-1)(.016s+1)(.02s+1)(.05s+1)} \tag{4.3}$$

A Bode plot of the frequency-dependent portion of this expression is shown in Figure 4.2. The plot indicates that cross-over can be accomplished at about 7.5 radians with an open loop gain of 8 dg = 6.3. This implies a K_a of 6.3/7.55 = .835. The phase margin of the system is about 22°, which is sufficient. A Nyquist plot can be

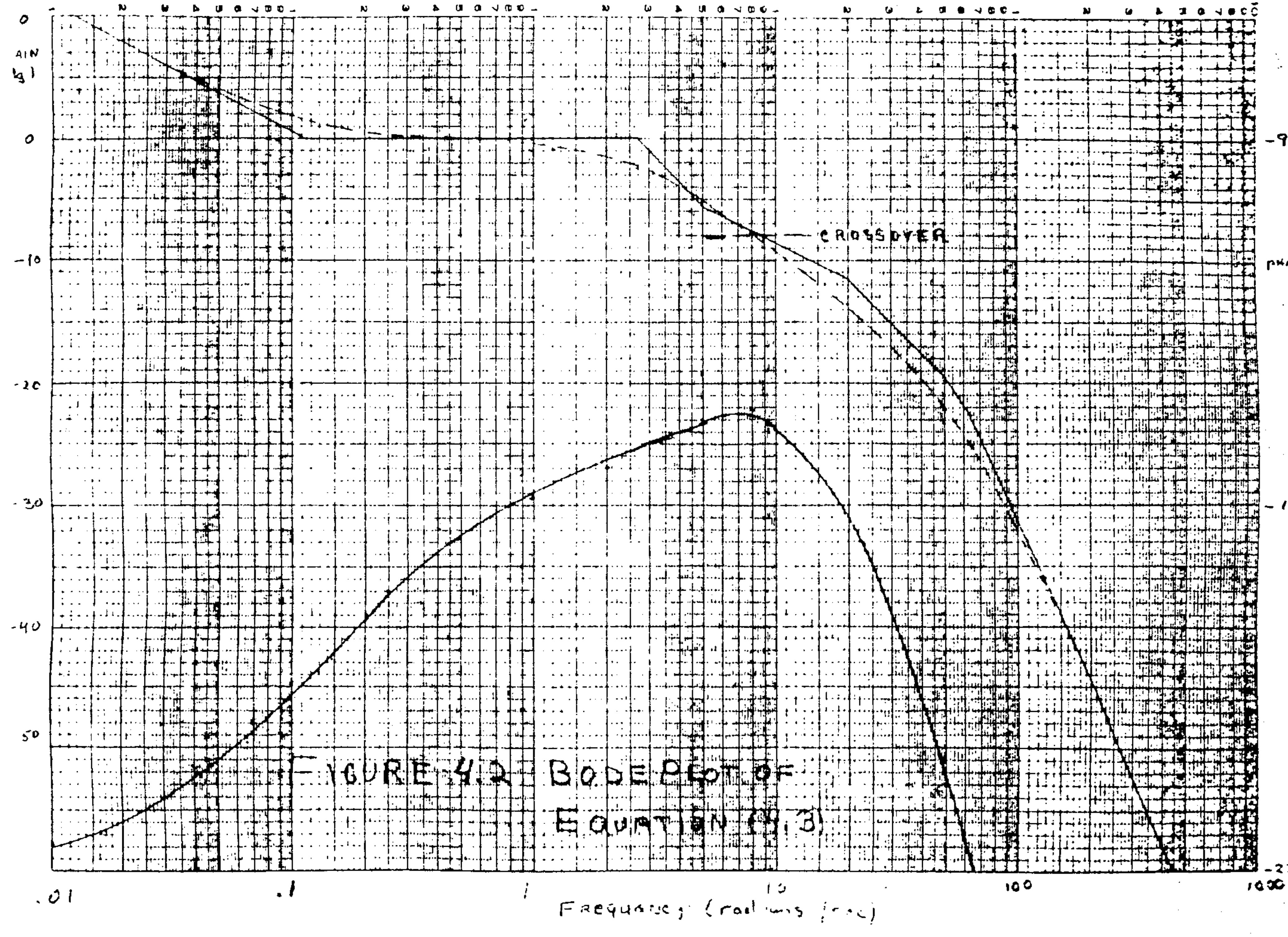

FIGURE 4.2 BODE PLOT OF EQUATION (4.3)

used to verify stability. The only difference between a Nyquist plot of this function and the one shown in Figure 2.3 is that for this function, Equation (4.3), the D.C. gain is infinite, and the D.C. phase shift is -270° as opposed to -180° in Figure 2.3. The net number of encirclements of the -1 point is still -1, thus indicating stability.

When the positive position feedback is included two complete minor loops are obtained with a gain K_p connecting them. One minor loop is the broom loop for which the transfer function is

$$\frac{s(s+.115)}{9.8(.378s+1)(.378s-1)} \tag{4.2}$$

The other minor loop is obtained by allowing K_f to be non-zero. If the previously determined value of K_a is chosen, this loop has a transfer function of

$$\frac{\dfrac{140(.033)(.835)(.204s+1)}{s^2(.02s+1)(.05s+1)(.016s+1)}}{1-\dfrac{K_f 140(.033)(.835)(.204s+1)}{s^2(.02s+1)(.05s+1)(.016s+1)}} \tag{4.4}$$

In actual operation of the device, a K_f of .75 volts/meter was found effective. Thus Equation (4.4) can be rewritten with K_f replaced by .75 as

$$\frac{140(.033)(.835)(.204s+1)}{s^2(.02s+1)(.05s+1)(.016s+1)-.75(140)(.033)(.835)(.204s+1)} \tag{4.5}$$

Combining expressions this becomes

$$\frac{1.33x(.204s+1)}{5.5x10^{-6}s^5+3.86x10^{-4}s^4+2.96x10^{-2}s^3+.345s^2-.204s-1} \qquad (4.6)$$

This can be factored as

$$\frac{1.33x(.204s+1)}{(.54s-1)(.66s+1)(.071s+1)((\ (s)^2/(67.5)^2+2(.41)s/67.5+1))} \qquad (4.7)$$

The total major open-loop transfer function may now be obtained by multiplying the product of Equations (4.7) and (4.2) by K_p. This yields

$$\frac{(2.18)(s)(.204s+1)(s+.115)}{(.378s+1)(.378s-1)(.54s-1)(.66s+1)(.071s+1)\left[\frac{s^2}{(67.5)^2}+\frac{2(.41)s}{67.5}+1\right]} \qquad (4.8)$$

This expression contains two open-loop poles in the right-half plane, one due to the broom and the other due to the positive position feedback. A Bode plot of this function is shown in Figure 4.3. Figure 4.4 is a Nyquist diagram of the function. The Nyquist diagram verifies the stability of the system. There are -2 net encirclements of the -1 point, and therefore sincethere are two open-loop poles in the right-half plane the system is stable. The consequence of introducing position feedback, necessary because the table is not of infinite length, becomes evident. Now there are two phase conditions to satisfy. The phase must be more negative than -180° at the low-

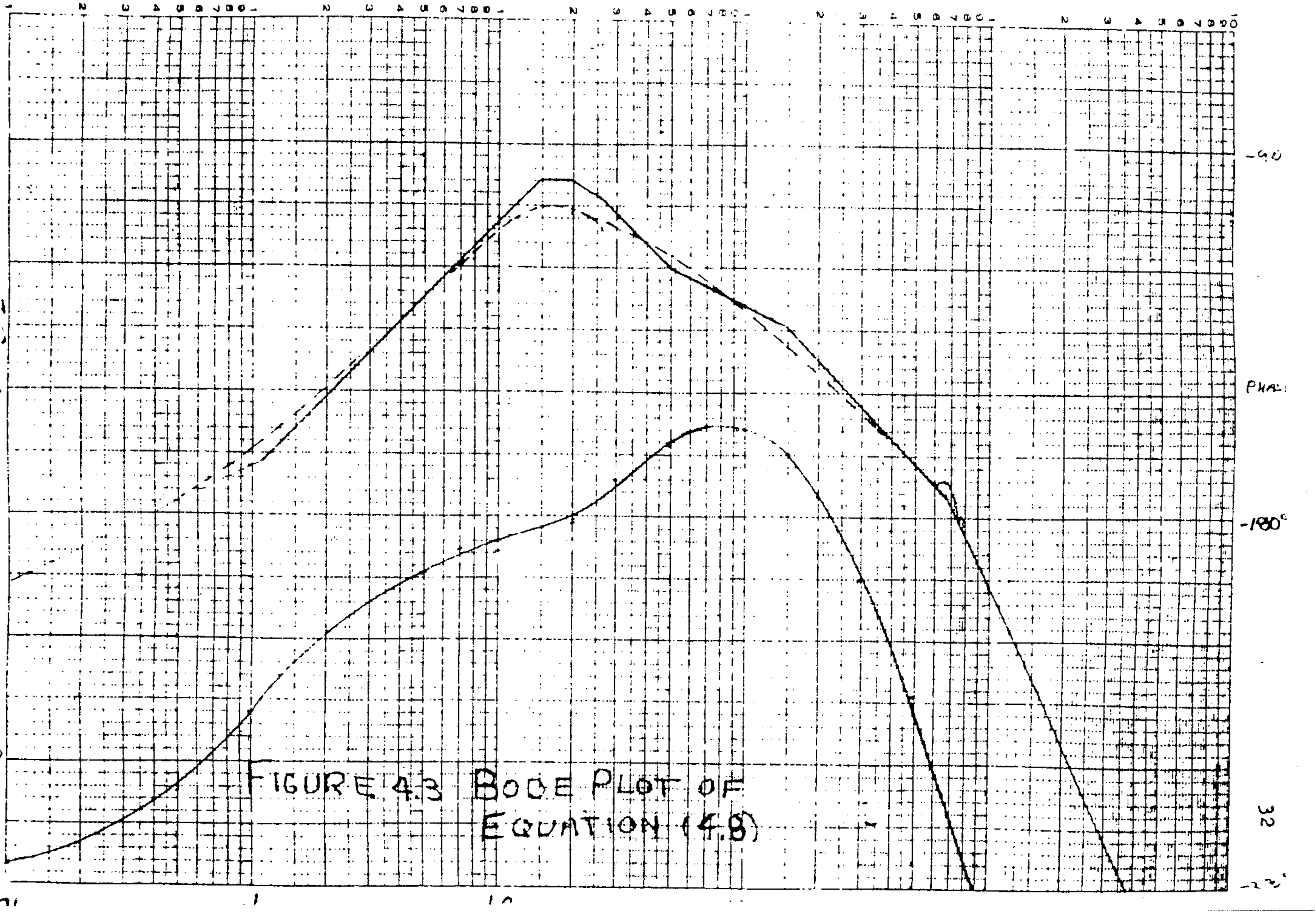

FIGURE 4.3 BODE PLOT OF EQUATION (4.8)

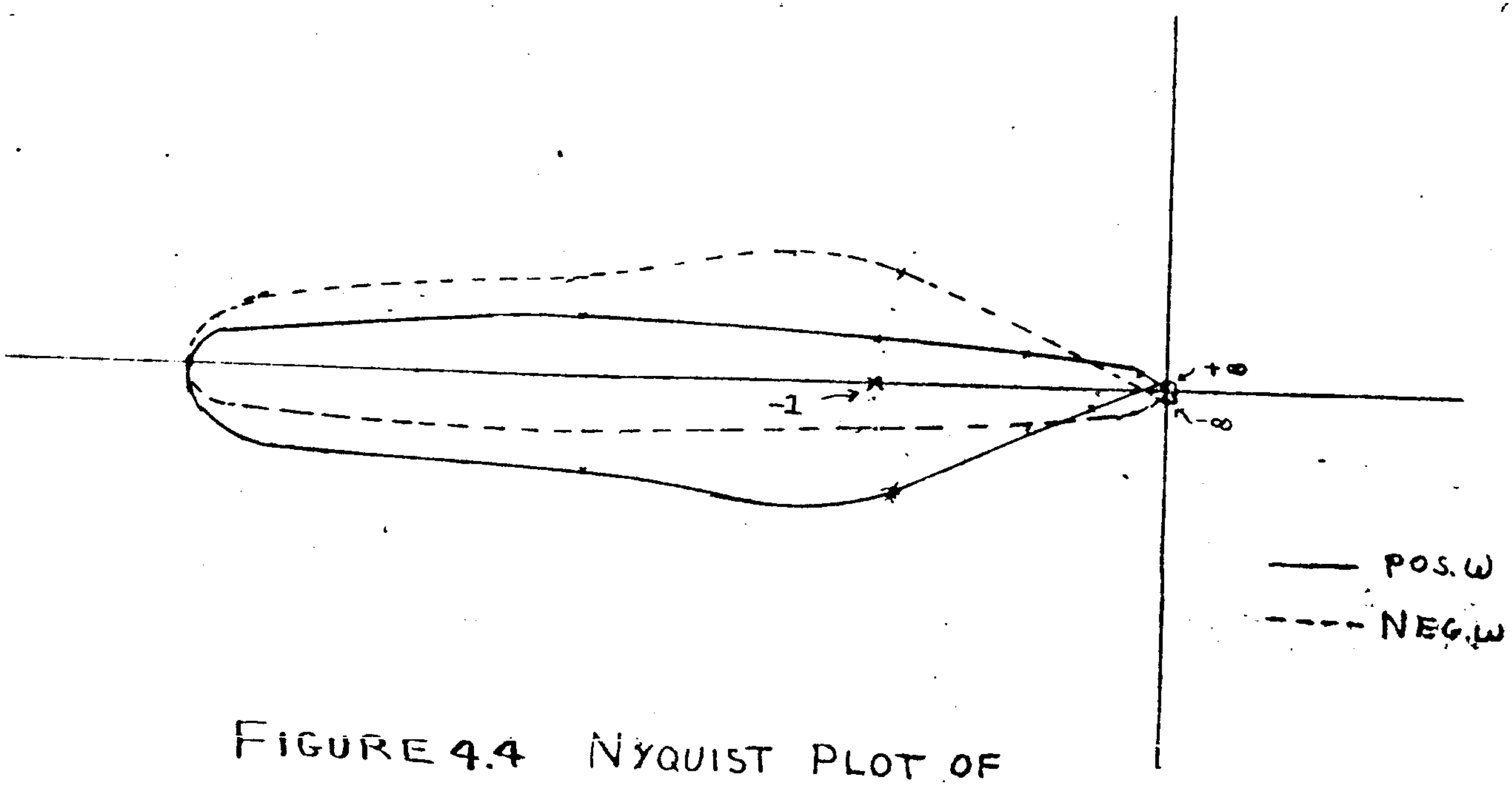

FIGURE 4.4 NYQUIST PLOT OF EQUATION (4.8)

frequency crossover and more positive than $-180°$ at the high-frequency crossover. If B were not present, stability could still be accomplished by utilizing a network which contributed negative phase shift at low frequencies.

Figures 4.3 and 4.4 indicate that there is a considerable amount of leeway as far as gain is concerned. K_p can be changed by 50% either way (both on the diagrams and with the actual machine) due to the relatively flat nature of the phase curve and relatively steep magnitude curve at both critical points. However, the phase margin is relatively low on both ends. The low end is particularly troublesome since B is really strongly velocity-dependent. The effects of this dependence will be discussed in greater detail in the following section.

V. RESULTS

To start the system the gain of the active network is increased from zero until stability occurs. The stable range of K_a is from about .75-1.25, which compares with about .835 predicted as optimum from the derivation of the preceding sections. In normal operation without any external disturbance the base of the broom oscillates at about 1 radian/second. By proper adjustment of the gain, K_a, and the feedback ratio, K_f, the peak-to-peak amplitude of this oscillation can be kept below .15 meters. The cause of this oscillation can be found from a consideration of Figure 4.3. From this Bode plot we notice that if the net phase shift at the low-frequency crossover (about 0.7 radians/second) becomes less negative than -180° the system will become unstable and oscillate at this frequency. The break-point which controls the magnitude of the phase shift at this point is dependent upon the assumed magnitude of B. Since air drag is proportional to the square of the velocity, if we represent B as viscous friction its magnitude will depend upon the velocity of travel. If the velocity of the top of the broom is low, the break-point in question will move toward lower frequencies. Eventually it will occur at a frequency such that the phase shift at the low-frequency crossover is not sufficient to maintain stability. The broom will start to oscillate, thus increasing velocity, and stability will be recovered. The magnitude of the oscillation will eventually reach a value just sufficient to maintain -180° of phase shift at the low-frequency crossover. The Bode plot predicts that this oscillation will occur at about .7 radians/second while in the actual system the frequency is closer to 1 radian/second.

From calculations involving the mass in the drive channel and the characteristics of the motor it can be shown that the maximum acceleration of the handle of the broom at one-half maximum drive-motor speed is about 0.2 g. Since Figure 4.3 indicated that a maximum gain in the channel of about 4 is necessary, linearity can be maintained for any initial offhang less than 3°. The actual unit can usually correct for a disturbance causing an offhang of close to 3°-4°.

When a correctible initial disturbance is introduced the base of the broom follows the head and overshoots possibly 50%-75%. This is typical of a system with only 20° of phase margin. After the initial high-frequency component of the transient has damped out, the broom slowly returns to equilibrium near the center of the table exhibiting lightly-damped low-frequency oscillation. Time for complete return to equilibrium behavior after a severe disturbance may take as long as 30 seconds. If too large an initial disturbance is applied, the base accelerates to the limit stops and control is lost.

If the gain (K_a) is adjusted slightly higher than 1.25 and a large initial disturbance is applied, the unit breaks into large amplitude oscillations at about 15 radians/second. If the loop gain is not quickly returned to normal the base oscillates into the limit stops.

As stated earlier, if the device is not disturbed it will perform for long periods of time. It has been in operation continuously for as long as three hours without any tendency toward instability noted. A warmup period of about ten minutes is necessary to stabilize the network and allow the motor to reach operating temperatures before results of this type can be expected.

BIBLIOGRAPHY

(Book) Brown, G.S. and Campbell, D.P.
"Principles of Servomechanisms"
New York: John Wiley & Sons, 1948

(Book) Chestnut, H. and Mayer, R.W.
"Servomechanisms and Regulating System Design" - Volume I
New York: John Wiley & Sons, 1959

(Memo) Mori, H.
"Description of Row 10 (Function Generators)"
DIC 6978, M-2.20-5. Dynamic Analysis and Control Laboratory, Massachusetts Institute of Technology. July 17, 1953

(Book) Newton, G.C. Jr., Gould, L.A., and Kaiser, J.F.
"Analytical Design of Linear Feedback Controls"
New York: John Wiley & Sons, 1957

(Memo) Prival, H.G.
"Investigation of the Performance of the Y Servo of Function Generator No. 3"
DIC 7457, M-33-40(1). Dynamic Analysis and Control Laboratory, Massachusetts Institute of Technology. August 8, 1956

APPENDIX

OPERATING INSTRUCTIONS

The components of the broom balancer are mounted on three levels which are part of the Z(X,Y) generator. The approximate locations of the major components and controls are shown in Figure A-1. To operate the device, the procedure is as follows.

1. Connect the green wire from the extreme right-hand side (viewed from the rear) of the terminal board on the back of the output amplifier to the minus terminal of the 6 volt battery. If there is any question as to this connection or the condition of the batteries, remove one of the 807s from the output amplifier and check for -28.5 volts from the grid pin to chassis ground. <u>This is extremely important</u>, since failure to maintain correct bias will destroy the 807s.

2. Connect the black shielded wire lying near the batteries to the + and -45 volt terminals on the batteries. The ground shield goes to -45. <u>Connect this lead first</u>, as application of the positive voltage first may destroy the transistors in the active network. Check to make sure that there is 90 volts between the two terminals.

3. First checking to insure that all four switches on the two power supplies (two switches on each) are turned off, plug in the line cord to 115 volts, 60 cps. The line cord comes from a plug strip on the right rear leg of the Z(X,Y) generator. All the A.C. power for the de-

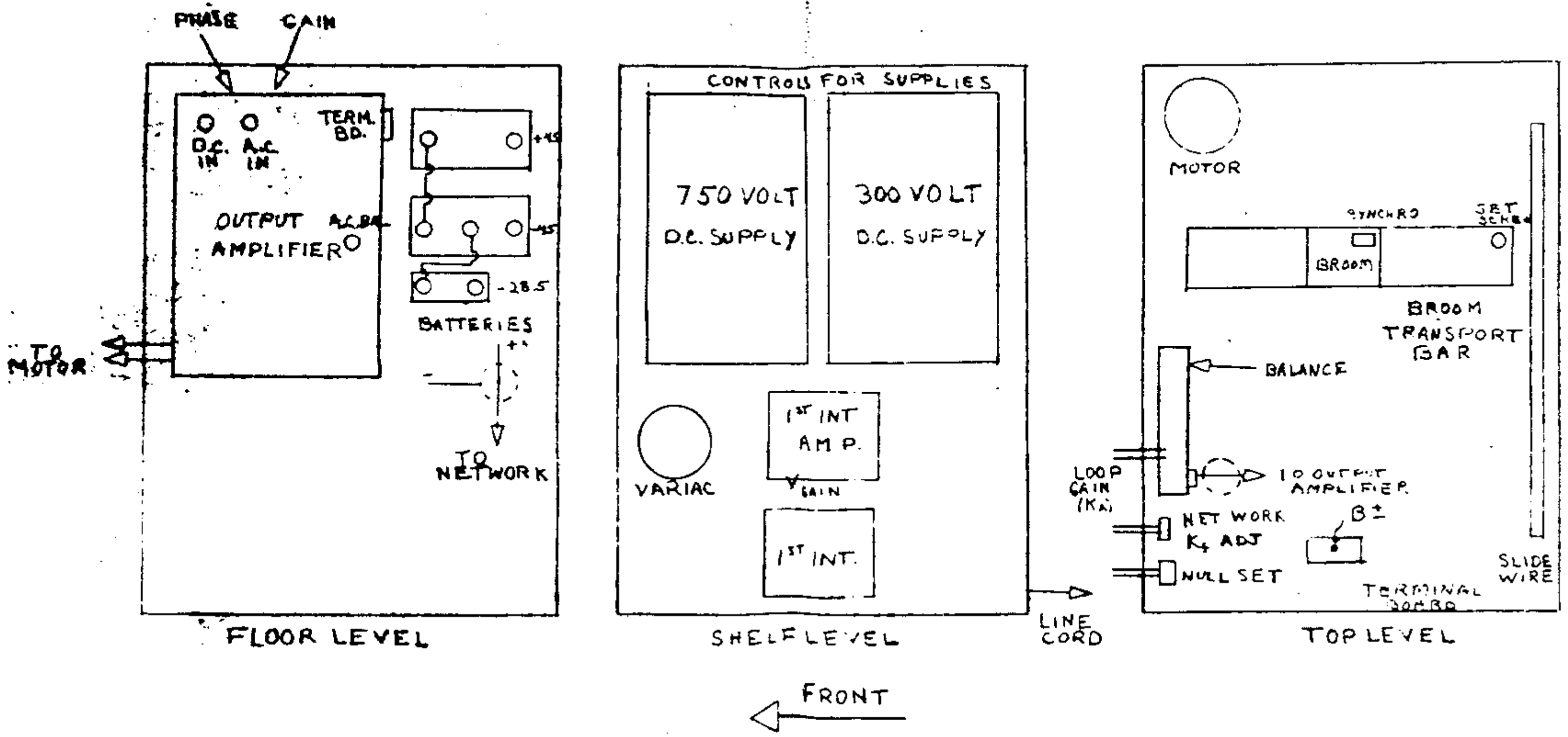

FIGURE A-1 . COMPONET LAYOUT (NOT TO SCALE) vice comes from this strip. At this point it is wise to check to be sure that:

A. The filaments in the output amplifier are lighted.

B. Both motors (the output and the first integrator) have 115 volts on the fixed windings.

C. The Brown Converter (gray can on the output amplifier) is operating. The operation can be noted by lifting off the can. If the reed is vibrating, the converter is receiving power.

4. Turn both the filament switches on the power supply on. <u>Be sure to leave the plate switches off</u>.

5. Allow ten minutes from the time the network was first connected for the transistors in the network to stabilize. Connect a D.C. voltmeter between the output terminal (center wire of the black shielded cable connected to the small terminal strip on the network) of the active network and the point labeled B+ on the terminal board. Adjust the screwdriver adjust balance pot on the back of the network for a voltage null on the meter. This adjustment should be made with the gain control (the knob furthest left on the front panel of the device) turned completely counter-clockwise. If the network is functioning properly the balance control should allow at least a ± 10 volt swing at the output.

6. Adjust the control on the top of the Variac to

100. With this setting the output of the small filament transformer connected directly to the Variac should be about 6.5 volts A.C. This is the synchro supply voltage.

7. Push the broom transport bar all the way to the left when viewed from the front. Lock the set screw securely.

8. Turn on the plate switch of the 750 volt power supply. Then turn on the plate switch of the 300 volt supply. <u>The power must be applied in this order</u>, or else the screens of the 807s will be ruined.

9. Check the gain setting on the first integrator amplifier. The 10 millivolt input gain pot should be set to 8. If everything to this point is operating correctly the first integrator should run into the stops on both ends as the broom is tilted to either side of vertical.

10. Set the K_f adjust (middle knob on the front panel) so that 0.5 volts is measured across the slidewire on a high impedance A.C. voltmeter. This corresponds to a setting about 1/3 of the way clockwise on the pot.

11. Set the null set (right-hand knob with large dial on the control panel) to about the center of its mechanical limits.

12. Loosen the set screw on the broom transport bar. Be sure to hold the broom handle, since drift in the network may cause the output integrator to drive quite quickly. If the speed of travel of the transport bar is greater than 0.1 meters/second, repeat Step 5. To check the

output integrator loop, try to move the transport bar against the direction of travel. Heavy resistance should be encountered. Force the bar to the center of the table and lock down the set screw. Be sure not to allow the broom to fall during these tests. Until familiarity is gained with the equipment it is probably wise to use two people to make these tests.

13. The first integrator should still go to either limit as the broom is tipped to either side of vertical. Check the null position on the first integrator. Motion should cease when the broom is perfectly vertical. If this is not the case, slip the large gear between the broom handle and the synchro on its shaft to position the synchro such that there is no first integrator motion when the broom is vertical. This adjustment is fairly critical.

14. Loosen the set screw and allow the broom transport bar to drift to either side. Turn gain (K_a) pot about 5° clockwise. At this point it should be possible to tip the broom to either side and have the broom transport bar follow.

15. Try to get the broom as nearly vertical as possible. Turn the K_a pot about 1/4 turn clockwise (to the indicated line) and let the broom fall simultaneously. This part requires a little bit of practice. If everything is working correctly the broom will oscillate for a few seconds and then stabilize. Do not allow the broom to fall over as a result of excessive initial offhang.

If difficulty is encountebed repeat Steps 14 and 15.

16. When the device is operating, if a tendency is noted for the broom to hunt about a point not in the center of the table, adjust the null set pot. Turning this pot clockwise makes the broom move right. If there is not enough latitude in this control, repeat step 13.

17. Adjust K_f and K_a slowly to minimize the magnitude of hunting oscillations. Tap the broom handle slightly. The response should be quick and have 2-3 high-frequency overshoots. If the overshoots are excessive, decrease K_a. If none are present increase K_a. To repeat a warning given previously, at no time should the broom ever be allowed to fall over for any reason whatsoever. If this occurs, the synchro shaft will be severely bent, and the required mechanical rebuilding on the synchro and platform takes about one hour.

A common cause of improper operation is slippage of the gear driving the first integrator pot. This gear can be tightened by the Allen Head screw on the gear collar.

If the above procedure fails to achieve operation, check the output integrator first. Remove the D.C. input jack from the output integrator and apply 1½ volts from a battery. This should result in a transport velocity of about .05 meters/second. Also lock the transport bar and measure the voltage on the motor-control winding. This voltage should be greater than 125 volts.

If it is ever necessary to replace a tube in the output amplifier the following adjustment procedure should be observed.

1. Remove the motor leads from the 84 ohm and 500 ohm taps on the output transformer. Also remove the tachometer output lead from the amplifier A.C. input.

2. Connect a 100 watt light bulb between the 84 and 250 ohm taps.

3. Remove the 807s from their sockets and apply the 300 volt supply to the amplifier.

4. Apply a 60 cps ½ volt signal in phase with the line to the A.C. input jack. With the gain pot turned up about ¼ of a turn clockwise adjust the A.C. balance control to drive both 807 grid pins an equal amount with respect to ground.

5. Replace the 807s and turn on the 750 volt supply. Put in both the A.C. signal and a 1½ volt D.C. signal. Observe the output waveform. Adjust the gain pot to yield a 100 volt peak-to-peak output. This waveform shoudd be the sum of a sine and square wave. Adjust the phase control so that the zero crossings of these two component waves coincide.

6. Remove the light bulb and replace the output drive motor. Adjust the gain control of the output amplifier to the highest possible setting consistant with stable operation of the loop.

If all of the above adjustments still fail to yield stable broom operation, the best advice is simply to become familiar with the theory of operation described earlier and start troubleshooting.