Noise Sources in Bulk CMOS

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The noise behavior of bulk CMOS devices is dominated primarily by two noise sources: thermal noise and flicker ($1/f$) noise. Other sources that are sometimes present in the noise spectrum are shot noise, generation/recombination noise, and "popcorn" noise. Of these sources, thermal noise and shot noise are physically fundamental to the operation of the device and are always present. The quality of the manufactured device (the number of defects in the bulk silicon, in the gate oxide, and in the various interfaces) determines the level of generation/recombination noise and popcorn noise. It is probable that flicker noise appears through both quality-dependent and fundamental noise processes.

1 Noise Notation

Following van der Ziel's [1] lead, common practice in the literature uses two-level subscripts, for example, S_{I_D} to denote the spectral density of the drain current. Unfortunately, this author can't stand the sight of two-level subscripts, so he reluctantly uses a different notation for noise variables.

In IEEE notation, as described in the Standard [2], variables are named with the following convention:

- Incremental small signal quantities are expressed in all lower case (i_d, v_{ds})
- Quiescent large signal quantities are expressed in all upper case (I_D, V_{DS})
- Instantaneous total values are expressed as a lower case variable with an upper case subscript ($i_D = I_D + i_d$, $v_{DS} = V_{DS} + v_{ds}$)
- Root-mean-square values are expressed as an upper case variable with a lower case subscript (I_d, V_{ds})

Note that all three editions of Gray and Meyer [3, page 31] label the total variable incorrectly (incorrectly using the form reserved for RMS values[1]). Roberge [4, page 3] uses the upper case variable with a lower case subscript to denote complex quantities, such as Laplace transforms, but uses the notation $I_d(s)$ to reinforce the fact that I_d is a function of the complex variable $s = \sigma + j\omega$.

Here, the convention for noise spectral density variables closely follows the notation used by Motchenbacher and Fitchen [5, page 2], using the IEEE standard for mean-square quantities. Thus, we use a capital letter squared for the current or voltage, with a three letter subscript, starting with n for "noise", followed by either e for excess noise, f for flicker noise, s for shot noise, or t for thermal noise, and ending with either g for gate-referred, d for drain-referred, or b for values in the body. A roman type face is used in the subscript to take advantage of kerning and ligatures so it looks nice. For example,

I^2_{ned} excess noise current at the drain in units of A^2/Hz

[1]Unfortunately, this mistake has probably miseducated a generation of electrical engineers.

I^2_{nfd} flicker noise current at the drain in units of A^2/Hz

I^2_{nsb} shot noise current in the body in units of A^2/Hz

I^2_{nsd} shot noise current at the drain in units of A^2/Hz

I^2_{ntd} thermal noise current at the drain in units of A^2/Hz

V^2_{neg} excess noise voltage referred to the gate in units of V^2/Hz

V^2_{nfg} flicker noise voltage referred to the gate in units of V^2/Hz

V^2_{nsb} shot noise voltage in the body in units of V^2/Hz

V^2_{ntg} thermal noise voltage referred to the gate in units of V^2/Hz

It is noted that there is some redundancy in this notation when used with MOSFETs (the input at the gate is always a voltage, and the output at the drain is always a current), but some redundancy in writing is well-known to be helpful for the reader [6].

For device elements, physical parameters are expressed in all upper case (C_{ox}, W, L), while incremental model parameters are expressed in all lower case (g_m, r_o). However, some widely used notation that does not correspond to the above rules is nevertheless still used in this paper, because it is so widely used (like I_{ii}). Thus, in addition to the above noise variables, the following are used:

α_H Hooge's empirical "constant"

C_D depletion region capacitance

C_I inversion layer capacitance

C_{SS} surface states capacitance

Δf noise bandwidth

f_c Lorentzian corner frequency

I_{ii} impact ionization current

K_F empirical flicker noise coefficient

μ_l mobility due to lattice scattering only

N number of free carriers

N_{ST} number of surface trap states

r_{js} incremental resistance of the body/source junction

Other variables are explained as they are introduced.

2 Flicker Noise

Flicker noise dominates the noise spectrum at low frequency. Flicker noise was first observed in vacuum tubes over seventy-five years ago [7]. It gets its name from the anomalous "flicker" that was seen in the plate current. Flicker noise is also commonly called $1/f$ noise, because the noise spectrum varies as $1/f^{\alpha}$, where the exponent α is very close to unity ($\alpha = 1 \pm 0.2$).

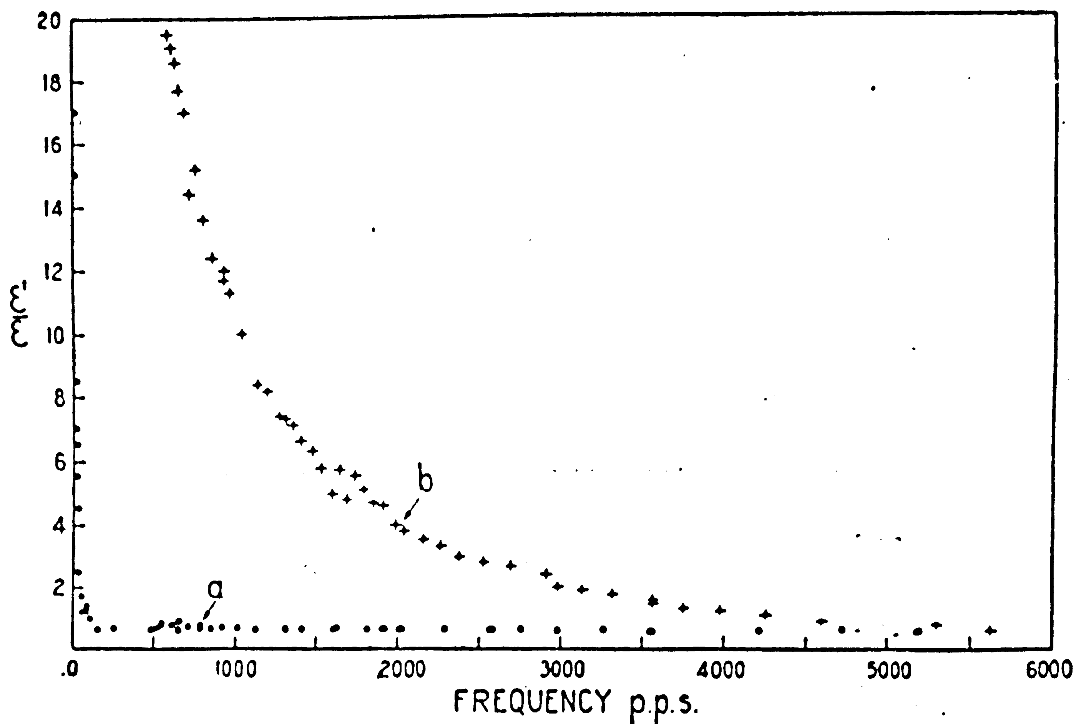

Figure 1: Plot of noise amplitude versus frequency (on linear scales) as measured in two dissimilar vacuum tubes by Johnson in 1925, from [7, Figure 7]. The vertical axis has been normalized by the expected shot noise amplitude.

Fluctuations with a $1/f$ power law have been observed in practically all electronic materials and devices, including homogenous semiconductors, junction devices, metal films, liquid metals, electrolytic solutions, and even superconducting Josephson junctions. In addition it has been observed in mechanical, biological, geological, and even musical systems. No entirely satisfactory physical explanation has been developed, and in fact, available evidence seems to suggest that the origins of flicker noise in different devices may be quite different [8, page 143].

Two competing models have appeared in the literature to explain flicker noise: the McWhorter number fluctuation theory and the Hooge mobility fluctuation theory. There is experimental evidence to support both theories, and thus the literature is mostly split into two camps over the issue.

2.1 The McWhorter Model (Number Fluctuations)

McWhorter, working with germanium filaments at MIT Lincoln Laboratory in 1957 [9], proposed that flicker noise is primarily a surface effect. He cites a number of experiments that showed that the $1/f$ noise in germanium is dependent on the ambient atmosphere of the filament. He writes, "recent results now leave little doubt that the noise is predominantly, if not entirely, a surface phenomenon."

Thus, the McWhorter number fluctuation (Δn) theory states that flicker noise is generated by fluctuations in the number of carriers due to charge trapping in surface states. McWhorter obtained the necessary $1/f$ spectrum by assuming that the time constant τ of the surface states varied with a $1/\tau$ distribution. Christensson et al [10, 11] were the first to apply the McWhorter theory to MOSFETs, using the assumption that the necessary time constants are caused by the tunneling of carriers from the channel into traps located inside the oxide.

A number of other applications of the McWhorter theory to MOSFETs have been done. Das and Moore [12] reviewed and compared a number of these theories and found that the basic assumptions behind any explanation affects the interpretation of experimental results. They said, "theoretical calculations of the MOSFET drain noise current have been performed by many investigators, which have led to different results, mainly due to the difference in the method of attack and nature of assumptions."

Reimbold [13] developed the McWhorter model further, taking into account all the capacitive components of the small-signal equivalent circuit, so as to account for all transistor operating regimes. This development was done to fit Reimbold's measurements in weak inversion. Ghibaudo [14] showed a shortcut through Reimbold's work and came up with the same result, where the spectral density of the drain current is

$$I_{\text{nfd}}^2 = \frac{K_F\, q^4 I_D^2}{n^2 kTWLf} \frac{N_{ST}}{(C_{\text{ox}} + C_{SS} + C_D + C_I)^2}.$$

where N_{ST} is the surface trap density, C_{SS} is the interface state capacitance, C_D is the depletion capacitance, and C_I is the inversion layer capacitance. The bias dependence of the flicker noise is folded into the effective mobility, μ_{eff}, rewriting the above equation as

$$I_{\text{nfd}}^2 = \left(\frac{\mu_{\text{eff}}}{\mu_0}\right)^2 \frac{K_F\, q^4 I_D^2}{n^2 kTWLf} \frac{N_{ST}}{(C_{\text{ox}} + C_{SS} + C_D + C_I)^2}$$

where μ_0 is the low-field mobility and μ_{eff} has a gate-voltage dependence defined as

$$\mu_{\text{eff}} = \frac{\mu_0}{1 + \theta(V_{GS} - V_T)}$$

where θ is the mobility attenuation factor [15, page 146].

2.2 The Hooge Model (Mobility Fluctuations)

In his paper "$1/f$ noise is no surface effect," F. N. Hooge [16] proposed that $1/f$ noise is essentially a bulk phenomenon. Working with metal films [17], he championed an empirical relation for $1/f$ noise in terms of resistance fluctuations, where the spectral density of the resistance is

$$R_{\mathrm{n}}^2 = \frac{\alpha_H}{N} \frac{R^2}{f}$$

where N is the total number of free carriers in the bulk, and α_H is known as "Hooge's constant" an empirical parameter with value about 2×10^{-3}. This equation fit his data for metal films very well (see his plots in Figure 2). Based on these results [18], he said "Investigations of noise... proved that the fluctuations in the conductivity are due to fluctuations in mobility and not in the number of charge carriers." Hooge summarized experimental support for his mobility fluctuation ($\Delta\mu$) theory in [19] and some theoretical support (a development of a phonon scattering theory) was provided by Jindal and van der Ziel [20].

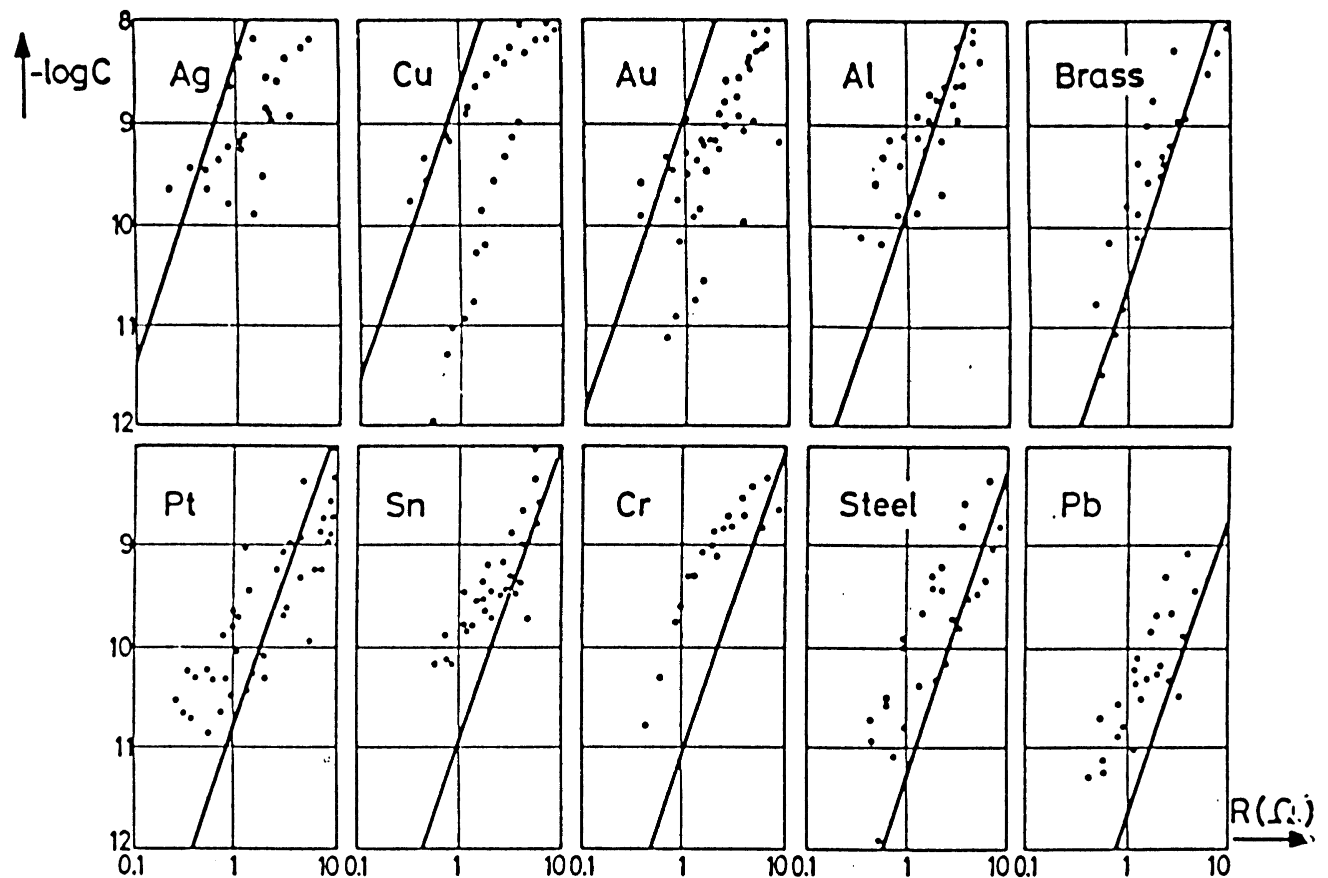

Figure 2: Flicker noise in ten metal films, from [17, Figure 1]. The solid lines correspond to a value of $\alpha_H = 2 \times 10^{-3}$ for Hooge's empirical parameter.

Vandamme [21] developed Hooge's theory for MOSFET devices, starting from the lattice scat-

tering theory presented in [18], calculating

$$I_{\text{nfd}}^2 = \frac{\mu_0}{\mu_{\text{eff}}}\left(\frac{\mu_0}{\mu_l}\right)^2 \frac{q\alpha_H I_D^2}{WLC_{\text{ox}}(V_{GS}-V_T)}\frac{1}{f}$$

where μ_l is the mobility if only lattice scattering were present. Again, significant bias dependences can be explained by changes in the assorted mobility parameters.

2.3 Combined Models

In an effort to fit as much published data as possible, some authors have combined the number fluctuation and mobility fluctuation theories into a single model. The main idea is that charges, when trapped, cause correlated surface mobility fluctuations (due to inversion layer carriers scattering off the trapped charges). Jayaraman and Sodini [22] were the first to propose this model[2]. They also showed that oxide band bending due to nonuniform oxide trap distributions give rise to a gate voltage dependence of the flicker noise coefficient, and of the $1/f^\alpha$ exponent. However, from their data, they were unable to determine if the gate voltage dependence they observed was due to correlated mobility fluctuations, or to nonuniform oxide trap distributions (with a corresponding higher trap density), or both.

With an eye on improving the flicker noise model in circuit simulators such as SPICE, the unified model idea was developed further by Hung et al [23, 24]. By examining the channel current modulation due to the trapping of a single electron (popcorn noise in a very small device), they were able to quantify the contribution of the correlated surface mobility fluctuations. This approach led to

$$I_{\text{nfo}}^2 = \frac{kTI_D^2}{\gamma fWL}(\frac{1}{N}+\alpha'\mu)^2 N_T(E_{fn}).$$

where γ is the attenuation coefficient of the electron wave function into the oxide, α' is their scattering coefficient, and $N_T(E_{fn})$ is the number of traps at the quasi-Fermi level. However, their model is geared towards numerical simulation.

This recent literature supports number fluctuations as the primary source of flicker noise in MOSFETs. Responding to this evidence, Hooge [25] has recently suggested that "in semiconductors there always is mobility $1/f$ noise with an α value of about 10^{-4}" but that "other types of $1/f$ noise may be present and may dominate the mobility noise."

Other authors, apparently in a peacemaking role, have suggested that $1/f$ noise in NMOS devices is caused by number fluctuations while in PMOS devices the noise is due to mobility fluctuations [26, 27]. This dichotomy makes sense if the NMOS transistors are surface channel devices (where the carriers interact more readily with Si-SiO_2 surface states), and the PMOS transistors are buried channel devices[3] (where the carriers interact only with the lattice). However, differences in the energy trap density between NMOS and PMOS transistors could also account for the differences in observed bias dependence [28].

[2] Although I have seen a similar theory attributed earlier to van der Ziel by Buckingham [8, page 137] but have been unable to locate the reference: A. van der Ziel. The oxide trap model of 1/f noise in MOSFETs. *Proc. Symp. on 1/f fluctuations, Orlando, Florida.* 1980.

[3] In modern deep-submicron technologies (0.35 μm and below), both NMOS and PMOS transistors are surface channel devices, so this dichotomy may no longer exist.

2.4 Simple Empirical Models

Many of the empirical models given by textbooks are much simpler than the theoretical and unified models above. Unfortunately, some of them are also questionable.

Tsividis [15, page 343] says that "in a common [bulk] CMOS fabrication process," one may find an input-referred voltage due to flicker noise of

$$V_{\mathrm{nfg}}^2 = \frac{K_F}{C_{\mathrm{ox}}^2 WL} \frac{1}{f}$$

with K_F equal to 5×10^{-9} $\mathrm{fC}^2/\mu\mathrm{m}^2$ for NMOS devices and 2×10^{-10} $\mathrm{fC}^2/\mu\mathrm{m}^2$ for buried channel PMOS devices. This equation is an empirical relation, and K_F is not entirely independent of operating point (and is not the same in weak inversion and strong inversion). This model is implemented by early versions of SPICE, and is generally considered to be oversimplified.

3 Thermal Noise

Thermal noise is the voltage fluctuations caused by the random Brownian motion of electrons in a resistive medium. It is broadband white noise, and it gets worse with increasing resistance and temperature. The spectral density of the thermal noise across a resistor with resistance R is given by

$$V_{\mathrm{nt}}^2 = 4kTR$$

A fifty ohm resistor has about 1 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ of thermal noise (modeled as a noise voltage source in series with the resistor).

The thermal noise in the channel of a MOSFET in strong inversion is

$$I_{\mathrm{ntg}}^2 = 4kT\left(\frac{2g_m}{3}\right)$$

where g_m is the small-signal transconductance at the bias point.

4 Shot Noise

Shot noise is caused by the fact that current flowing across a junction isn't smooth, but is comprised of individual electrons arriving at random times. This non-uniform flow gives rise to broadband white noise that gets worse with increasing average current. The spectral density of the shot noise associated with a junction current I is given by

$$I_{\mathrm{ns}}^2 = 2qI.$$

A one milliamp current has about 2 $\mathrm{pA}/\sqrt{\mathrm{Hz}}$ of shot noise (modeled as a noise current source in parallel with the junction).

MOSFETs in subthreshold exhibit shot noise (instead of thermal noise) due to the current flowing in the channel. This noise source has the standard form

$$I_{\mathrm{nsd}}^2 = 2qI_D.$$

since we are in subthreshold, the effective noise voltage at the gate can be written as

$$V_{\mathrm{nsg}}^2 = \frac{I_{\mathrm{nso}}^2}{g_m^2} = \frac{2q}{I_D}\left(\frac{nkT}{q}\right)^2.$$

For low drain-to-source voltage ($V_{DS} < 5kT/q$), the shot noise grows in value [29], approaching

$$I_{\mathrm{nsd}}^2 = 4qI_{\mathrm{sat}}$$

where I_{sat} is the saturation current of the transistor in subthreshold, defined where the drain current is

$$I_D = I_{\mathrm{sat}}\left[1 - \exp\left(-\frac{qV_{DS}}{kT}\right)\right].$$

For complete equations for I_{sat} see [15, page 139].

5 Generation/Recombination Noise

In addition to the flicker noise caused by traps in the oxide, trapping centers in the bulk of the device can cause generation/recombination (G/R) noise. The trapping of carriers by these traps causes fluctuations in the number of carriers, and thus fluctuation in the resistance. The spectral density of the resistance fluctuation is [25]

$$R_{\mathrm{n}}^2 = \frac{\sigma^2}{N^2} \frac{4\tau R^2}{1+\omega^2\tau^2}$$

where τ is the trap relaxation time, and the variance σ^2 is given by

$$\frac{1}{\sigma^2} = \frac{1}{N} + \frac{1}{N_O} + \frac{1}{N_E}$$

where N_O is the number of occupied traps and N_E is the number of empty traps. If there is more than one kind of trap, the equations are significantly more complicated. Note that G/R noise creates a Lorentzian noise spectrum.

6 Popcorn Noise

Popcorn noise, sometimes called burst noise or random-telegraph-signal (RTS) noise, is a discrete modulation of the channel current caused by the capture and emission of a channel carrier. See Figure 3.

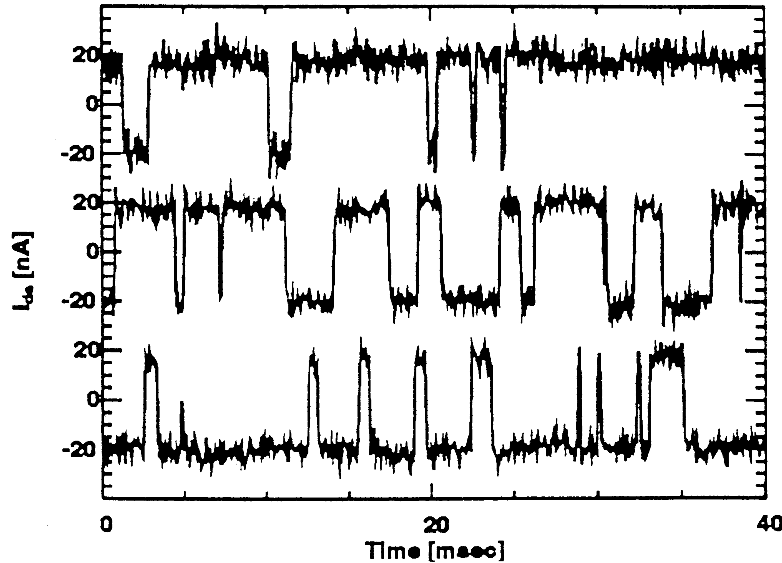

Figure 3: Typical popcorn noise, showing discrete levels of channel current modulation due to the trapping and release of a single carrier, for three different bias conditions, from [30, Figure 1].

7 kT/C Noise

kT/C noise[4] really isn't a fundamental noise source, but is actually thermal noise in the presence of a filtering capacitor.

Consider a passive low-pass filter composed of a resistor and a capacitor, with the output voltage measured across the capacitor. When the input is shorted to ground, the broadband thermal noise of the resistor will be shaped by the low-pass filter. The spectral density of the thermal noise of the resistor is

$$V_{\mathrm{nt}}^2 = 4kTR.$$

In order to find the total output noise, we must integrate the above spectral density over the noise bandwidth, Δf. Expressed in Hertz, this noise bandwidth is

$$\Delta f = \frac{1}{2\pi} \int_0^{\infty} \frac{d\omega}{1 + (\omega RC)^2} = \frac{1}{2\pi} \frac{\pi}{2RC} = \frac{1}{4RC}$$

Thus, the total output noise voltage that is measurable across the capacitor, in volts, is simply

$$v_{no} = \sqrt{V_{\mathrm{nt}}^2 \, \Delta f} = \sqrt{\frac{kT}{C}}.$$

This result is not a new noise source, but a repackaging of thermal noise. It is simply the result of low-pass filtering the noise from a resistor. It doesn't matter if the filter above is a discrete resistor and capacitor, or the channel resistance and drain capacitance of a MOSFET. Note that to include thermal noise and kT/C noise in a single noise calculation would be double-counting.

[4] CCD designers [31], who count electrons instead of volts, talk instead about kTC noise ($Q_n^2 = kTC$, usually expressed in units of electrons-squared per Hertz).

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