Klein backscattering and Fabry-Perot resonances in graphene p-n-p junctions

Leonid Levitov

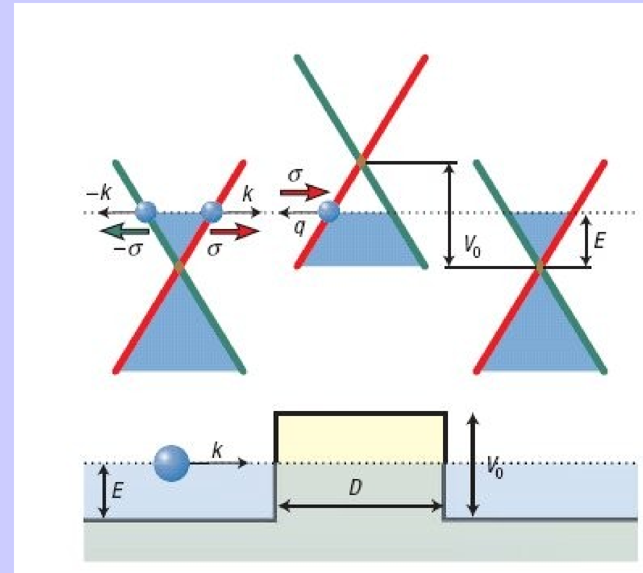
collaboration with:
Andrei Shytov (Utah) and
Mark Rudner (MIT)

August 2008

Klein tunneling

Klein paradox: transmission of relativistic particles is unimpeded even by highest barriers
Reason: negative energy states;
Physical picture: particle/hole pairs

Katsnelson, Novoselov, Geim



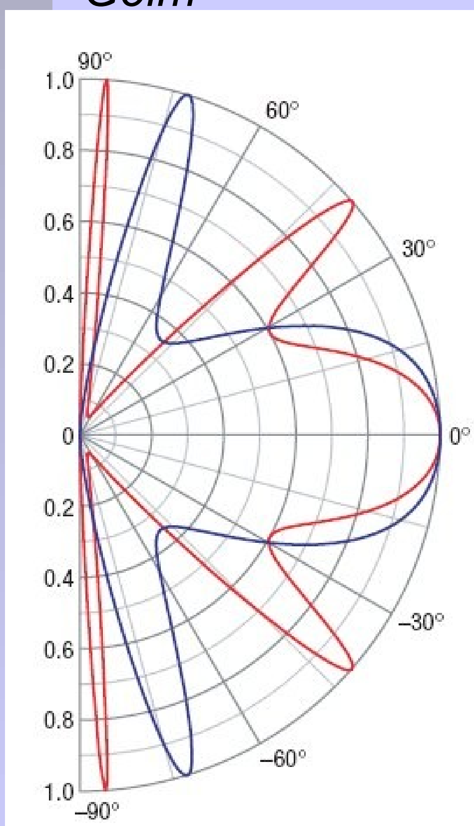
Example: potential step

$$V(x) = \begin{cases} V_0, & 0 < x < D, \\ 0 & \text{otherwise.} \end{cases}$$

$$\psi_1(x,y) = \begin{cases} (e^{ik_x x} + r e^{-ik_x x}) e^{ik_y y}, & x < 0, \\ (a e^{iq_x x} + b e^{-iq_x x}) e^{ik_y y}, & 0 < x < D, \\ t e^{ik_x x + ik_y y}, & x > D, \end{cases}$$

$$\psi_2(x,y) = \begin{cases} s(e^{ik_x x + i\phi} - r e^{-ik_x x - i\phi}) e^{ik_y y}, & x < 0, \\ s'(a e^{iq_x x + i\theta} - b e^{-iq_x x - i\theta}) e^{ik_y y}, & 0 < x < D, \\ s t e^{ik_x x + ik_y y + i\phi}, & x > D, \end{cases}$$

Transmission, angular dependence



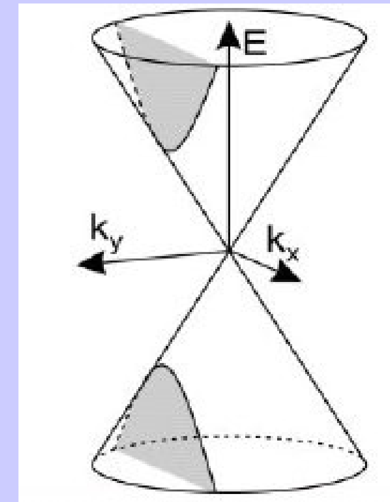
Chiral dynamics of massless Dirac particles: no backward scattering (perfect transmission at zero angle)

Limit of extremely high barrier: finite T

$$T = \frac{\cos^2\phi}{1 - \cos^2(q_x D)\sin^2\phi}.$$

Electron confinement in a p-n-p junction

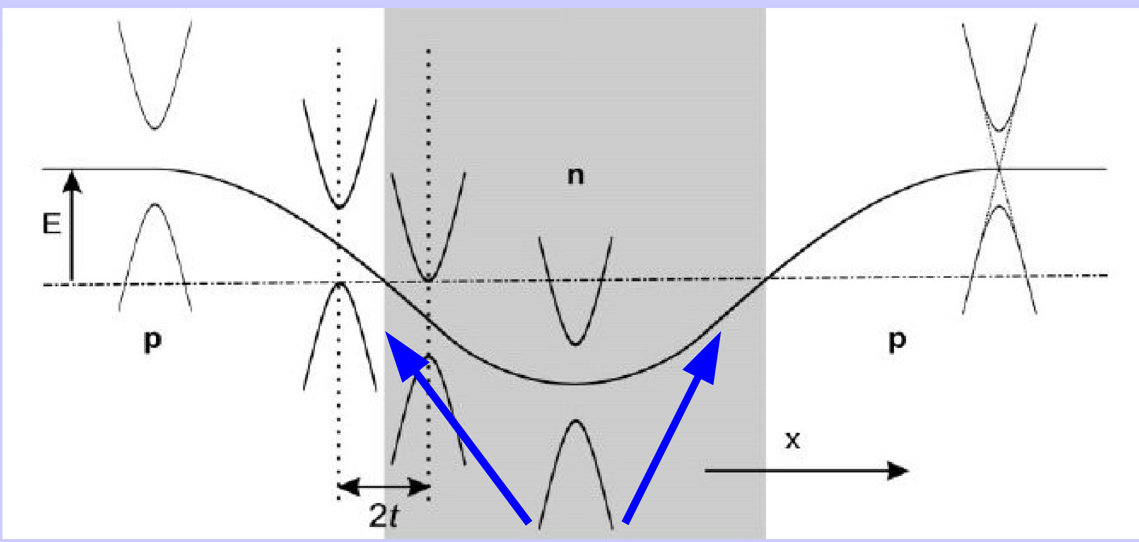
Gate-induced potential well, e.g. V(x) = ax^2 + E



Momentum conserved along y-axis:

Effective D=1 potential

$$H_{\text{eff}} = \varepsilon = \pm c\sqrt{p_x^2 + p_y^2} + V(x).$$



Klein tunneling

Savchenko & Guinea

<u>Confinement by gates difficult!</u>

No discrete spectrum, instead:
(i) quasistationary states (resonances);
(ii) collimated transmission

Quasiclassical treatment

Silvestrov, Efetov

Potential V(x) = U(x/x₀)^2 + E

$$H_{\rm eff} = \varepsilon = \pm c\sqrt{p_x^2 + p_y^2} + V(x).$$

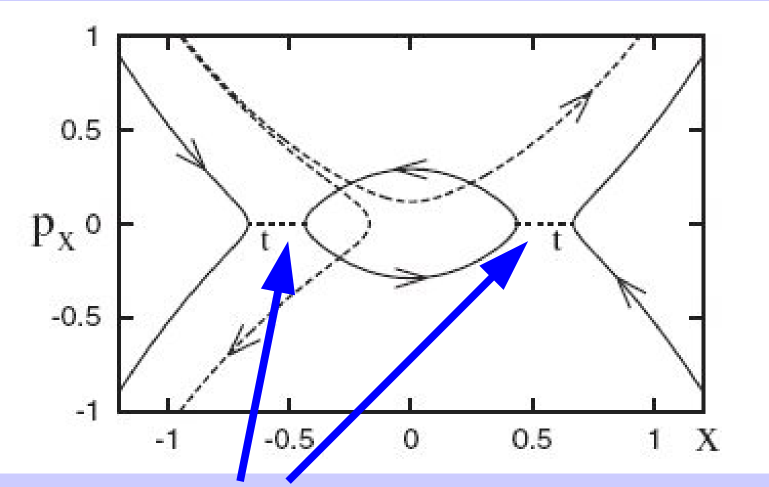
Bohr-Sommerfeld quantization

$$\int_{x_{\rm in-}}^{x_{\rm in+}} \sqrt{[\varepsilon_N - V(x)]^2 - c^2 p_y^2}\,\frac{dx}{c} = \pi\hbar\left(N + \frac{1}{2}\right).$$

Finite lifetime

$$\Gamma_N = \frac{\hbar}{\Delta t} w = \frac{\hbar v_0}{2x_0}\sqrt{\frac{U}{-2\varepsilon_N}}\exp\left(-\frac{\pi c p_y^2 x_0}{\hbar\sqrt{-2\varepsilon_N U}}\right).$$

Classical trajectories



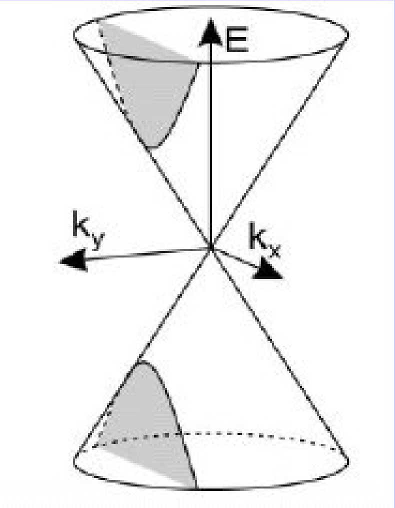
Tunneling

Turning points:

$$\frac{x_{\rm out_\pm}}{x_0} = \pm\sqrt{2\frac{c|p_y| - \varepsilon}{U}}, \qquad \frac{x_{\rm in_\pm}}{x_0} = \pm\sqrt{2\frac{-c|p_y| - \varepsilon}{U}}.$$

Degree of confinement can be tuned by gates; BUT:
no confinement for p_y=0

Geometric confinement in ribbons and dots

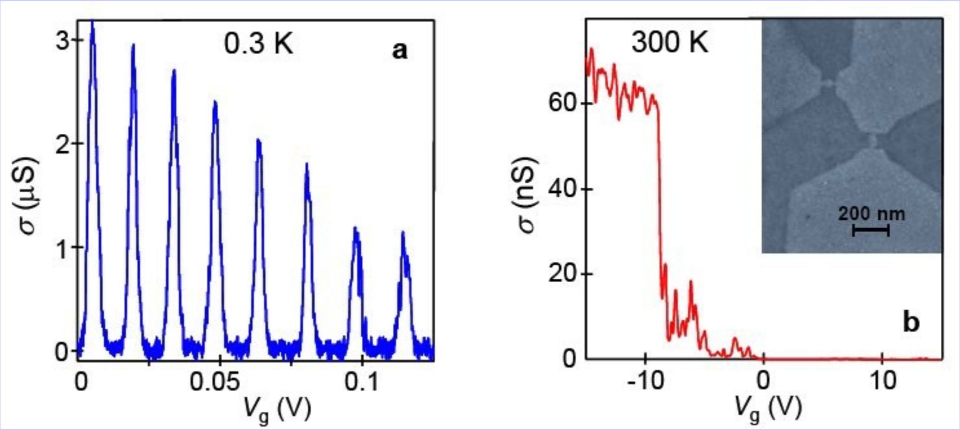


Nanoribbons: quantized $k_y = \pi/\text{width}$

Geometric energy gap $\Delta = hv_F/\text{width}$

Coulomb blockade in graphene

Geim, Novoselov; Ensslin group



Graphene p-n junctions: collimated transmission

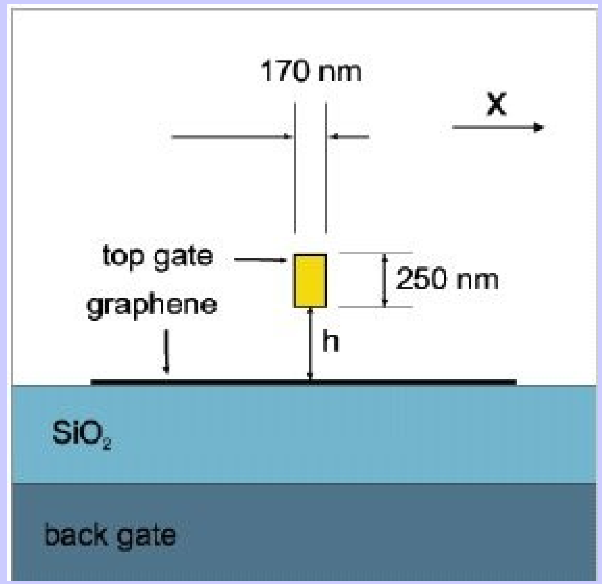
- Ballistic transmission at normal incidence (contrast tunneling in conventional p-n junctions);
- Ohmic conduction (cf. direct/reverse bias asymmetry in conventional p-n junctions)
- No minority/majority carriers

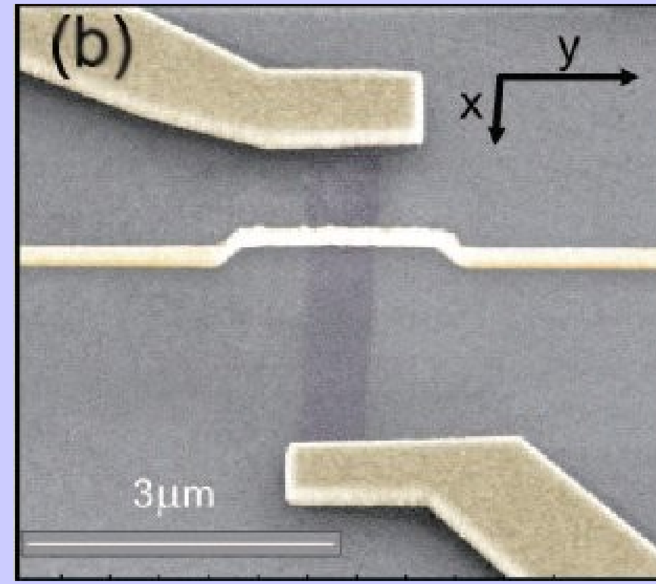
Signatures of collimated transmission in pnp structures

Exeter group:
narrow gate (air bridge)

simulated electrostatic potential, density profile

compare expected and measured resistance, find an excess part

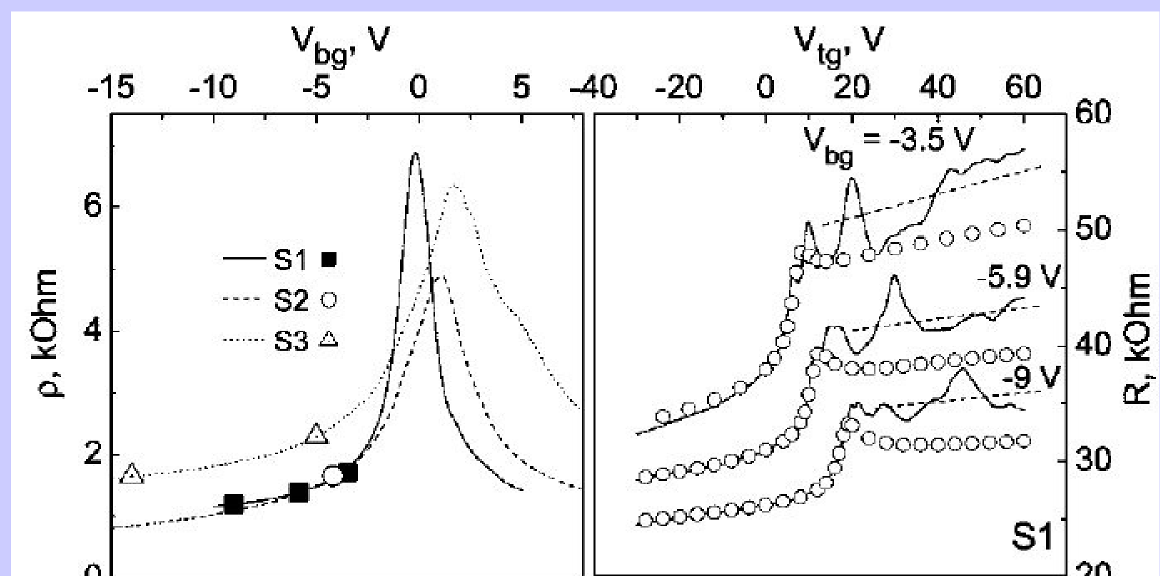




Stanford group:
sharp confining potential
(the top gate ~10 times closer)

analyze the antisymmetric bipolar/unipolar part of resistance

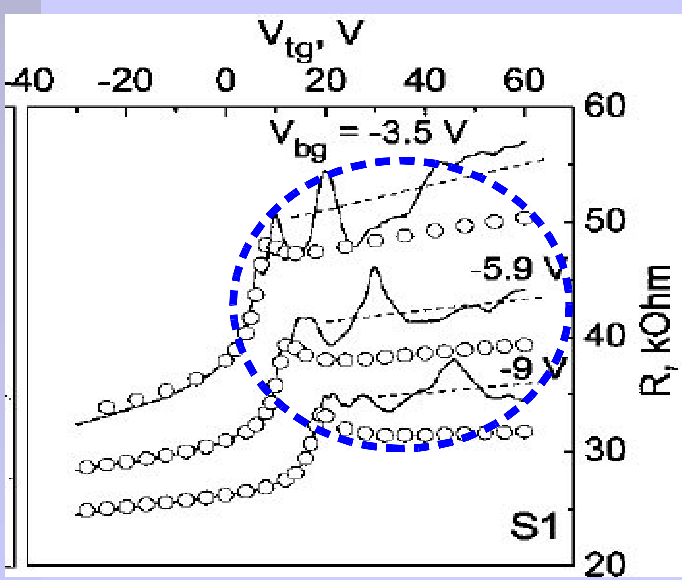
ΔR agrees w. Klein picture, BUT:
a small effect, model-sensitive



Besides collimated transmission, are there any other observable signatures of the Klein physics?

Negative refraction and electron lense (Cheianov, Falko, Altshuler);
Magnetoresistance (Cheianov & Falko)

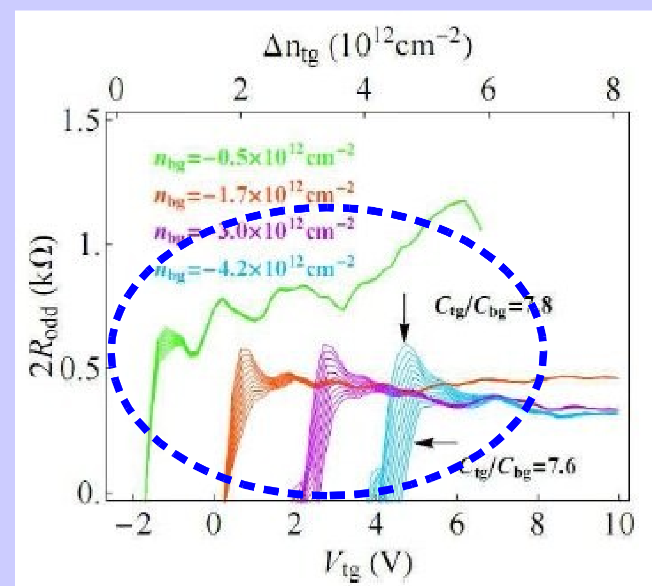
Shytov, Rudner & LL, arXiv:0808.0488



Fabry-Perot resonances mixed with UCF?

Exeter group

Stanford group

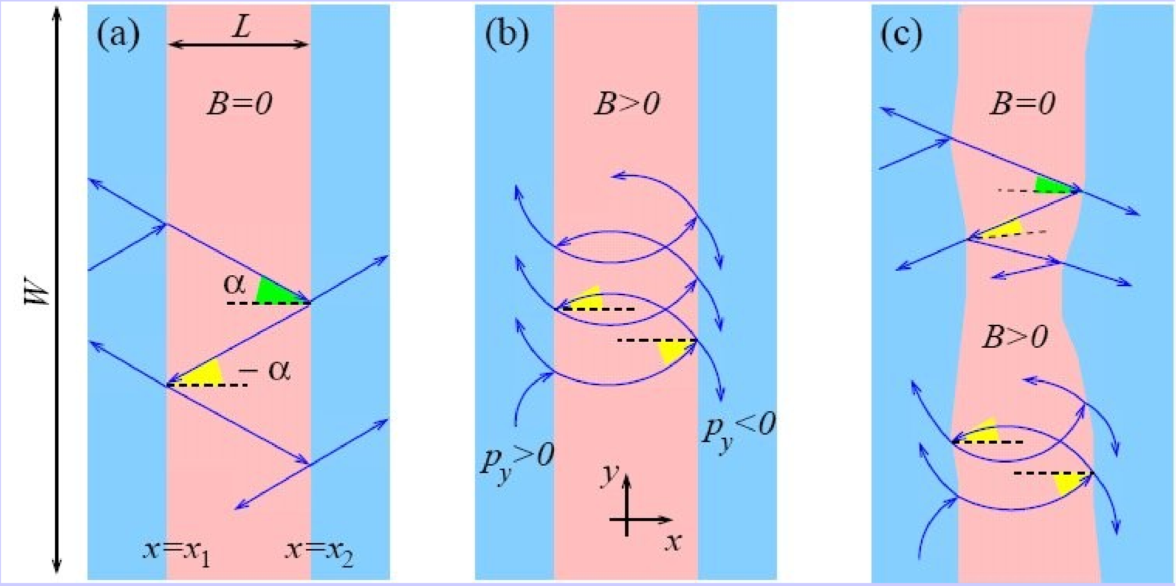


Klein backscattering and Fabry-Perot resonances

Momentum-conserving tunneling, no disorder

$$T(\varepsilon, p_y) = \frac{t_1 t_2}{\left|1 - \sqrt{r_1 r_2} e^{i\Delta\theta}\right|^2}$$

$r_{1(2)} = 1 - t_{1(2)}$ reflection coefficients,



$$\Delta\theta = 2\theta_{\mathrm{WKB}} + \Delta\theta_1 + \Delta\theta_2,$$

$$\theta_{\mathrm{WKB}} = \frac{1}{\hbar}\int_1^2 p_x(x')dx'$$

$\Delta\theta_{1(2)}$ the backreflection phases

Phase of backreflection:

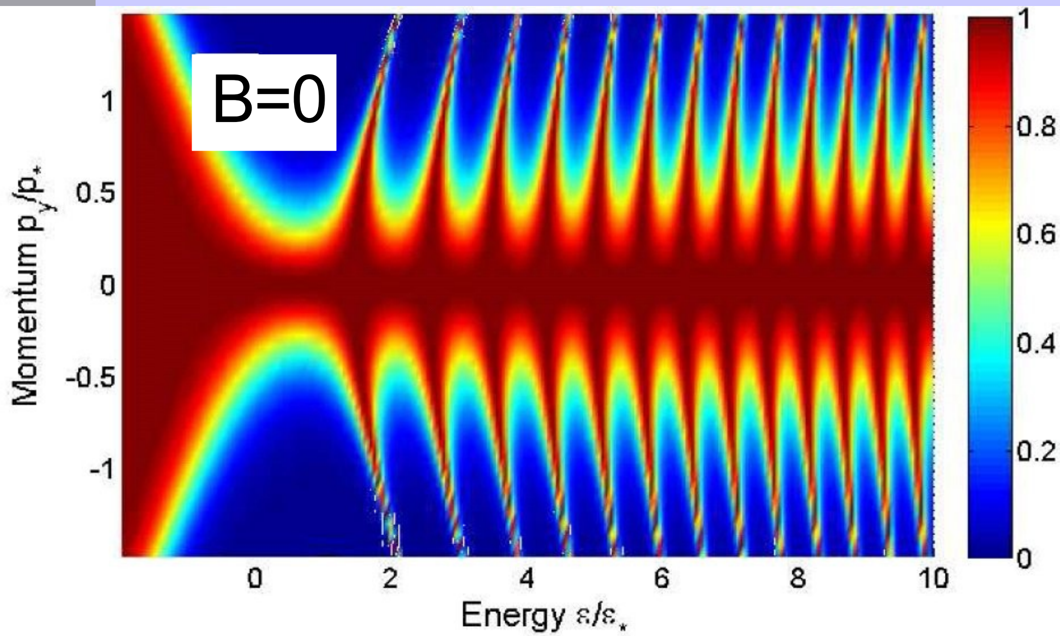
(i) phase jump by π at normal incidence shows up in FP interference;
(ii) the net FP phase depends on the sign of inner incidence angles;
(iii) CAN BE ALTERED by B field

$$p_y(x) = p_{y,0} - eBx,$$

$$-eBL/2 < p_{y,0} < eBL/2$$

$p_y(x_1) > 0$ and $p_y(x_2) < 0$

Transmission at B=0 and B>0



Top-gate potential;
Dirac hamiltonian

$$U(x) = ax^2 - \varepsilon,$$

$$\text{p-n interfaces at } x = \pm x_\varepsilon, \quad x_\varepsilon \equiv \sqrt{\varepsilon/a}$$

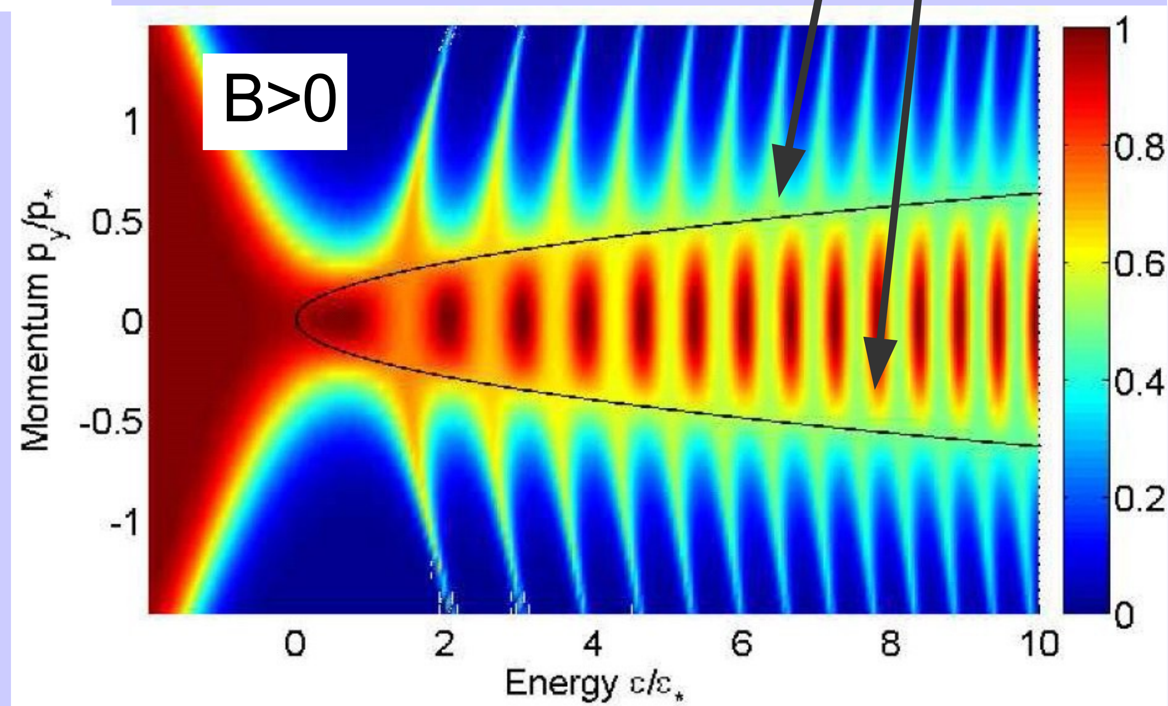
$$\mathcal{H} = v_F\sigma_3 p_x + v_F\sigma_2(p_y - eBx) + U(x)$$

Reversal of fringe contrast on the lines $p_y = \pm eB\sqrt{\varepsilon/a}$

Interpretation of scattering problem: fictitious time t=x; repeated Landau-Zener transitions; Stuckelberg oscillations

$$i\partial_x\psi = (U(x)\sigma_3 - i(p_y - eBx)\sigma_1)\,\psi,$$

$$\Delta\theta = -2\int_{-x_\varepsilon}^{x_\varepsilon} U(x)dx = \tfrac{4}{3}\varepsilon x_\varepsilon$$



Quasiclassical analysis

Confining potential and Dirac hamiltonian

$$U(x) = ax^2 - \varepsilon,$$

$$\text{p-n interfaces at } x = \pm x_\varepsilon, \quad x_\varepsilon \equiv \sqrt{\varepsilon/a}$$

$$\mathcal{H} = v_F \sigma_3 p_x + v_F \sigma_2 (p_y - eBx) + U(x)$$

WKB wavefunction

$$\psi \sim \frac{e^{\pm i \int^x p_x(x')dx'}}{\sqrt{2}|U(x)|} \begin{pmatrix} -U(x) \\ \tilde{p}_y(x) \pm i p_x(x) \end{pmatrix}$$

$$p_x(x) = \sqrt{U^2(x) - \tilde{p}_y^2(x)}, \; \tilde{p}_y(x) \equiv p_y - eBx$$

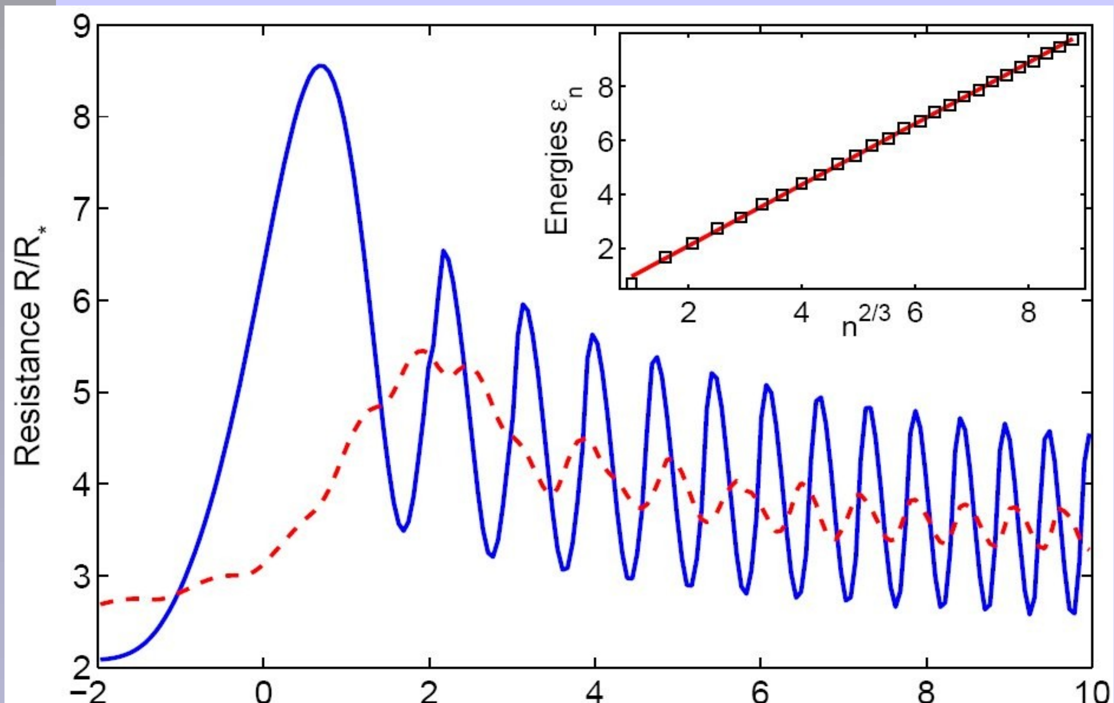
Transmission and reflection amplitudes

$$t_1 = e^{-2\mathrm{Im} \int_{x_1}^{x_1'} p_x(x')dx'} \approx e^{-\lambda(p - eBx_\varepsilon)^2}, \quad \lambda = \frac{\pi}{2ax_\varepsilon}$$

Sign change (phase jump)

$$\mathrm{sgn}(p \pm eBx_\varepsilon)\, e^{i\theta_{\mathrm{reg}}(p)} \sqrt{1 - e^{-\lambda(p \pm eBx_\varepsilon)^2}}$$

FP contrast in conductance



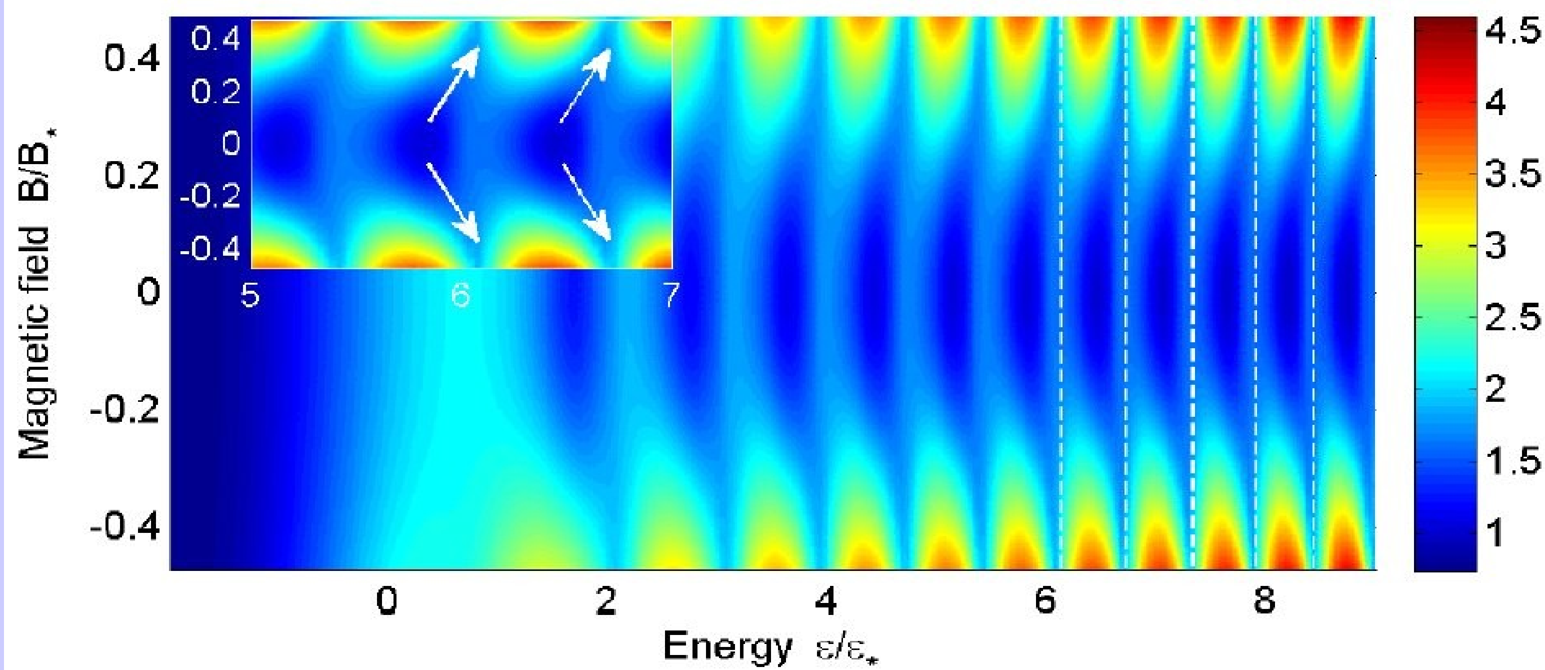
Landauer formula:

$$R(\varepsilon) = G^{-1}, \quad G = \frac{4e^2}{h} W \int_{-\infty}^{\infty} T(\varepsilon, p_y) \frac{dp_y}{2\pi}$$

Signature of π:
Half-a-period phase shift induced by magnetic field

MR same as in Cheianov, Falko

FP phase contrast not washed out after integration over p_y

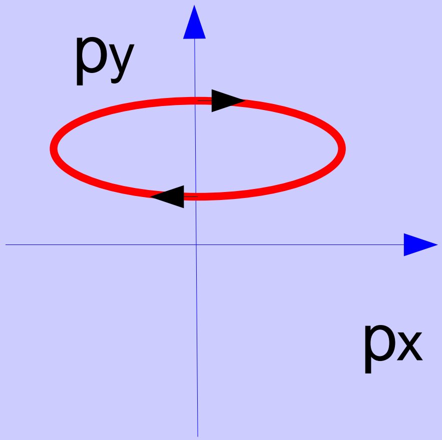


Interpretation of the π-shift as a Berry's phase

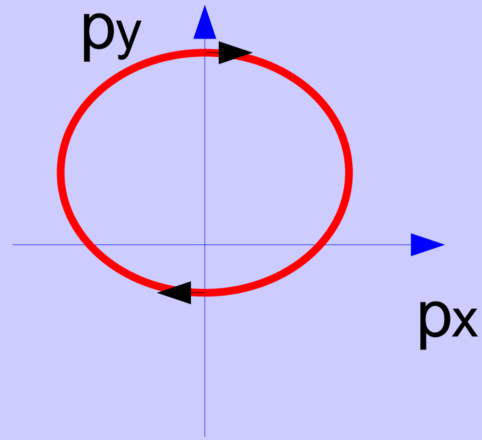
Trajectory in momentum space yields
an effective time-dependent “Zeeman” field

$$H = v\, \sigma . p(t)$$

Weak B:
zero not enclosed, $\Delta\theta = 0$



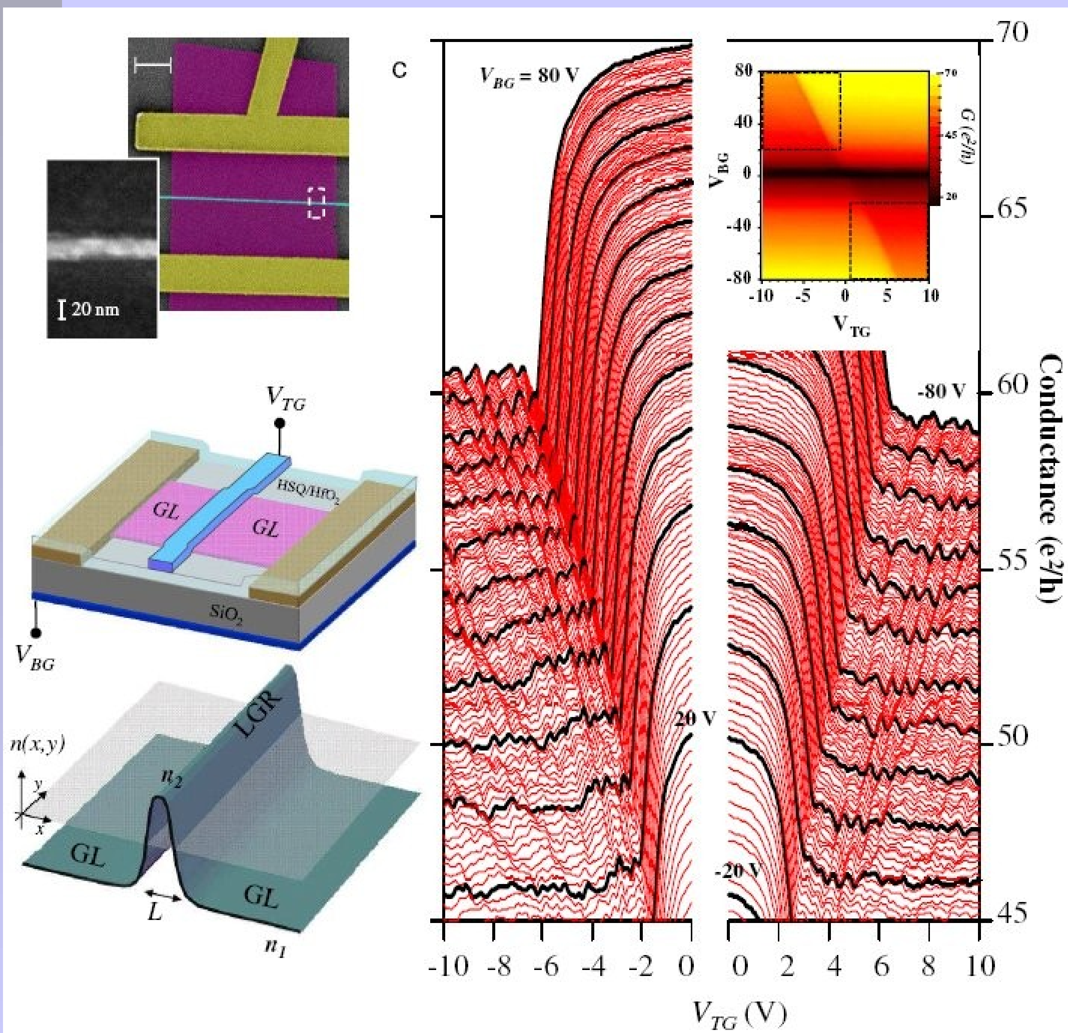
Strong B:
zero enclosed, $\Delta\theta = \pi$

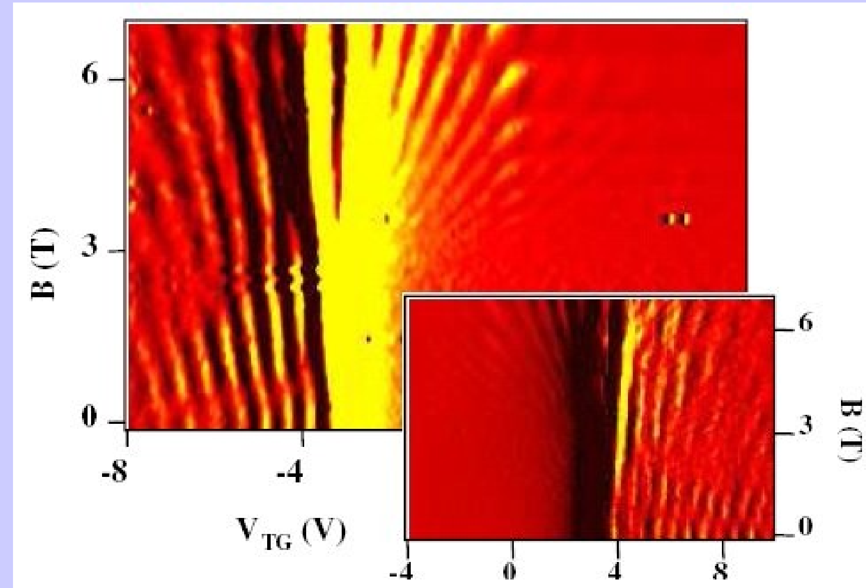


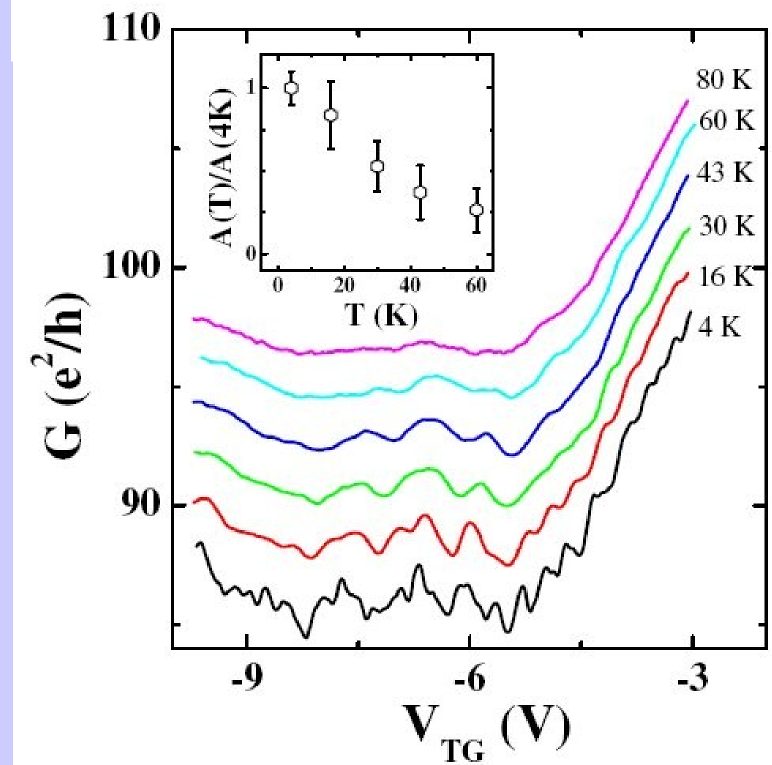
Berry's phase must be added to the WKB phase

FP oscillations (experiment)

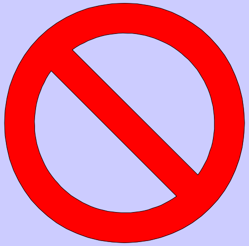
Columbia group (2008):
FP resonances in zero B; crossover to Shubnikov-deHaas oscillations at B>1T







Scattering on disorder: Shubnikov - de Haas effect in a p-n-p structure

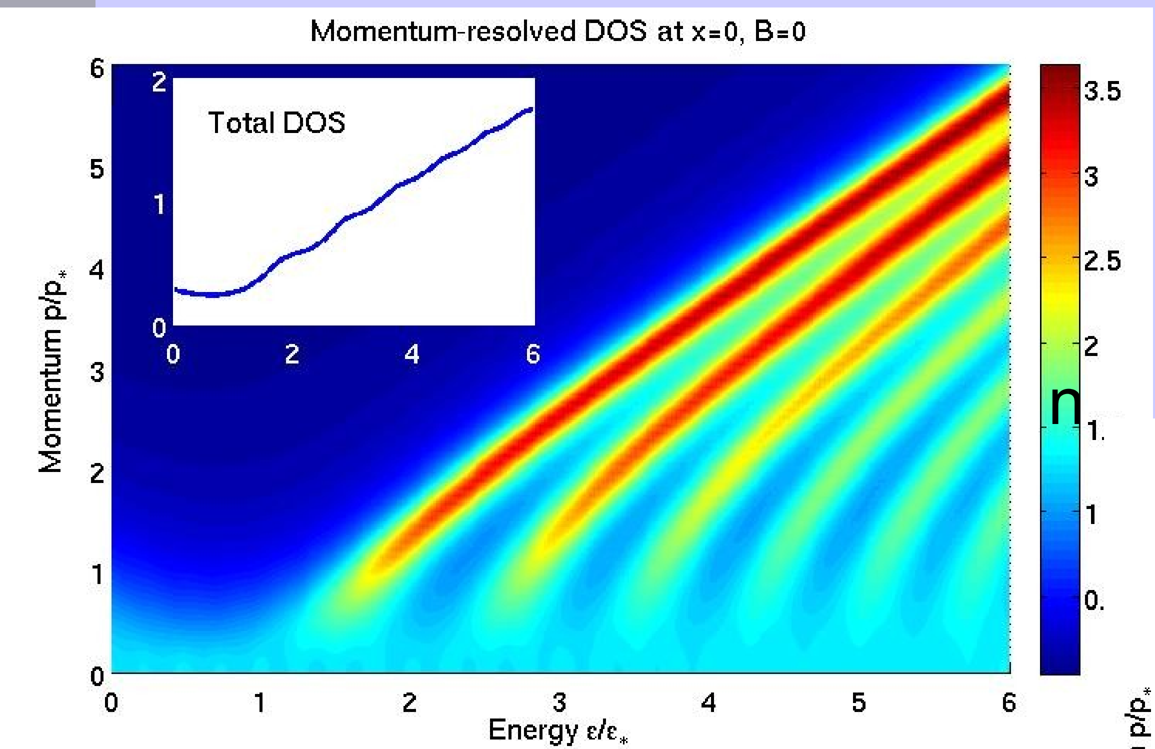


Men working!

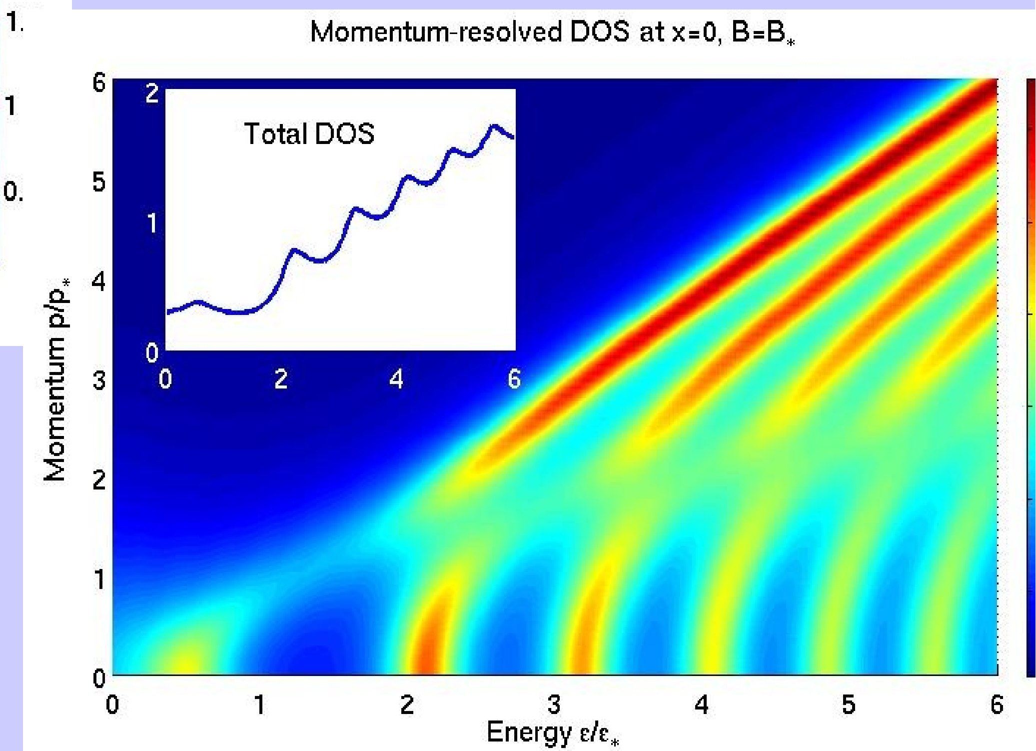
Oscillations: LDOS, impurity scattering, conductance

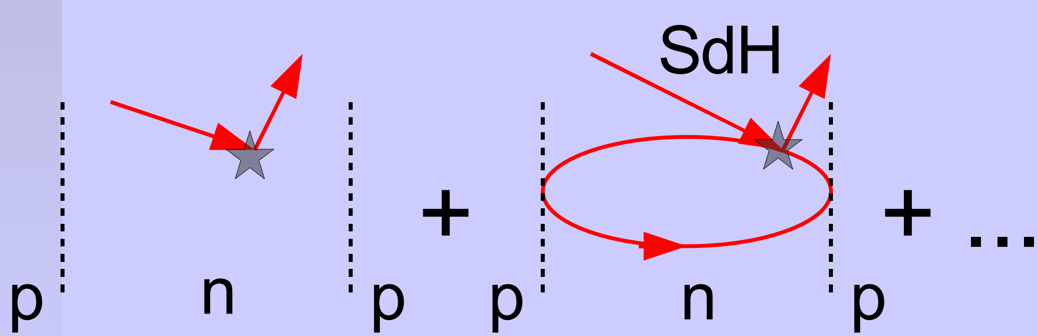
Density of states (momentum-resolved) within the p-n-p structure

B=0

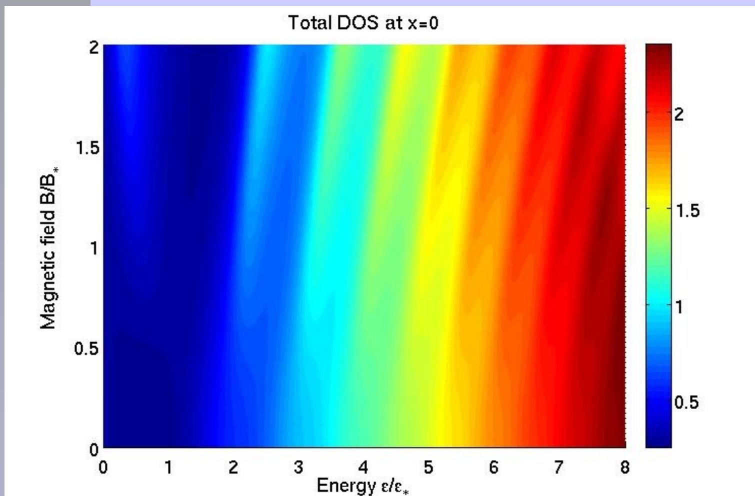


B>0

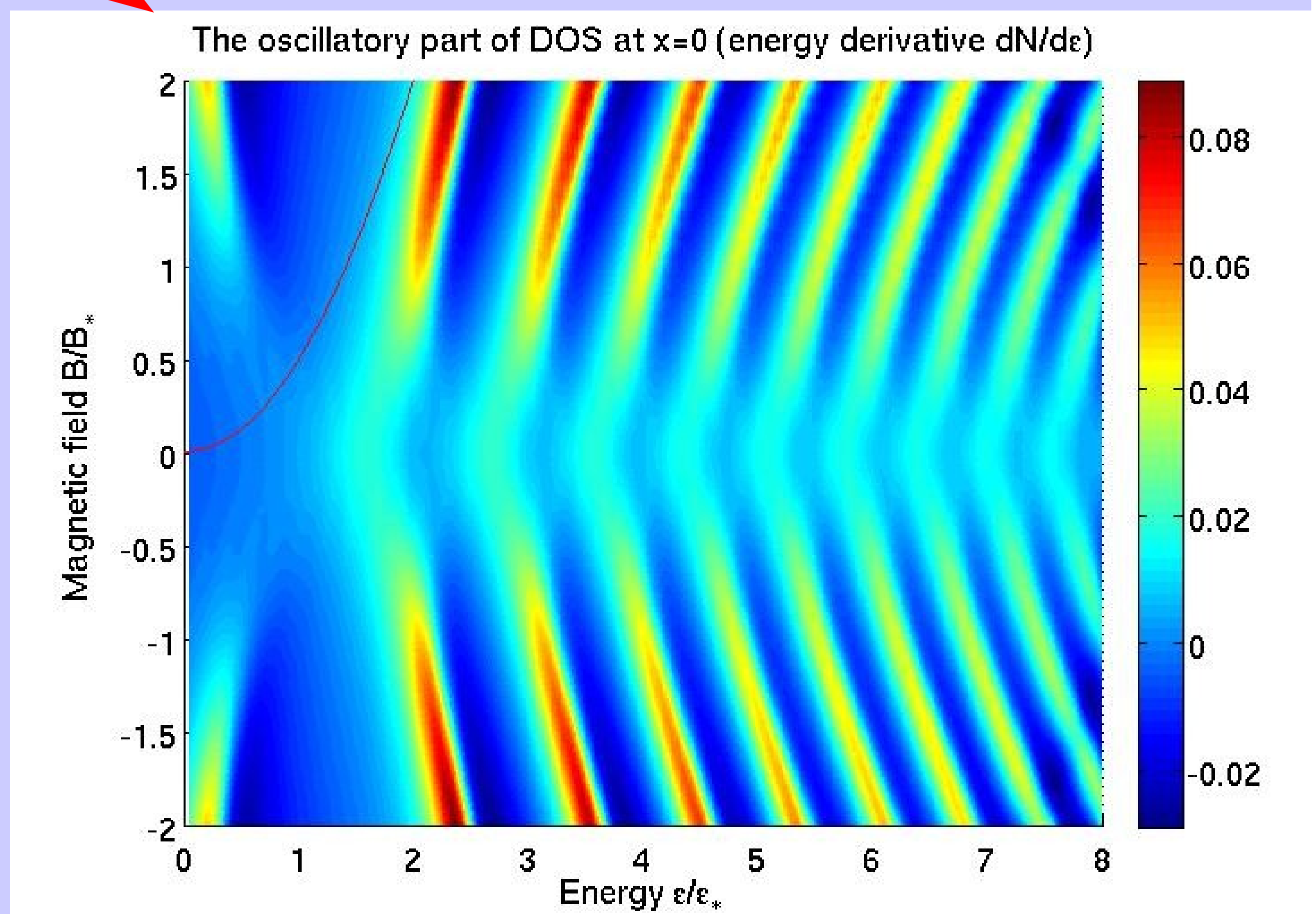




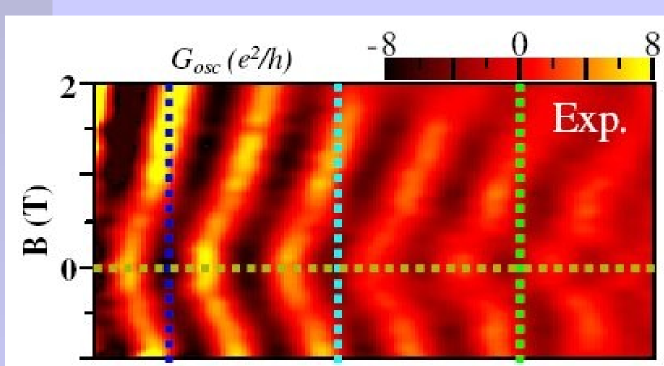
Total density of states



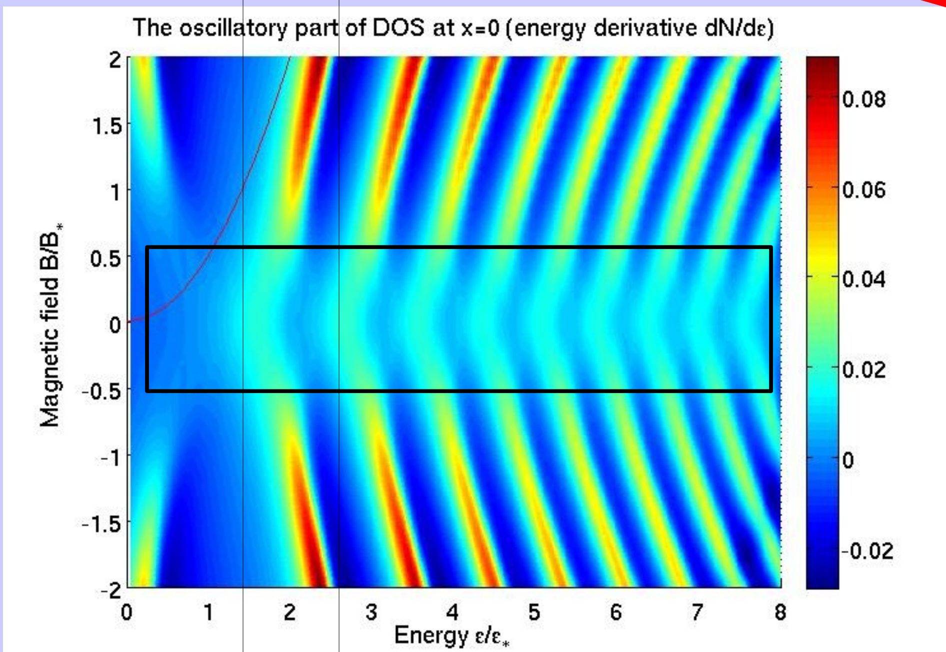
Energy-derivative

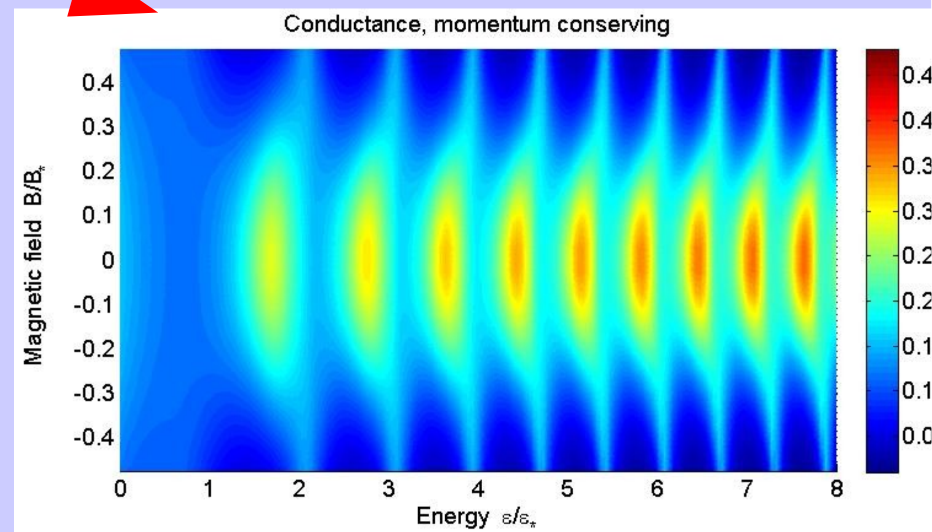


AGREES WITH EXPERIMENT?

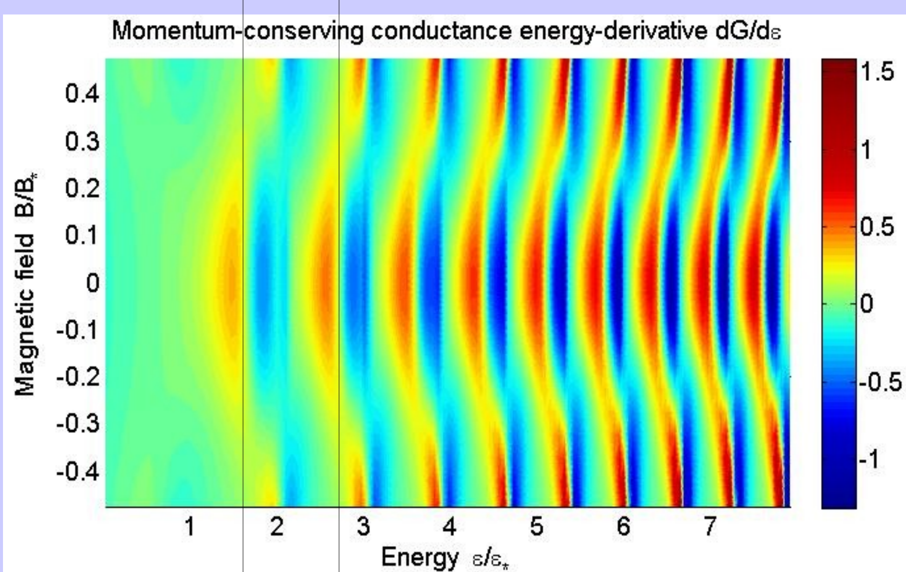


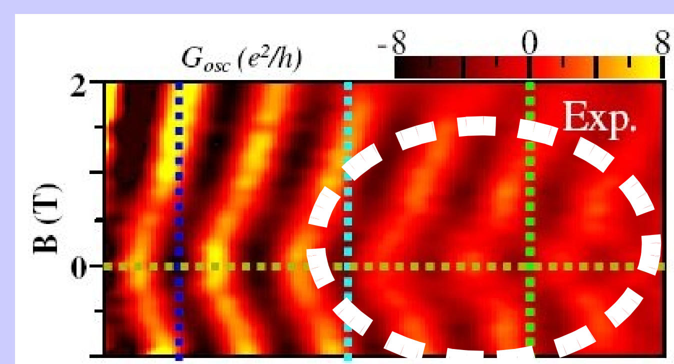
Adding momentum-conserving contribution to SdH conductance





Energy-derivative





Part II

Lorentz boost and magnetoresistance of p-n junctions

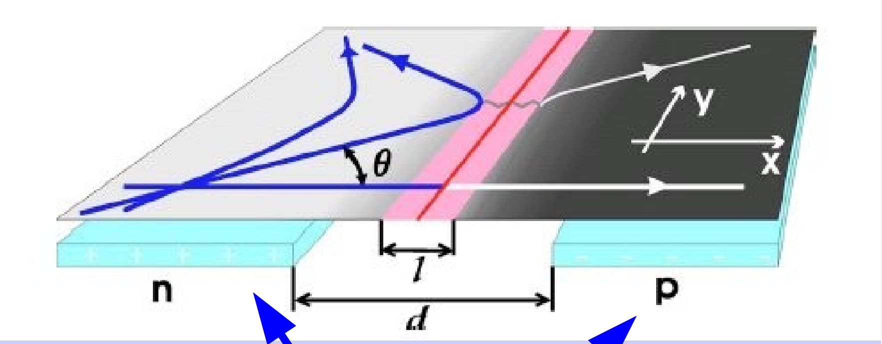
Electron in a single p-n junction

Potential step instead of a barrier (smooth or sharp)

Cheianov, Falko

$$H = e\varphi(\mathbf{x}) + v_F \xi \begin{pmatrix} 0 & p_+ \\ p_- & 0 \end{pmatrix}, \quad p_\pm = p_1 \pm i p_2,$$

p-n junction schematic:



+1(-1) for points K(K')

gates

smooth step:

$$w(\theta) = e^{-\pi(k_F d)\sin^2\theta}.$$

(nontrivial)

sharp step:

$$w_{\text{step}}(\theta) = \cos^2\theta$$

(straightforward)

In both cases, perfect transmission in the forward direction: manifestation of chiral dynamics

Exact solution in a uniform electric field ("Landau-Zener")

Use momentum representation (direct access to asymptotic plane wave scattering states)

Evolution in a fictitious time with a hermitian 2x2 Hamiltonian

$$-ieE\,d\psi/dp_2 = \tilde{H}\psi, \quad \tilde{H} = v_F(p_1\sigma_1 - p_2\sigma_2) - \varepsilon.$$

Equivalent to Landau-Zener transition at an avoided level crossing;
Interpretation: interband tunneling for $p_2(t)=vt$

Transmission equals to the LZ probability of staying in the diabatic state:

$$T(p_1) = \exp(-\pi\hbar v_F p_1^2/|eE|),$$

Exact transmission matches the WKB result

Single p-n junction in B field

Recall relativistic motion in crossed E, B fields

Andrei Shytov, Nan Gu & LL

Two regimes:

Lorentz invariants $\mathbf{E}^2 - \mathbf{B}^2$, $\mathbf{E}.\mathbf{B}$

(i) electric case E>B ("parabolic" trajectories)
(ii) magnetic case B>E (cyclotron motion + drift)

Analogous regimes in graphene p-n junction:

Dirac equation (4) in a Lorentz-invariant form

$$\gamma^\mu (p_\mu - a_\mu) \psi = 0, \quad \{\gamma_\mu, \gamma_\nu\}_+ = 2g_{\mu\nu}, \qquad (7)$$

where γ^μ are Dirac gamma-matrices, $\gamma^0 = \sigma_3$, $\gamma^1 = -i\sigma_2$, $\gamma^2 = -i\sigma_1$, and ψ is a two-component wave function.

$$a_0 = -\frac{e}{v_F} E y, \quad a_1 = -\frac{e}{c} B y, \quad a_2 = 0.$$

$c/v_F = 300$

Electric regime (scattering T-matrix, G>0) $\quad B < (c/v_F)E,$

Magnetic regime (Quantum Hall Effect, G=0) $\quad B > (c/v_F)E$

Lorentz transformation

Electric regime B<B$_*$, critical field $B = B_* \equiv (c/v_F)E.$

Eliminate B using Lorentz boost: $$\Lambda = \begin{pmatrix} \gamma & \gamma\beta & 0 \\ \gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Aronov, Pikus 1967

Transmission coefficient is Lorentz invariant:

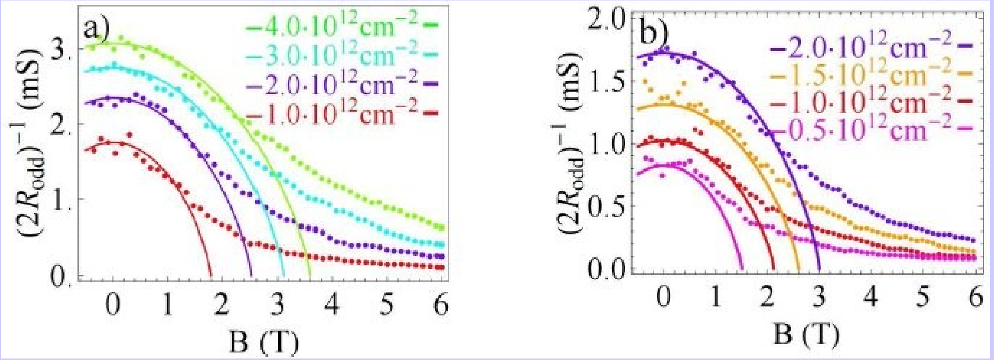
$$T(p_1) = e^{-\pi\gamma^3 d^2 (p_1 + \beta\varepsilon)^2}, \quad d = (\hbar v_F/|eE|)^{1/2}.$$

experiment in Stanford:

Net conductance (Landauer formula)

$$G = \frac{e^2}{h} \sum_{-k_F<p_1<k_F} T(p_1) = \frac{we^2}{2\pi h}\int_{-k_F}^{k_F} T(p_1)dp_1$$

$$G(B \leq B_*) = \frac{e^2}{2\pi h}\frac{w}{d}\left(1-(B/B_*)^2\right)^{3/4}$$



Suppression of G in the electric regime precedes the formation of Landau levels and edge states at p-n interface

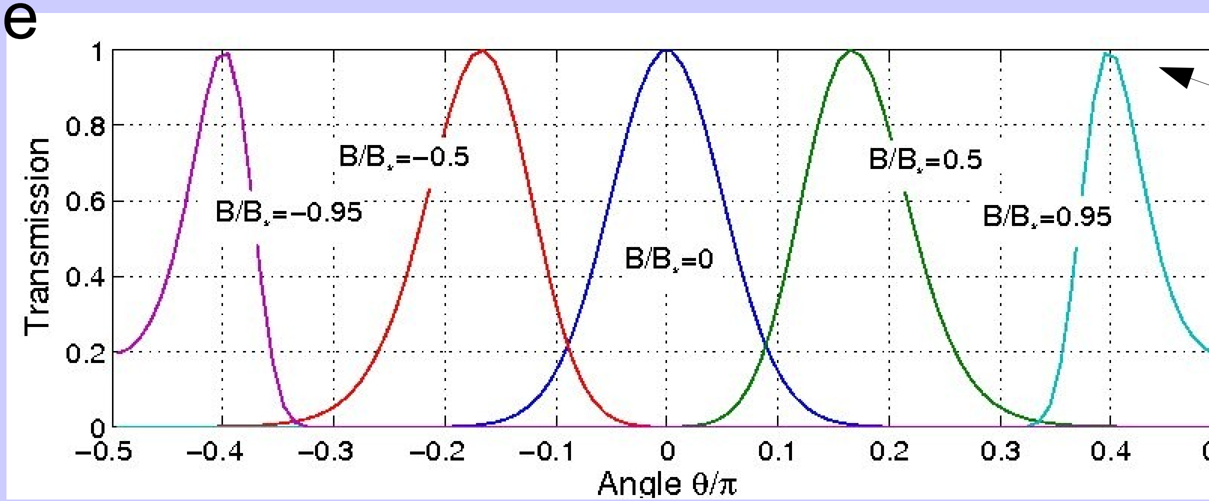
At larger *B*: no bulk transport, only edge transport

Collimated transmission for subcritical B

Electric regime B<B*

- Perfect transmission at a finite angle

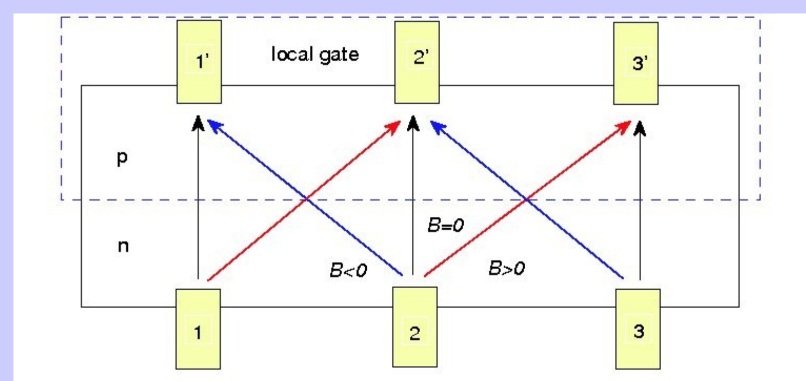
$$\theta_B = \arcsin B/B_*$$



T=1

- Collimation angle reduced by Lorentz contraction

- Current switch controled by B



Mapping to the Landau-Zener transition problem

Quasiclassical WKB analysis

$$i\partial_x\psi(x) = ((\varepsilon + ax)\sigma_2 + i(p_1 + bx)\sigma_3)\,\psi(x).$$

Evolution with a non-hermitian Hamiltonian

Eigenvalues:

$$\kappa(x) \;=\; \sqrt{(\varepsilon + ax)^2 - (p_1 + bx)^2}$$

$$S = 2\int_{x_1}^{x_2} \mathrm{Im}\,\kappa(x)dx = \pi\frac{(p_1 a - \varepsilon b)^2}{(a^2 - b^2)^{3/2}}.$$

$$T(p_1) = \exp(-\pi\hbar v_F p_1^2/|eE|),$$

Exact solution: use momentum representation (gives direct access to asymptotic plane wave scattering states)

$$-ieE\,d\psi/dp_2 = \tilde{H}\psi, \quad \tilde{H} = v_F(p_1\sigma_1 - p_2\sigma_2) - \varepsilon.$$

Equivalent to the Landau-Zener transition
Interpretation: interband tunneling for
$p_2(t)=vt$
L-Z result agrees with WKB

Classical trajectories

a comment by Haldane, 2007

Electron ("comet") orbits the Dirac point ("Sun")

$$\mathcal{H}(p,r) = \epsilon(\mathbf{p}) - eEx, \quad \mathbf{p} = \tilde{\mathbf{p}} - e\mathbf{A}, \quad \mathbf{A} = (0, Bx)$$

$$\text{Energy integral}: \ \epsilon(\mathbf{p}) - \mathbf{v}_D.\mathbf{p} = \epsilon_0, \quad \mathbf{v}_D = \mathbf{E} \times \mathbf{B}/B^2$$

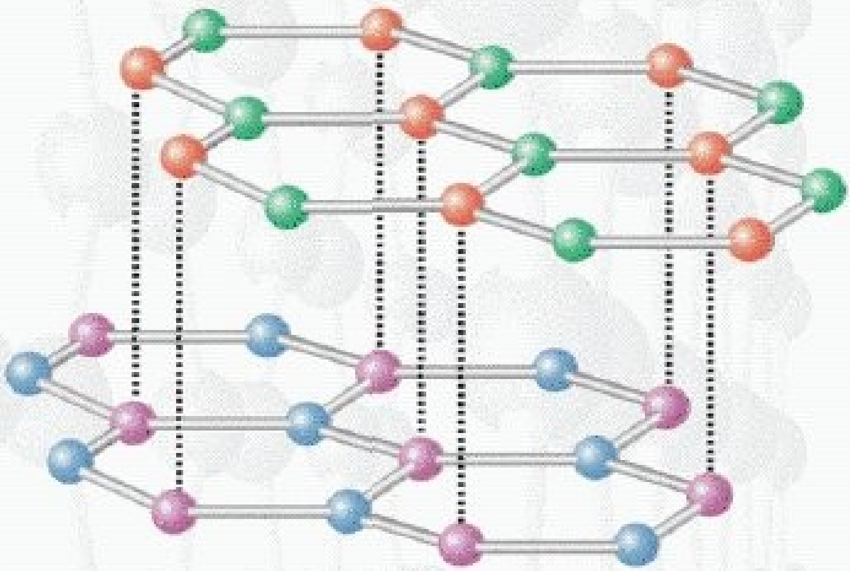
$$\text{Poisson brackets}: [p_1, p_2] = e\hbar B$$

$$\text{Graphene}: \ \epsilon(\mathbf{p}) = v_F|\mathbf{p}|, \quad p(\theta) = \frac{\epsilon_0}{v_F - v_D \cos\theta}$$

Two cases, open and closed orbits:

$$v_D > v_F: \ \text{hyperbola}; \quad v_D < v_F: \ \text{ellipse}$$

Graphene bilayer: electronic structure and QHE

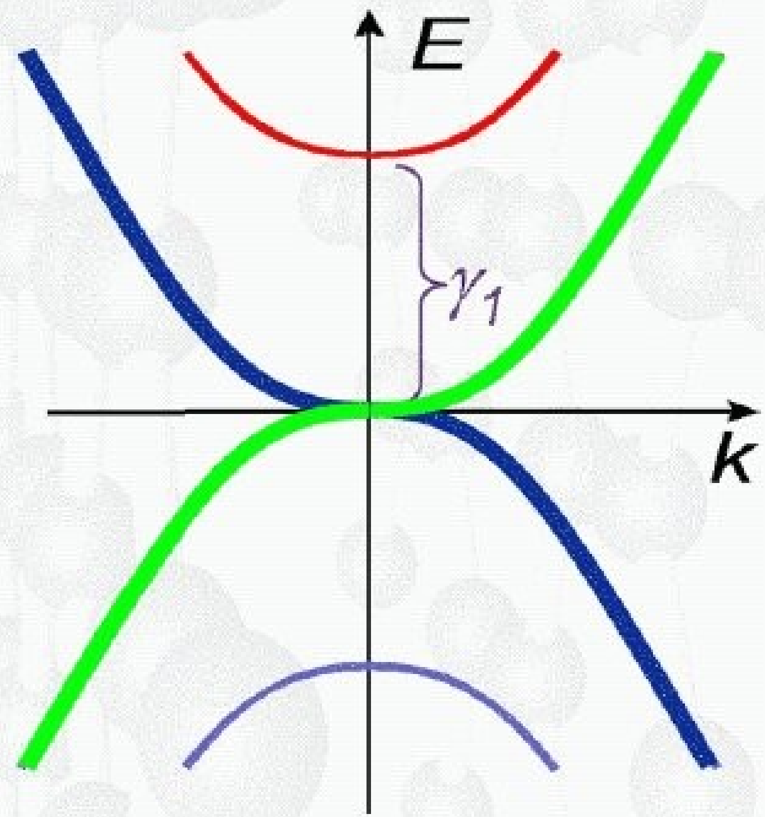


$$E(p) = \pm \tfrac{1}{2}\gamma_1 \pm \sqrt{\tfrac{1}{4}\gamma_1^2 + v_F^2 p^2}$$

$$\hat{H} = -\frac{1}{2m}\begin{pmatrix} 0 & (\hat{p}_x + i\hat{p}_y)^2 \\ (\hat{p}_x - i\hat{p}_y)^2 & 0 \end{pmatrix}$$

$$E_N = \pm\hbar\omega_c\sqrt{N(N-1)}$$

McCann & Falko 2006



p-n junction in graphene bilayer

Bilayer Dirac Hamiltonian with vertical field and interlayer coupling

$$H = v_F p_1 \sigma_1 - v_F p_2 \sigma_2 + \frac{1}{2} u \tau_3 + \frac{\Delta}{2} (\tau_1 \sigma_1 + \tau_2 \sigma_2).$$

Dirac eqn with fictitios pseudospin-dependent gauge field:

$$\gamma^\mu (p_\mu - a_\mu - g_\mu)\psi = 0, \quad g_\mu = \left(\tilde{u}\tau_3, -\tilde{\Delta}\tau_1, \tilde{\Delta}\tau_2 \right)$$

After Lorentz boost (B eliminated):

$$H_k(p_1', p_2') = \frac{1}{2}\gamma \left(u\tau_3 - \beta\Delta\tau_1 \right) +$$
$$\left(v_F p_1' - \frac{1}{2}\gamma(\beta u \tau_3 - \Delta\tau_1) \right) \sigma_1 - \left(v_F p_2' - \frac{1}{2}\Delta\tau_2 \right) \sigma_2.$$

Transmission characteristics

4x4 transfer matrix in momentum space (effectively 2x2)

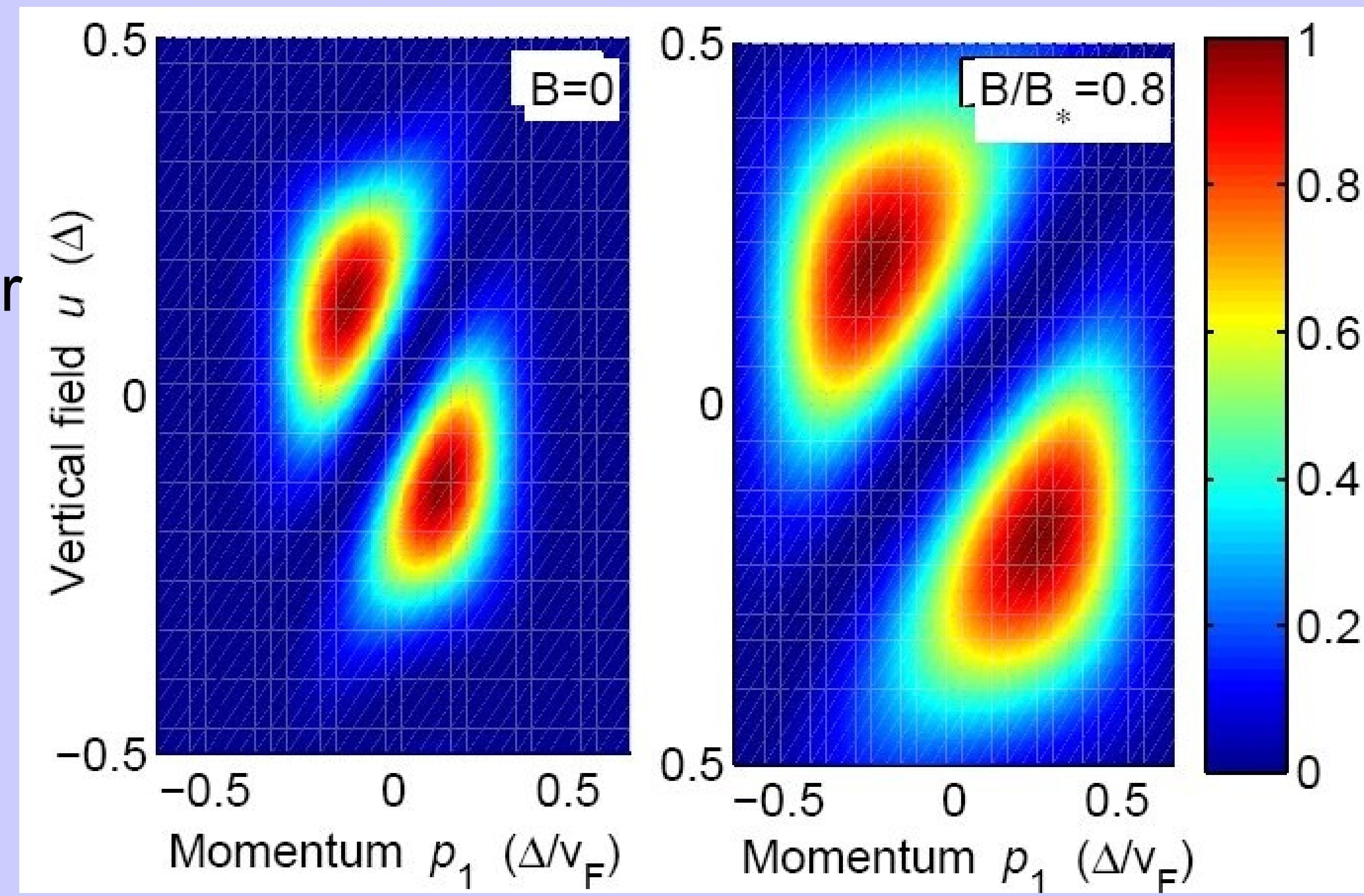
$$ieE'\,d\psi/dp_2' = (H_k(p_1', p_2') - \varepsilon')\,\psi$$

Gapped spectrum at finite vertical field

Zero transmission near *u*=0 --- tunable!

Perfect transmission for certain *u* and *p*

Tunneling at small *p* suppressed by *B* field



Transport in pn junctions, Manifestations of relativistic Dirac physics:

- Klein backreflection contributes a π phase to interference ;
- Bilayers: a 2π phase;
- Half a period phase shift a hallmark of Klein scattering
- electric and magnetic regimes B<300E and B>300E ($300=c/v_F$)
- Consistent with FP oscillations and magnetoresistance of existing p-n junctions

:0 ?

:) !

Also: a momentum-conserving contribution to conductance