

EFFECT OF CHANNEL ESTIMATION ERROR ON THE PERFORMANCE OF SVD-BASED MIMO COMMUNICATION SYSTEMS

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Abstract - Recent information-theoretic work has shown the potential capacity increase of Multiple-input Multiple-output (MIMO) systems compared to Single-input Single-output (SISO) systems [1] [2]. The Singular Value Decomposition (SVD) has proven very useful in MIMO communication systems but its performance depends on the accuracy of the Channel State Information (CSI) at both sides of the link. This paper presents a study on the influence of channel estimation errors and noise on the probability of error of the system when no error correction mechanisms are provided.

Keywords - MIMO, SVD, Singular Value Decomposition, Wireless Communication Systems.

I. INTRODUCTION AND NOTATION

A linear Multiple-input Multiple-output (MIMO) system with N_t inputs (transmitters), $\mathbf{x} = (x_1 \dots x_{N_t})^T$, and N_r outputs (receivers), $\mathbf{y} = (y_1 \dots y_{N_r})^T$, can be characterized by the following expression

$$\begin{pmatrix} y_1 \\ \vdots \\ y_{N_r} \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_1 \\ \vdots \\ x_{N_t} \end{pmatrix} + \begin{pmatrix} n_1 \\ \vdots \\ n_{N_r} \end{pmatrix} \quad (1)$$

where $\mathbf{n} = (n_1 \dots n_{N_r})^T$ are noise samples and the channel matrix \mathbf{A} can be either constant, in the case of a frequency non-selective channel, or a function in the frequency domain, in the case of a frequency-selective channel.

In the following analysis, we will consider that the number of receive antennas is at least the number of transmit antennas, so that, in the case where the channel matrix is full rank, all the eigenmodes can be decoded.

The Singular Value Decomposition (SVD) of a rank- r matrix \mathbf{A} is $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, where \mathbf{U} and \mathbf{V} are unitary matrices, $(\cdot)^H$ denotes conjugate transpose and $\mathbf{\Sigma} = \text{diag}(\sigma_1 \dots \sigma_r)$, where $\sigma_1 \geq \dots \geq \sigma_r > 0$ are the channel singular values [3]. In the case of a frequency selective channel, they would be defined for every frequency. The importance of SVD in MIMO communication systems relies on the fact that the presteering matrix \mathbf{V} at the transmitter and the steering matrix \mathbf{U}^H at the receiver decompose the MIMO channel into r SISO orthogonal

modes of excitation or eigenchannels [4].

$$\mathbf{U}^H \mathbf{y} = \mathbf{U}^H \mathbf{A} \mathbf{V} \mathbf{x} + \mathbf{U}^H \mathbf{n} = \mathbf{\Sigma} \mathbf{x} + \mathbf{U}^H \mathbf{n} \quad (2)$$

Let us suppose that the estimate of the channel matrix is slightly different from the actual channel matrix by an amount defined as an additive perturbation matrix \mathbf{E} ; that is, the estimated channel matrix is $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{E}$. This estimation error can be the result of both quantization error and time variation of the channel. We want the influence of this error matrix to be low enough not to change the decisions of the decoder. To this end, we will consider an 8-PSK constellation and we will ignore the influence of coding. In this situation,

$$\hat{\mathbf{X}} = \mathbf{\Sigma}^{-1} \mathbf{U}^H (\tilde{\mathbf{A}} \tilde{\mathbf{X}} + \mathbf{N}), \quad \tilde{\mathbf{X}} = \mathbf{V} \mathbf{X} \quad (3)$$

Therefore, the error function for each component, $\Upsilon = \hat{x}_i - x_i$, can be expressed as

$$\Upsilon = \frac{1}{\sigma_i} \mathbf{u}_i^H \begin{pmatrix} \uparrow & & \uparrow \\ \mathbf{e}_1 & \dots & \mathbf{e}_{N_t} \\ \downarrow & & \downarrow \end{pmatrix} \begin{pmatrix} \uparrow & & \uparrow \\ \mathbf{v}_1 & \dots & \mathbf{v}_{N_t} \\ \downarrow & & \downarrow \end{pmatrix} \mathbf{x} + \frac{\hat{n}_{ii}}{\sigma_i} \quad (4)$$

where $\hat{\mathbf{n}} = \mathbf{U}^H \mathbf{n}$ has the same statistic properties as \mathbf{n} for \mathbf{U} being a unitary matrix. Let us define $\epsilon_i = \mathbf{u}_i^H \mathbf{E}$. Then

$$\Upsilon = \frac{1}{\sigma_i} (\epsilon_i \mathbf{V} \mathbf{x} + \hat{n}_{ii})$$

We will not have a decoding error if the error component Υ makes \hat{x}_i remain in the region assigned to x_i by the decoder (see Figure 1). Without loss of generality we can assume that $x_i = 1$. Then, we can express this condition mathematically in the following way

$$\begin{cases} \langle \Upsilon, S_1 \rangle < \sin(\frac{\pi}{8}) \\ \langle \Upsilon, S_2 \rangle < \sin(\frac{\pi}{8}) \end{cases} \quad (5)$$

$$S_1 = (\cos(\frac{5\pi}{8}), \sin(\frac{5\pi}{8})) = (-\sin(\frac{\pi}{8}), \cos(\frac{\pi}{8}))$$

$$S_2 = (\cos(\frac{-5\pi}{8}), \sin(\frac{-5\pi}{8})) = (-\sin(\frac{\pi}{8}), -\cos(\frac{\pi}{8}))$$

where the function $\langle \cdot, \cdot \rangle$ is the usual \mathbb{R}^2 -scalar product, $\langle a, b \rangle = \sum a_i b_i$. If a or b are complex variables, they will be substituted by their image in the real space, via the canonical isometry.

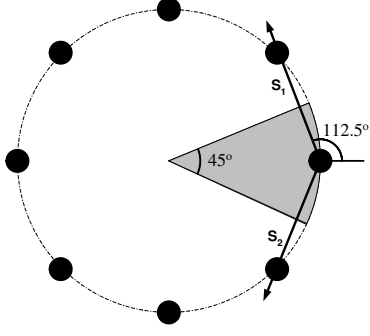


Fig. 1

It is not possible to decouple the influence of channel variation and that of the noise nor treat them jointly unless we do some assumptions or assign values to the parameters in the system. Therefore, a study of the two extreme cases, i.e., the case when the influence of the noise term is dominant and the case when its influence is negligible, will be carried out. Later on, a joint study will be done under certain assumptions.

II. THE NOISE TERM IS DOMINANT

In this case

$$\Upsilon = \hat{x}_i - x_i = \frac{\hat{n}_{ii}}{\sigma_i} \quad (6)$$

and we must guarantee Equation (5). If we define the signal to noise ratio of the i th channel as $SNR_i = \mathbf{E}\left(\frac{\sigma_i^2}{\hat{n}_{ii}^2}\right) = \frac{\sigma_i^2}{\sigma_n^2}$, the performance of the channel will be good if

$$\frac{1}{\sqrt{SNR_i}} \max \left(\langle \alpha, S_1 \rangle, \langle \alpha, S_2 \rangle \right) < \sin\left(\frac{\pi}{8}\right) \quad (7)$$

where $\alpha = \alpha_R + i\alpha_I$ is a sample from a circularly symmetric complex Gaussian random variable with variance 1. The above inequality can be rewritten as

$$\Theta = -\alpha_R + \max \left(\pm \alpha_I \cot\left(\frac{\pi}{8}\right) \right) < \sqrt{SNR_i} \quad (8)$$

By means of simulation, the distribution of Θ has been found. The probability of symbol error is the probability that $\Theta < \frac{\sigma_i}{\sigma_n}$. Figure 2 shows the probability of error as a function of $\frac{\sigma_i}{\sigma_n}$ in semilogarithmic axis.

In Table 1, the necessary signal to noise ratios for probabilities of error comprised in $[10^{-6}, 10^{-1}]$ are given. Besides, a third order interpolating polynomial has been used to extrapolate the values that would be needed for higher reliability.

III. THE ESTIMATION ERROR IS DOMINANT

In this case,

$$\Upsilon = \frac{1}{\sigma_i} (\epsilon_i \mathbf{V} \mathbf{x}) \quad (9)$$

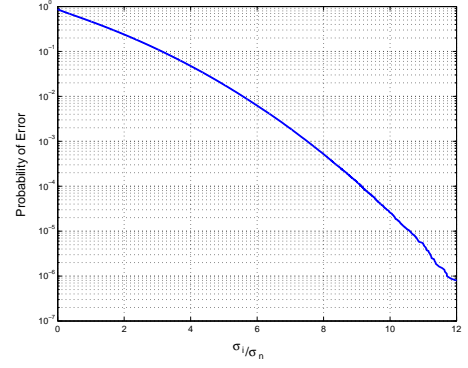


Fig. 2

Table 1

Required values of SNR_i for some error probabilities.

\tilde{P}	$SNR_i (dB)$
10^{-1}	9.9341
10^{-2}	14.928
10^{-3}	17.504
10^{-4}	19.188
10^{-5}	20.509
10^{-6}	21.428
10^{-7}	22.051
10^{-8}	22.692
10^{-9}	23.242
10^{-10}	23.723

Since \mathbf{V}^H is a unitary matrix, its columns form a basis of \mathbb{C}^{N_t} so we can write $\epsilon_i = \delta_1 v_1^H + \delta_2 v_2^H + \dots + \delta_{N_t} v_{N_t}^H$.

$$\Upsilon = \frac{1}{\sigma_i} \delta \mathbf{x} = \frac{1}{\sigma_i} (\delta_1 x_1 + \dots + \delta_{N_t} x_{N_t}) \quad (10)$$

A. General approach

Let us study necessary conditions for the first inequality in Equation (5) to hold. Since $x_j, j \neq i$ are independent from x_i and the elements δ_k are also out of our control, one possible approach is to find an upper bound over all the possible combinations of those parameters for a given length $\|\epsilon_i\| = \|\delta\| = K$ of the vector ϵ_i .

$$\max_{\|\epsilon_i\|=K} \langle \Upsilon, S_1 \rangle = \max_{\|\epsilon_i\|=K} \left\langle \frac{1}{\sigma_i} \delta \mathbf{x}, S_1 \right\rangle = \quad (11)$$

$$\max_{\delta \in S^{N_t-1}(\mathbb{C})} \frac{K}{\sigma_i} \langle \tilde{\delta} \mathbf{x}, (-\sin(\frac{\pi}{8}), \cos(\frac{\pi}{8})) \rangle$$

By using Lagrange multipliers it is easy to show that the result of this maximization problem is $\frac{K}{\sigma_i} \sqrt{N_t}$. So, a sufficient condition for the first inequality in (5) to hold is

$$\frac{\sqrt{N_t} \|\epsilon_i\|}{\sigma_i} < \sin \frac{\pi}{8} \Leftrightarrow \|\epsilon_i\| < \frac{\sin \frac{\pi}{8}}{\sqrt{N_t}} \sigma_i = \nu_0 \sigma_i \quad (12)$$

If we perform the same analysis for the second inequality in (5), the above condition on the perturbation matrix is again obtained.

Let us recall the definition of ϵ_i

$$\epsilon_i = \mathbf{u}_i^H \mathbf{E}$$

Since \mathbf{U} is unitary, we can write $\mathbf{e}_i = e_i^1 \mathbf{u}_1 + \dots + e_i^{N_r} \mathbf{u}_{N_r}$ and $\epsilon_i = (e_i^1, e_i^2, \dots, e_i^{N_t})$. Therefore, if we take the square of (12), we obtain

$$\sum_{j=1}^{N_t} (e_j^i)^2 < \nu_0^2 \sigma_i^2 \quad (13)$$

In the above expression, we can upper bound the left hand term as

$$\sum_{j=1}^{N_t} (e_j^i)^2 \leq \sum_{j=1}^{N_t} \sum_{i=1}^{N_r} (e_j^i)^2 = \|\mathbf{E}\|_F^2 \quad (14)$$

where $\|\cdot\|_F$ is the Fröbenius norm of the error matrix. Taking into account that, if the channel matrix is full-rank, the smallest nonzero singular value is σ_{N_t} and using this expression in (13) we conclude that a sufficient condition for the error matrix is

$$\|\mathbf{E}\|_F < \nu_0 \sigma_{N_t} \quad (15)$$

Note that the process we have followed so far is only slightly dependent on the constellation. In fact, for any other constellation, we could do the same analysis, the only difference being the value of the constant ν_0 . A straightforward result of the previous procedure is that ν_0 for an N-PSK constellation is $\frac{\sin(\frac{\pi}{N_t})}{\sqrt{N_t}}$.

B. Probabilistic approach

Certainly, extremely pessimistic and possibly unnecessary bounds have been used to provide us with condition (15), which is valid over all the possible realizations of the error matrix. Another approach which provides less stringent conditions is the probabilistic one. In other words, we will find the necessary conditions so that optimal performance—i.e., condition (5)—is not achieved only with a small probability P_e , which can be written as

$$\mathcal{P}\left[\langle \Upsilon, S_1 \rangle \leq \sin \frac{\pi}{8}, \langle \Upsilon, S_2 \rangle \leq \sin \frac{\pi}{8} \mid \mathbf{E}\right] > 1 - P_e \quad (16)$$

A Matlab routine has been developed in order to compute the empiric distribution of the function $\Theta(\mathbf{x}, \delta) = \max\left(\langle \delta \mathbf{x}, (-1, \cot \frac{\pi}{8}) \rangle, \langle \delta \mathbf{x}, (-1, -\cot \frac{\pi}{8}) \rangle\right)$ for the case $N_t = 4$, $N_r = 20$. Using this distribution we can express the restriction for $\|\epsilon_i\|$ as

$$\|\epsilon_i\| \leq \frac{\sigma_i}{K_{P_e}} \quad (17)$$

where K_{P_e} is the value of Θ such that only P_e of the realizations of E can produce higher values of Θ . Some of the values of K_{P_e} are listed in Table 2.

As it may be expected, $K_{P_e}^{-1} \rightarrow \nu_0 = 0.19134$. Another important conclusion of this analysis is that if we have

Table 2
Values of $K_{P_e}^{-1}$ and $G_{P_e}^{-1}$ for some error probabilities.

$P_e(\%)$	$K_{P_e}^{-1}$	$G_{P_e}^{-1}$
5	0.28715	1.23717
2.5	0.26143	1.08520
1	0.23982	0.94784
0.5	0.22885	0.87250
0.1	0.21293	0.74961
0.05	0.20856	0.71108
0.01	0.20171	0.63955
0.001	0.19651	0.56913
0.0001	0.19376	0.51951
0.00001	0.19252	0.47995
0.000001	0.19185	0.46249

means to overcome a certain P_e , the restrictions over \mathbf{E} can be substantially loosened.

At this point we could proceed as in equation (15) and write

$$\|\mathbf{E}\|_F \leq \frac{\sigma_i}{K_{P_e}} \quad (18)$$

Another option, however, is to extend the previous steps to include the random behavior of $\|\epsilon_i\|$ in the empirical distribution and express the result as a function of $\|\mathbf{E}\|_F$. Recall that our goal is that with high probability $\|\epsilon_i\| \Theta = \|\mathbf{E}\|_F \|\tilde{\epsilon}_i\| \Theta \leq \sigma_i$, where $\tilde{\epsilon}_i$ is restricted to the matrices with Fröbenius norm equal to unity. As before, we can use the empirical distribution of $\|\tilde{\epsilon}_i\| \Theta$ to express a restriction depending on the mean probability of error. This dependency is represented in the function G_{P_e} , whose definition is analogous to the one given for K_{P_e} . Therefore, a new condition can be expressed as

$$\|\mathbf{E}\|_F \leq G_{P_e}^{-1} \sigma_i = \nu_{P_e} \sigma_i \quad (19)$$

Some values of ν_{P_e} can be found in Table 2. Note that we obtain an even further loosening in the conditions to be satisfied. Nevertheless, the result for $P_e = 0$ —recall that there is no noise—would still be $\|\mathbf{E}\|_F \leq \nu_0 \sigma_i$ since there exist realizations of the channel for which all the inequalities involved in this analysis become equalities.

It should be noted that the conditions obtained herein can be considered from two different standpoints. Indeed, if we know the type of estimation errors for the channel matrix, we can decide how many eigenmodes are suitable for transmitting information along the channel—those for which $\|\mathbf{E}\|_F \leq \nu_{P_e} \sigma_i$. On the other hand, if we are restricted to maintain a fixed number of eigenmodes active, we must monitor \mathbf{E} so that the condition in (19) is valid for all of them and adjust accordingly the design parameters involved such as, for instance, the elapsed time between two consecutive channel estimations.

To continue this analysis we need to suppose a model for the error matrix. Let us assume that we can express the

temporal channel matrix behavior as $\mathbf{A}(n) = \mathbf{B}(n) + \mathbf{M}$, where the entries of $\mathbf{B}(n)$ are zero mean circularly symmetric complex Gaussian variables with variance σ_b^2 , and that $\mathbf{B}(n+1) = \alpha\mathbf{B}(n) + \mathbf{W}(n+1)$, where α is a constant modelling the resemblance of the channel matrices in two consecutive symbol times and $\mathbf{W}(n)$ is a matrix of zero mean circularly symmetric complex Gaussian random variables with variance σ_w^2 . The matrix \mathbf{M} is introduced to account for the fact that, although we assume that the channel matrix elements have zero mean, in a short-term analysis they would have nonzero mean and would be conditioned by the previous temporal evolution.

Under these assumptions, let us define the error matrix $\mathbf{E}(n, s) = \mathbf{A}(n+s) - \mathbf{A}(n)$. It can be easily proven by induction that

$$\mathbf{E}(n, s) = (\alpha^s - 1)\mathbf{B}(n) + \sum_{i=0}^{s-1} \alpha^i \mathbf{W}(n+s-i) \quad (20)$$

Given the fact that the random variables in the sum in the definition of $\mathbf{E}(n, s)$ are zero mean circularly symmetric complex normal random variables, the elements in $\mathbf{E}(n, s)$ are random variables of the same type with variance

$$\sigma_E^2 = (\alpha^s - 1)^2 \sigma_b^2 + \sum_{i=0}^{s-1} \alpha^{2i} \sigma_w^2 = (1 - \alpha^s)^2 \sigma_b^2 + \frac{1 - \alpha^{2s}}{1 - \alpha^2} \sigma_w^2 \quad (21)$$

Furthermore, since \mathbf{M} is constant, and $\mathbf{A}(n) = \mathbf{B}(n) + \mathbf{M}$, $\sigma_a = \sigma_b$ and

$$\sigma_a^2 = \mathbf{E}\{|b_{ij}(n)|^2\} = \mathbf{E}\{|b_{ij}(n+1)|^2\} = \quad (22)$$

$$\mathbf{E}\{(\alpha b_{ij}(n) + w_{ij}(n+1))(\alpha b_{ij}(n) + w_{ij}(n+1))^*\}$$

$$\sigma_a^2 = \alpha^2 \sigma_a^2 + \sigma_w^2 \Leftrightarrow (1 - \alpha^2) \sigma_a^2 = \sigma_w^2$$

$$\sigma_E^2 = (1 - \alpha^s)^2 \sigma_a^2 + (1 - \alpha^{2s}) \sigma_a^2 = 2(1 - \alpha^s) \sigma_a^2 \quad (23)$$

Note also that if X is a circularly symmetric normal complex random variable, then $|X|^2$ is the sum of the squares of two normal random variables with the same mean and variance. Therefore

$$\|\mathbf{E}(n, s)\|_F^2 = \sum_{j=1}^{N_t} \sum_{i=1}^{N_r} |e_i^j|^2 = \sigma_E^2 \sum_{k=1}^{2N_t N_r} (X_k)^2 = \sigma_E^2 \Phi^2 \quad (24)$$

where X_k are independent identically distributed normal random variables with zero mean and unity variance. Hence, Φ^2 follows a chi-squared distribution with $2N_t N_r$ degrees of freedom, $\Phi^2 \sim \chi_{2N_t N_r}^2$.

Recall that to ensure the proper performance of the decoder we need condition (15) to be satisfied. If we introduce the model for the error matrix, that amounts to

$$\|\mathbf{E}(n, s)\|_F \|\tilde{\mathbf{e}}_i\|_\Theta \leq \sigma_i \Rightarrow \Phi \|\tilde{\mathbf{e}}_i\|_\Theta \leq \frac{\sigma_i}{\sigma_E} \quad (25)$$

Note that this distribution is directly related to the probability of error of the system since if we guarantee $F_{\Phi \|\tilde{\mathbf{e}}_i\|_\Theta}^{-1}(1-p) \leq \frac{\sigma_i}{\sigma_E}$, then the probability of error is less than p . Taking into account the fact that $F_{\Phi \|\tilde{\mathbf{e}}_i\|_\Theta}^{-1}(1-p)$ is a strictly decreasing function of p we can conclude that the probability of error p of the i th channel can be upper-bounded as

$$(1-p) \geq F_{\Phi \|\tilde{\mathbf{e}}_i\|_\Theta}\left(\frac{\sigma_i}{\sigma_E}\right) \quad (26)$$

$$p \leq 1 - F_{\Phi \|\tilde{\mathbf{e}}_i\|_\Theta}\left(\frac{\sigma_i}{\sigma_E}\right) = 1 - F_{\Phi \|\tilde{\mathbf{e}}_i\|_\Theta}\left(\frac{\sigma_i}{\sqrt{2(1-\alpha^s)}\sigma_a}\right)$$

where σ_i is the singular value associated to the i th eigenmode, σ_a is the variance of the elements in the matrix $\mathbf{A}(n)$ and s is the number of symbol intervals elapsed since the last estimation of the channel matrix. The next figure shows the dependence of the bound for the probability of error as a function of $\frac{\sigma_i}{\sigma_E}$ for the case of four transmit antennas and twenty receive antennas.

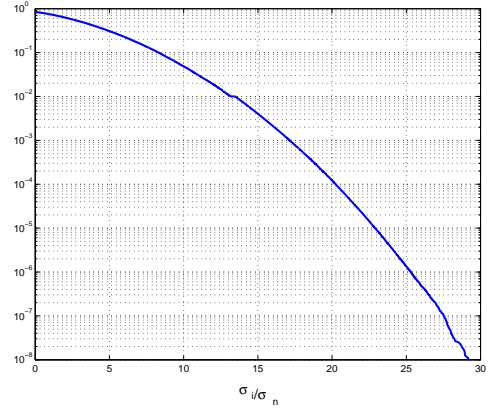


Fig. 3

IV. JOINT STUDY

Let us expand completely the expression in Equation (4)

$$\hat{x}_i - x_i = \frac{1}{\sigma_i} \left[\sum_{j=1}^{N_t} \sum_{k=1}^{N_t} \sum_{l=1}^{N_r} u_i^l e_k^l v_j^k x_j + \hat{n}_{ii} \right] \quad (27)$$

If we assume as in the previous section that the error matrix can be modelled as $\mathbf{E}(n, s) = (\alpha^s - 1)\mathbf{B}(n) + \sum_{i=0}^{s-1} \alpha^i \mathbf{W}(n+s-i)$, being \mathbf{B} and \mathbf{W} Gaussian matrices, then all the entries e_k^l are also samples from a circularly symmetric complex Gaussian distribution with variance σ_E . Hence, the expression above consists of the sum of $N_t N_r$ zero mean circularly symmetric complex Gaussian random variables whose variance has not been yet determined plus one random variable of the same kind with variance σ_n .

Since the term accompanying each of the e_k^l is $\sum_{j=1}^{N_t} x_j u_i^l v_j^k$ and all these random variables are independent

and zero mean normal-distributed, the overall variance of the $N_t N_r$ variables is

$$\tilde{\sigma}^2 = \sigma_E^2 \sum_{k,l} |u_i^l \sum_j x_j v_j^k|^2 \quad (28)$$

If we take the expectation of the expression above with respect to the constellation, keeping in mind that the symbols are assumed independent in this analysis, then

$$\begin{aligned} \langle \tilde{\sigma}^2 \rangle &= \sigma_E^2 \sum_{k,l} |u_i^l|^2 \sum_j \sum_{j'} \mathbf{E}\{x_j x_{j'}^* \} v_j^k (v_{j'}^k)^* = \quad (29) \\ \sigma_E^2 \sum_l |u_i^l|^2 \sum_{k,j} |v_j^k|^2 &= \sigma_E^2 \|\mathbf{u}_i\|^2 \|\mathbf{V}\|_F^2 = N_t \sigma_E^2 \end{aligned}$$

Finally, if we include the effect of the noise in this analysis, we obtain

$$\hat{x}_i - x_i = \frac{\sigma}{\sigma_i} \xi \quad (30)$$

where $\xi = \xi_R + i\xi_I$ is a zero mean circularly symmetric complex random variable with unity variance and

$$\sigma = \sqrt{N_t \sigma_E (\alpha, s)^2 + \sigma_n^2} = \sqrt{2N_t (1 - \alpha^s) \sigma_a^2 + \sigma_n^2} \quad (31)$$

Recall that this is the same formulation as that of Section II. In that section, the conditions for correct performance were found to be

$$\max \frac{\sigma}{\sigma_i} \left(\langle \xi, S_1 \rangle, \langle \xi, S_2 \rangle \right) < \sin\left(\frac{\pi}{8}\right) \quad (32)$$

If we define $\widehat{SNR}_i = \frac{\sigma_i^2}{\sigma^2}$, then the above inequality can be rewritten as

$$\Theta = -\xi_R + \max \left(\pm \xi_I \cot\left(\frac{\pi}{8}\right) \right) < \sqrt{\widehat{SNR}_i} \quad (33)$$

Let us define the signal-to-noise ratio at the receiver as

$$SNR = \frac{\mathbf{E}\{(\mathbf{y} - \mathbf{n})^H (\mathbf{y} - \mathbf{n})\}}{\mathbf{E}\{\mathbf{n}^H \mathbf{n}\}} = \frac{\mathbf{E}\{(A\mathbf{V}\mathbf{x})^H (A\mathbf{V}\mathbf{x})\}}{\mathbf{E}\{\mathbf{n}^H \mathbf{n}\}} \quad (34)$$

Taking into account

$$\begin{aligned} \mathbf{E}\{(A\mathbf{V}\mathbf{x})^H (A\mathbf{V}\mathbf{x})\} &= N_t N_r \sigma_a^2 + \|\mathbf{M}\|_F^2 \\ \mathbf{E}\{\mathbf{n}^H \mathbf{n}\} &= N_r \sigma_n^2 \end{aligned}$$

and also, that if we consider the original channel matrix,

$$\mathbf{E}\{(A\mathbf{V}\mathbf{x})^H (A\mathbf{V}\mathbf{x})\} = \sum_{k=1}^{N_t} \sigma_k^2$$

we can express the signal-to-noise ratio in the following way

$$SNR = \frac{N_t \sigma_a^2 + \frac{\|\mathbf{M}\|_F^2}{N_r}}{\sigma_n^2} = \frac{\sum_{k=1}^{N_t} \sigma_k^2}{N_r \sigma_n^2} \quad (35)$$

Defining $\rho_i = \frac{\sigma_i^2}{\sum_{k=1}^{N_t} \sigma_k^2}$,

$$\widehat{SNR}_i = \frac{\sigma_i}{2N_t(1 - \alpha^s)\sigma_a^2 + \sigma_n^2} \geq \frac{N_r \rho_i}{2(1 - \alpha^s) + \frac{1}{\widehat{SNR}}} \quad (36)$$

If we assume that the matrix is full rank and we are using all the eigenmodes, then

$$SNR = \frac{N_t E_s}{N_0} \quad (37)$$

In the case of 8-PSK and 4 transmit antennas $SNR = 12 \frac{E_b}{N_0}$ so, finally,

$$\widehat{SNR}_i \geq \frac{N_r \rho_i}{2(1 - \alpha^s) + \frac{1}{12(E_b/N_0)}} \quad (38)$$

In Figure 4, the probability of error for $\rho_i = \frac{1}{20}$ and $N_r = 20$, with $\alpha^s = 0.9, 0.95, 0.97, 0.98, 0.99, 1$ as a function of E_b/N_0 (dB) is shown.

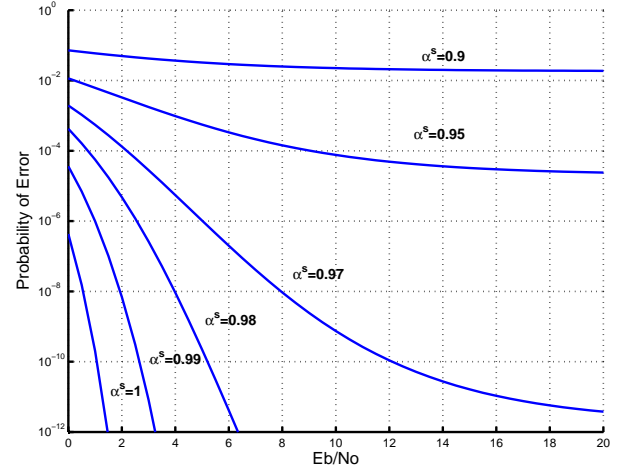


Fig. 4

V. CONCLUSION

A study of the impact of channel estimation errors on the performance of SVD-based systems is performed. Bounds relating the Fröbenius norm of the difference between the channel matrix considered and the actual channel matrix with the active eigenmodes' singular values are obtained both from a strict and from a probabilistic viewpoint. For a given estimation error model, those bounds can be used to find the maximum number of active eigenmodes or to determine the maximum elapsed time between two consecutive channel estimations to maintain a certain probability of error.

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