A Trio of Sliding Block Puzzles

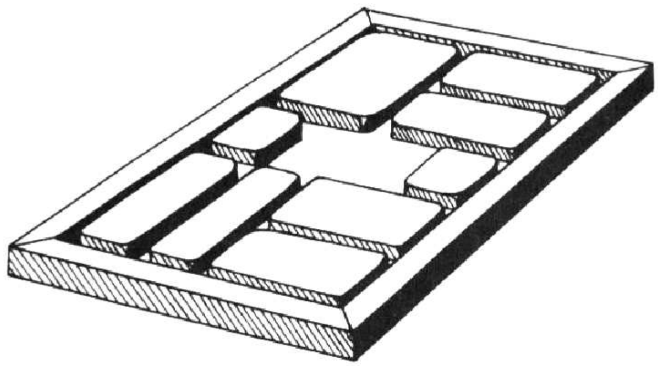
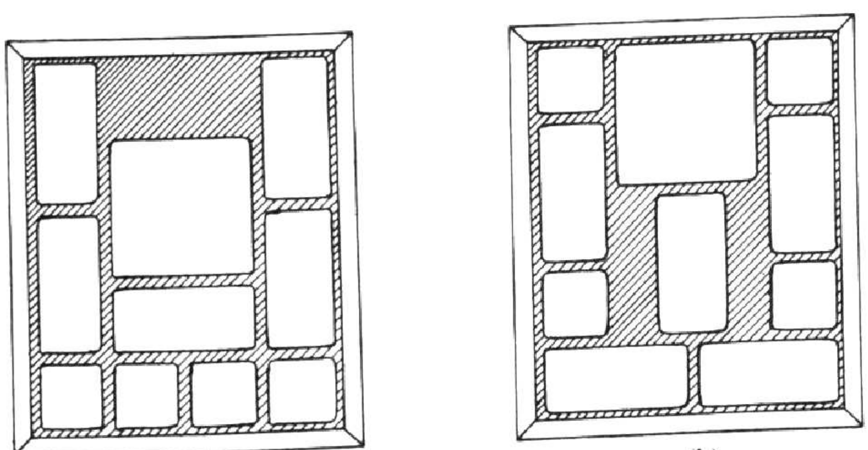


Figure 36. Dad's Puzzler.

Dad's Puzzler (Fig. 36) is unfortunately almost the only sliding block puzzle that's generally available from toy stores, although it goes under many different names. The problem is to slide the pieces without lifting any out of the tray, until the 2×2 square arrives in the lower left hand corner. Fifty years ago the puzzle represented Dad's furniture-removing difficulties, and the 2×2 block was the piano; at other times it has been depicted as a pennant, a car, a mountain, or space capsule but the puzzle has remained unchanged, probably for a hundred years. Some more enterprising manufacturer should sell a set containing one 2×2, four 1×1 and six 2×1 pieces which can be used either for Dad's Puzzler or for the following more interesting puzzles.

In the **Donkey** puzzle the initial arrangement is as in Fig. 37(a) and the problem is to move the 2×2 square to the middle of the bottom row. The name arises from the picture of a red donkey which adorned the 2×2 square in the original French version (L'Âne Rouge, which probably goes back to the last century) but we think that our choice of starting position already looks quite like a donkey's face.



(a) (b)

Figure 37. The Donkey and The Century (and a Half).

The **Century Puzzle**, published for the first time in *Winning Ways*, was discovered by one of us several years ago as a result of a systematic search for the hardest puzzle of this size. Start from Fig. 37(b) and, as in the Donkey, end with the 2 × 2 block in the middle of the bottom row. Or, if you're a real expert, you might try the **Century-and-a-Half Puzzle** in which you're to end in the position got by turning Fig. 37(b) upside-down.

Tactics for Solving Such Puzzles

As in our previous sliding puzzles the basic idea is to see what can be done while quite a lot of the pieces are kept fixed. In all three of these examples one occasionally sees one of the configurations of Fig. 38 somewhere, and any of these can be exchanged for any other, moving

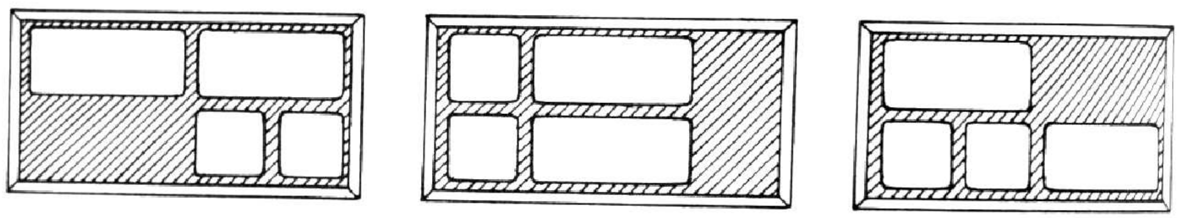


Figure 38. A Micropuzzle.

only the pieces in the area shown. They form a kind of micro-puzzle within the larger one. Figure 40 is a complete "map" of Dad's Puzzler showing how it consists of a dozen of these micro-puzzles joined by various paths of moves that are more or less forced. Using this map, you'll find it easy to get from anywhere to anywhere else.

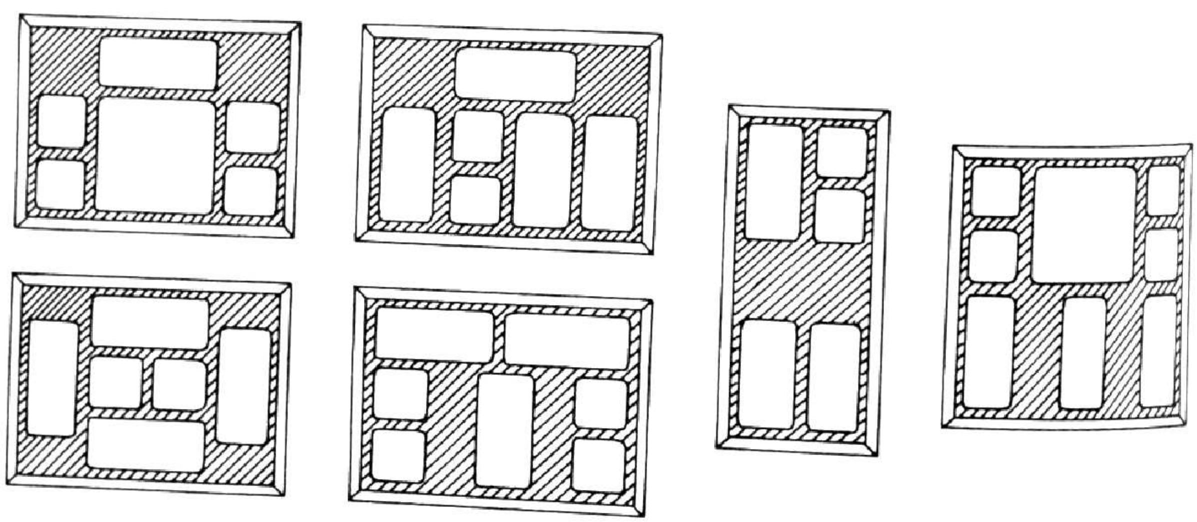


Figure 39. Micro- and Mini-puzzles Found in Donkey and Century.

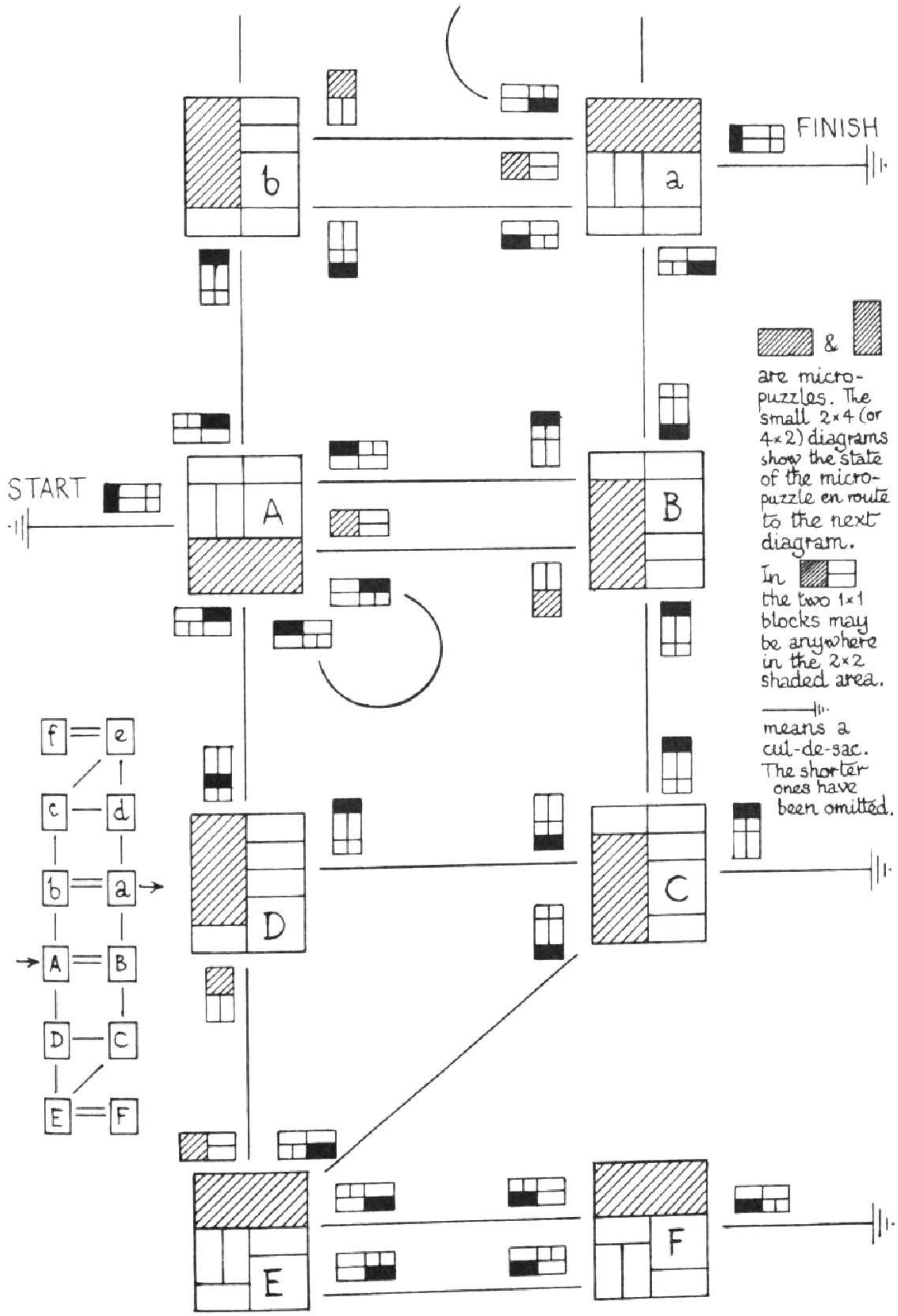


Figure 40. Map of Dad's Puzzler.

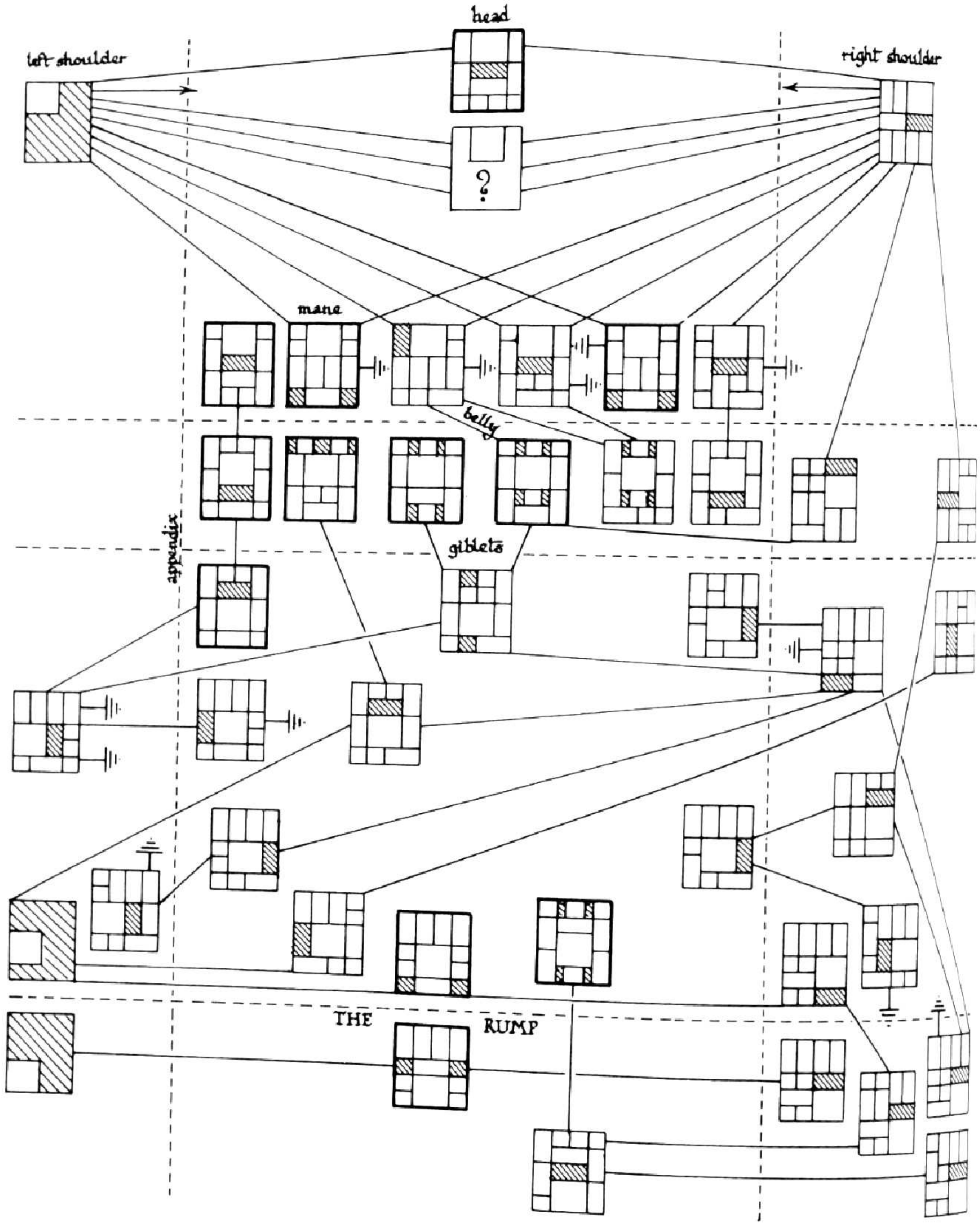


Figure 41. Map of the Donkey.

In the Donkey and Century puzzles there are several micro- and mini-puzzles: see what moves you can make inside the regions shown in Fig. 40. The Century and Donkey puzzles will never become easy but it will help if you become an adept at these minipuzzles. Figure 41 is our map of the Donkey. The positions are classified according to the location of the 2 × 2 square and in most cases we have only drawn one of a left-right mirror-image pair. Some unimportant culs-de-sac will be found in the directions indicated by the signs ⫞|||·, and the rectangle containing (?) represents many positions connected to the left and right shoulders. The arrows indicate other connexions to the shoulders. Left-right symmetric positions are boldly bordered.

The Century puzzle is very much larger, and we need more abbreviations to draw its map within a reasonable compass. The positions are best classified by the position of the large square together with information about which of the two horizontal pieces should be counted as "above" or "below" the square. We remark that in Fig. 42 both horizontal pieces should be counted as *below* the square despite their appearance, because the only way to move these pieces takes the horizontals *down* and the square *up*.

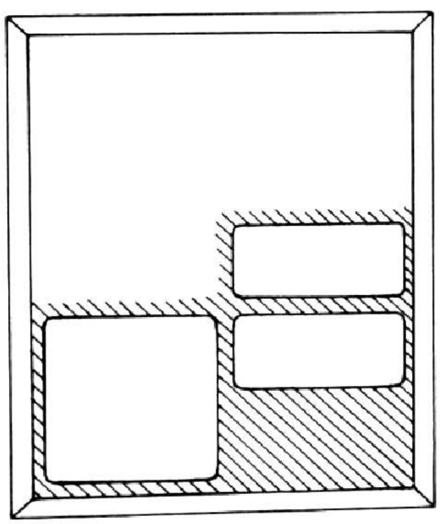


Figure 42. The Two Horizontal Pieces are Below the Square!

The key to the puzzle is to find one of the two possible **narrow bridges** in the map at which the first horizontal piece changes from *below* the square to *above* it. In fact it's best to think out the possible configurations in which this can happen and then work the puzzle backwards and forwards from one of these. Very few people have ever solved the puzzle by starting at the initial configuration and moving steadily towards its end. A much abbreviated map appears as Fig. 43.

Our maps were prepared with much help from some computer calculations made by David Fremlin at the University of Essex, who found incidentally that the Donkey pieces may be placed in the tray in 65880 positions and the Century pieces in 109260 ways. Although the Century puzzle can be inverted (this is our Century-and-a-Half problem) Fremlin's computer found that the Donkey cannot. It would be nice to have a more perspicuous proof of this.

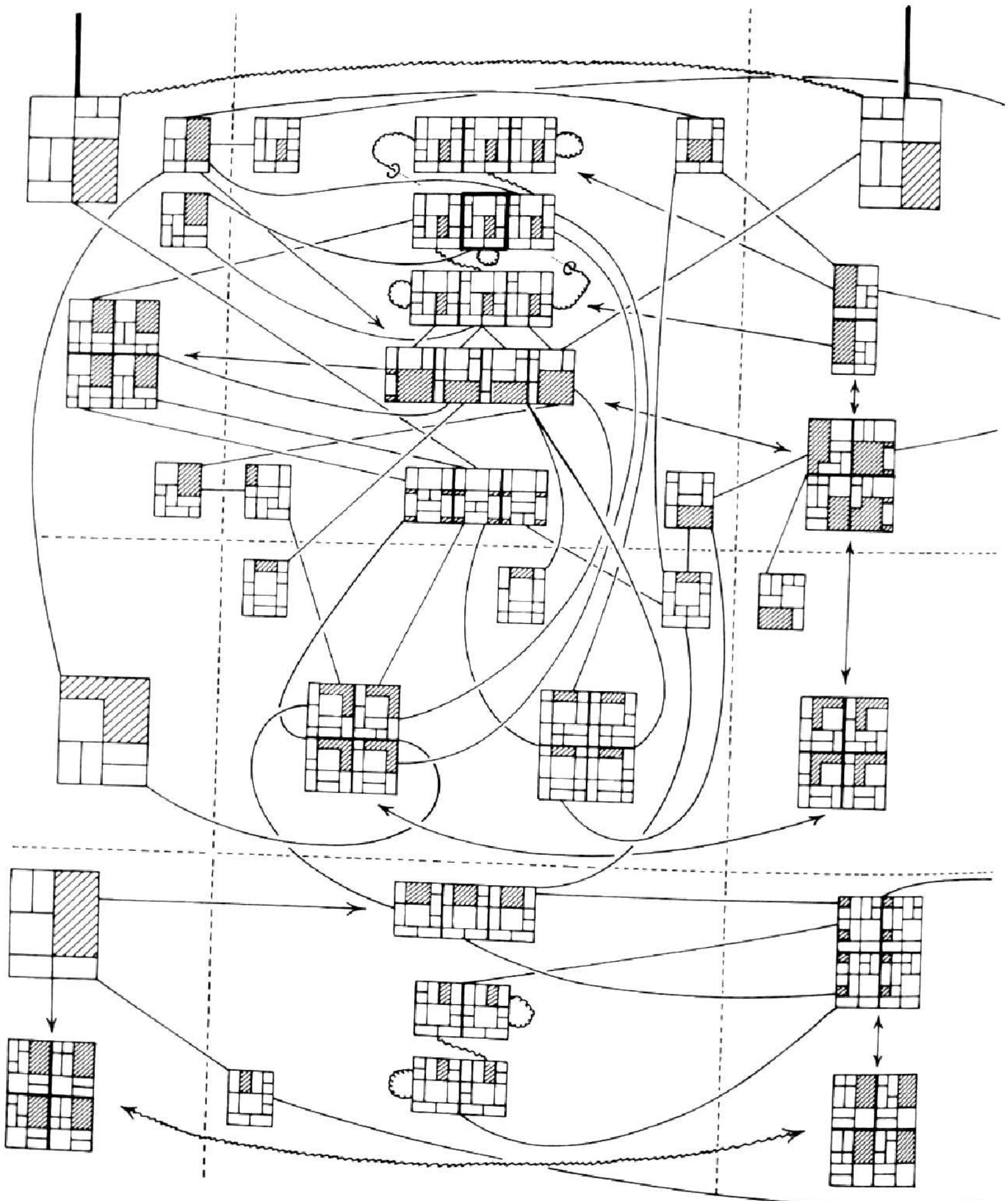
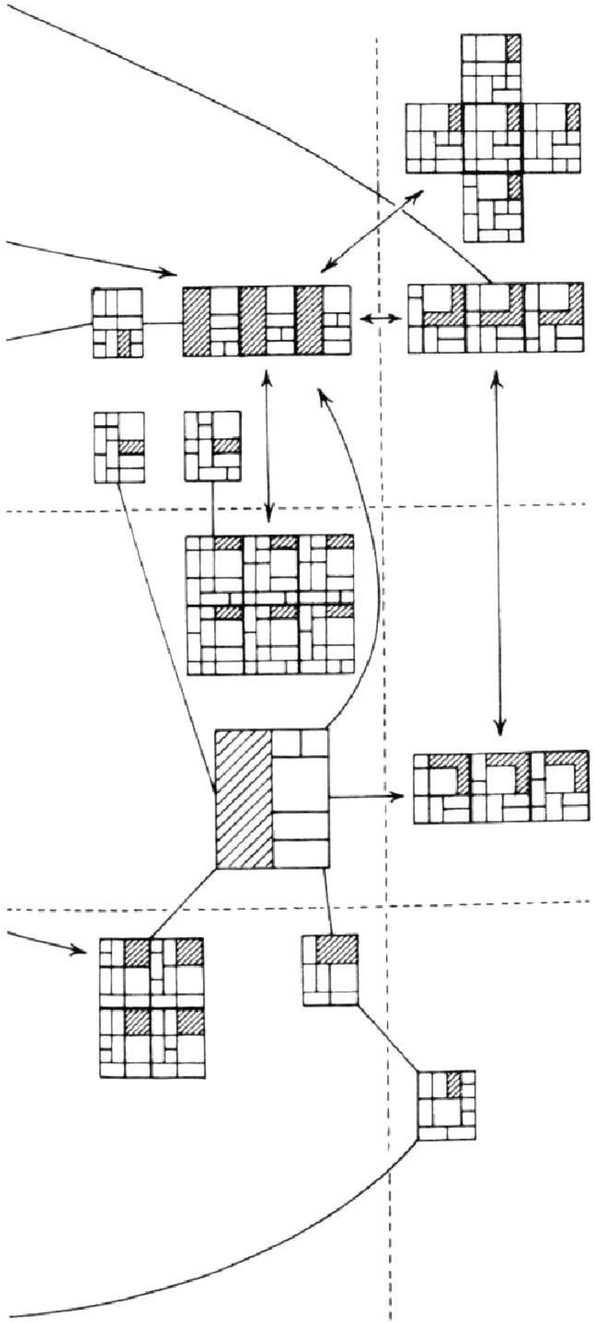


Figure 43. Map ....

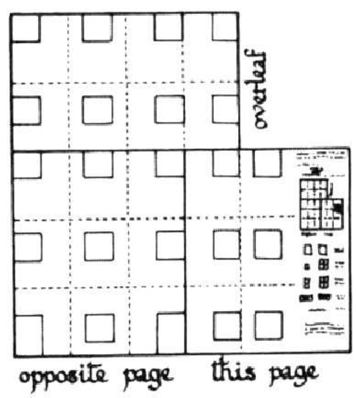


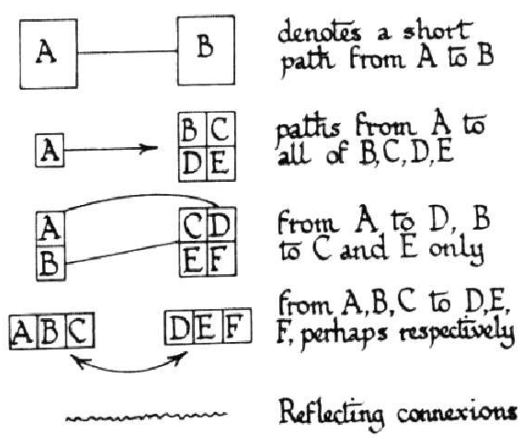
The starting position is heavily outlined; see centre column of opposite page, near top

Positions are classified thus:

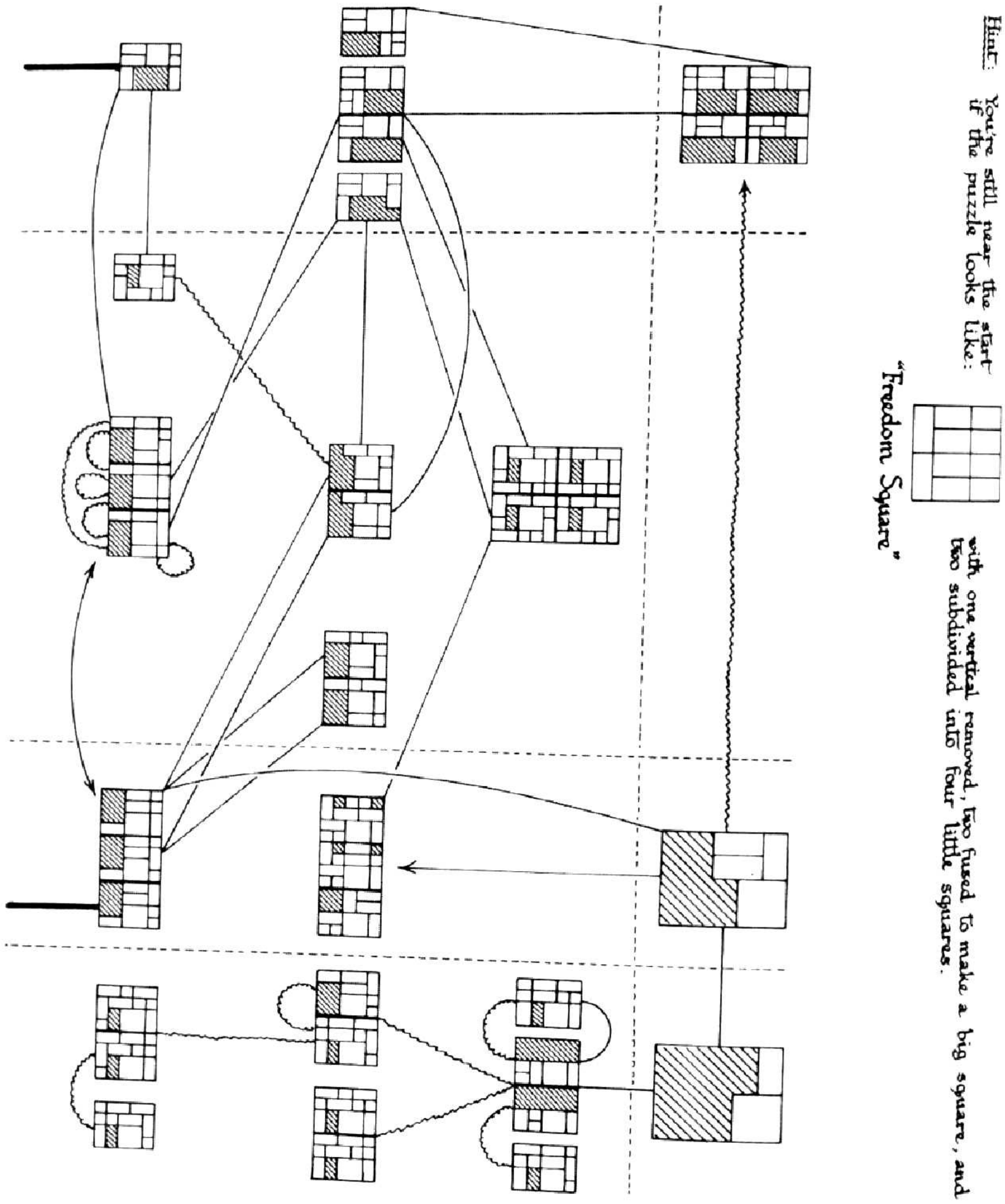
overleaf: Square "between" horizontals.
opposite: Two verticals left, one right.
this page: Three verticals left.

They are further classified according to the location of the square:





The "narrow bridges" are the two thick connexions between the top of the opposite page and the left of overleaf.


Hint: You're still near the start if the puzzle looks like:
with one vertical removed, two fused to make a big square, and two subdivided into four little squares.
"Freedom Square"

. . . . Century.

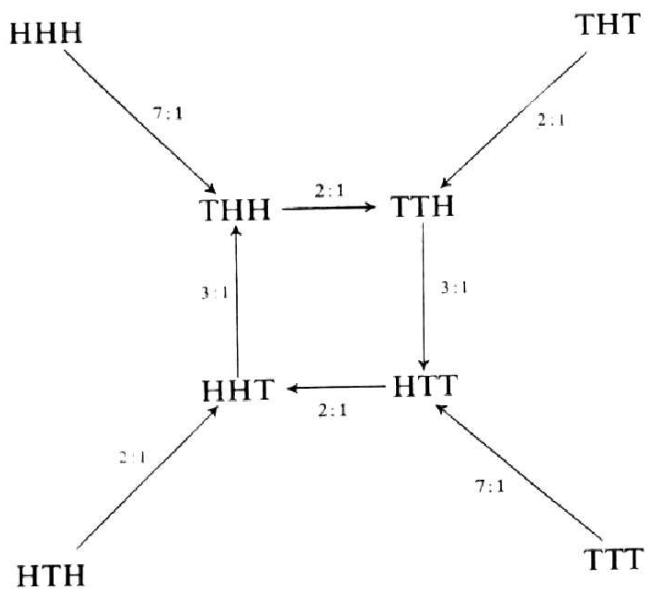
Counting Your Moves

It's customary to follow Martin Gardner and declare that any kind of motion involving just one piece counts as a single move. It takes 58 moves to solve Dad's Puzzler and 83 to solve the Donkey. How many do you need to solve the Century puzzle? And how many for the Century-and-a-Half?

Paradoxical Pennies

You tell me your favorite sequence of three Heads or Tails and then I'll tell you mine. We then spin a penny until the first time either of our sequences appears as the result of three consecutive throws. I bet you 2 to 1 it's mine!

The graph



shows the sequence I'll choose for each possible sequence of yours, together with the odds that I win. You'll see that it's always at least 2 to 1 in my favor.

Here's a rule for computing the odds. Given two Head-Tail sequences a and b of the same length, n, we compute the **leading number**, $a\mathrm{L}b$, by scoring 2^{k-1} for every positive k for which the last k letters of a coincide with the first k of b. Then we can show that the odds, that b beats a in Paradoxical Pennies, are exactly

$$a\mathrm{L}a - a\mathrm{L}b \text{ to } b\mathrm{L}b - b\mathrm{L}a.$$

Leo Guibas and Andy Odlyzko have proved that, given a, the best choice for b is one of the two sequences obtained by dropping the last digit of a and prefixing a new first digit. Notice the paradoxical fact that in the length 3 game:

$$\text{THH beats HHT beats HTT beats TTH beats THH.}$$