

$$\begin{aligned}
1 &= 1^2 \\
2 &= 1^2 + 1^2 \\
3 &= 1^2 + 1^2 + 1^2 \\
4 &= 2^2 \\
5 &= 2^2 + 1^2 \\
6 &= 2^2 + 1^2 + 1^2 \\
7 &= 2^2 + 1^2 + 1^2 + 1^2 \\
8 &= 2^2 + 2^2 \\
9 &= 3^2 \\
10 &= 3^2 + 1^2 \\
11 &= 3^2 + 1^2 + 1^2 \\
12 &= 2^2 + 2^2 + 2^2 \\
13 &= 3^2 + 2^2 \\
14 &= 3^2 + 2^2 + 1^2 \\
15 &= 3^2 + 2^2 + 1^2 + 1^2 \\
16 &= 4^2 \\
17 &= 4^2 + 1^2 \\
18 &= 3^2 + 3^2 \\
19 &= 3^2 + 3^2 + 1^2 \\
20 &= 4^2 + 2^2 \\
21 &= 4^2 + 2^2 + 1^2 \\
22 &= 3^2 + 3^2 + 2^2 \\
23 &= 3^2 + 3^2 + 2^2 + 1^2 \\
24 &= 4^2 + 2^2 + 2^2 \\
25 &= 5^2 \\
26 &= 5^2 + 1^2 \\
27 &= 3^2 + 3^2 + 3^2 \\
&= 5^2 + 1^2 + 1^2 \\
28 &= 3^2 + 3^2 + 3^2 + 1^2 \\
&= 4^2 + 2^2 + 2^2 + 2^2 \\
&= 5^2 + 1^2 + 1^2 + 1^2 \\
29 &= 5^2 + 2^2 \\
30 &= 5^2 + 2^2 + 1^2 \\
31 &= 3^2 + 3^2 + 3^2 + 2^2 \\
&= 5^2 + 2^2 + 1^2 + 1^2 \\
32 &= 4^2 + 4^2 \\
33 &= 4^2 + 4^2 + 1^2 \\
&= 5^2 + 2^2 + 2^2 \\
34 &= 5^2 + 3^2 \\
35 &= 5^2 + 3^2 + 1^2 \\
36 &= 6^2 \\
37 &= 6^2 + 1^2 \\
38 &= 5^2 + 3^2 + 2^2 \\
&= 6^2 + 1^2 + 1^2 \\
39 &= 5^2 + 3^2 + 2^2 + 1^2 \\
&= 6^2 + 1^2 + 1^2 + 1^2 \\
40 &= 6^2 + 2^2 \\
41 &= 5^2 + 4^2 \\
42 &= 5^2 + 4^2 + 1^2 \\
43 &= 5^2 + 3^2 + 3^2 \\
44 &= 6^2 + 2^2 + 2^2 \\
45 &= 6^2 + 3^2 \\
46 &= 6^2 + 3^2 + 1^2 \\
47 &= 5^2 + 3^2 + 3^2 + 2^2 \\
&= 6^2 + 3^2 + 1^2 + 1^2 \\
48 &= 4^2 + 4^2 + 4^2 \\
49 &= 7^2 \\
50 &= 5^2 + 5^2 \\
&= 7^2 + 1^2 \\
51 &= 5^2 + 5^2 + 1^2 \\
&= 7^2 + 1^2 + 1^2 \\
52 &= 6^2 + 4^2 \\
53 &= 7^2 + 2^2 \\
54 &= 5^2 + 5^2 + 2^2 \\
&= 6^2 + 3^2 + 3^2 \\
&= 7^2 + 2^2 + 1^2 \\
55 &= 5^2 + 5^2 + 2^2 + 1^2 \\
&= 6^2 + 3^2 + 3^2 + 1^2 \\
&= 7^2 + 2^2 + 1^2 + 1^2 \\
56 &= 6^2 + 4^2 + 2^2 \\
57 &= 5^2 + 4^2 + 4^2 \\
&= 7^2 + 2^2 + 2^2 \\
58 &= 7^2 + 3^2 \\
59 &= 5^2 + 5^2 + 3^2 \\
&= 7^2 + 3^2 + 1^2 \\
60 &= 5^2 + 5^2 + 3^2 + 1^2 \\
&= 6^2 + 4^2 + 2^2 + 2^2 \\
&= 7^2 + 3^2 + 1^2 + 1^2 \\
61 &= 6^2 + 5^2 \\
62 &= 6^2 + 5^2 + 1^2 \\
&= 7^2 + 3^2 + 2^2 \\
63 &= 5^2 + 5^2 + 3^2 + 2^2 \\
&= 6^2 + 3^2 + 3^2 + 3^2 \\
&= 6^2 + 5^2 + 1^2 + 1^2 \\
&= 7^2 + 3^2 + 2^2 + 1^2 \\
64 &= 8^2 \\
65 &= 7^2 + 4^2 \\
&= 8^2 + 1^2 \\
66 &= 5^2 + 5^2 + 4^2 \\
&= 7^2 + 4^2 + 1^2 \\
&= 8^2 + 1^2 + 1^2 \\
67 &= 7^2 + 3^2 + 3^2 \\
68 &= 8^2 + 2^2 \\
69 &= 7^2 + 4^2 + 2^2 \\
&= 8^2 + 2^2 + 1^2 \\
70 &= 6^2 + 5^2 + 3^2 \\
71 &= 6^2 + 5^2 + 3^2 + 1^2 \\
&= 7^2 + 3^2 + 3^2 + 2^2 \\
72 &= 6^2 + 6^2 \\
73 &= 8^2 + 3^2 \\
74 &= 7^2 + 5^2 \\
75 &= 5^2 + 5^2 + 5^2 \\
&= 7^2 + 5^2 + 1^2 \\
76 &= 6^2 + 6^2 + 2^2 \\
77 &= 6^2 + 5^2 + 4^2 \\
&= 8^2 + 3^2 + 2^2 \\
78 &= 7^2 + 5^2 + 2^2 \\
79 &= 5^2 + 5^2 + 5^2 + 2^2 \\
&= 6^2 + 5^2 + 3^2 + 3^2 \\
&= 7^2 + 5^2 + 2^2 + 1^2 \\
80 &= 8^2 + 4^2 \\
81 &= 9^2 \\
82 &= 9^2 + 1^2 \\
83 &= 7^2 + 5^2 + 3^2 \\
&= 9^2 + 1^2 + 1^2 \\
84 &= 8^2 + 4^2 + 2^2 \\
85 &= 7^2 + 6^2 \\
&= 9^2 + 2^2 \\
86 &= 6^2 + 5^2 + 5^2 \\
&= 7^2 + 6^2 + 1^2 \\
&= 9^2 + 2^2 + 1^2 \\
87 &= 6^2 + 5^2 + 5^2 + 1^2 \\
&= 7^2 + 5^2 + 3^2 + 2^2 \\
&= 7^2 + 6^2 + 1^2 + 1^2 \\
&= 9^2 + 2^2 + 1^2 + 1^2 \\
88 &= 6^2 + 6^2 + 4^2 \\
89 &= 8^2 + 5^2 \\
90 &= 9^2 + 3^2 \\
91 &= 9^2 + 3^2 + 1^2 \\
92 &= 6^2 + 6^2 + 4^2 + 2^2 \\
&= 7^2 + 5^2 + 3^2 + 3^2 \\
&= 9^2 + 3^2 + 1^2 + 1^2 \\
93 &= 8^2 + 5^2 + 2^2 \\
94 &= 7^2 + 6^2 + 3^2 \\
&= 9^2 + 3^2 + 2^2 \\
95 &= 6^2 + 5^2 + 5^2 + 3^2 \\
&= 7^2 + 6^2 + 3^2 + 1^2 \\
&= 9^2 + 3^2 + 2^2 + 1^2 \\
96 &= 8^2 + 4^2 + 4^2 \\
97 &= 9^2 + 4^2 \\
98 &= 7^2 + 7^2 \\
99 &= 7^2 + 5^2 + 5^2 \\
&= 7^2 + 7^2 + 1^2 \\
&= 9^2 + 3^2 + 3^2 \\
100 &= 10^2 \\
101 &= 10^2 + 1^2 \\
102 &= 7^2 + 7^2 + 2^2 \\
&= 10^2 + 1^2 + 1^2 \\
103 &= 7^2 + 5^2 + 5^2 + 2^2 \\
&= 7^2 + 6^2 + 3^2 + 3^2 \\
&= 7^2 + 7^2 + 2^2 + 1^2 \\
&= 9^2 + 3^2 + 3^2 + 2^2 \\
&= 10^2 + 1^2 + 1^2 + 1^2 \\
104 &= 10^2 + 2^2 \\
105 &= 8^2 + 5^2 + 4^2 \\
&= 10^2 + 2^2 + 1^2 \\
106 &= 9^2 + 5^2 \\
107 &= 7^2 + 7^2 + 3^2 \\
&= 9^2 + 5^2 + 1^2 \\
108 &= 6^2 + 6^2 + 6^2 \\
&= 10^2 + 2^2 + 2^2 \\
109 &= 10^2 + 3^2 \\
110 &= 7^2 + 6^2 + 5^2 \\
&= 9^2 + 5^2 + 2^2 \\
&= 10^2 + 3^2 + 1^2 \\
111 &= 6^2 + 5^2 + 5^2 + 5^2 \\
&= 7^2 + 6^2 + 5^2 + 1^2 \\
&= 7^2 + 7^2 + 3^2 + 2^2 \\
&= 9^2 + 5^2 + 2^2 + 1^2 \\
&= 10^2 + 3^2 + 1^2 + 1^2 \\
112 &= 6^2 + 6^2 + 6^2 + 2^2 \\
&= 8^2 + 4^2 + 4^2 + 4^2 \\
&= 10^2 + 2^2 + 2^2 + 2^2 \\
113 &= 8^2 + 7^2 \\
114 &= 7^2 + 7^2 + 4^2 \\
&= 8^2 + 5^2 + 5^2 \\
&= 8^2 + 7^2 + 1^2 \\
115 &= 9^2 + 5^2 + 3^2 \\
116 &= 10^2 + 4^2 \\
117 &= 9^2 + 6^2 \\
118 &= 9^2 + 6^2 + 1^2 \\
&= 10^2 + 3^2 + 3^2 \\
119 &= 7^2 + 6^2 + 5^2 + 3^2 \\
&= 9^2 + 5^2 + 3^2 + 2^2 \\
&= 9^2 + 6^2 + 1^2 + 1^2 \\
&= 10^2 + 3^2 + 3^2 + 1^2 \\
120 &= 10^2 + 4^2 + 2^2 \\
121 &= 11^2
\end{aligned}$$