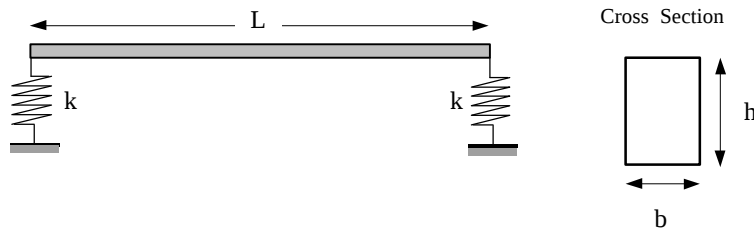


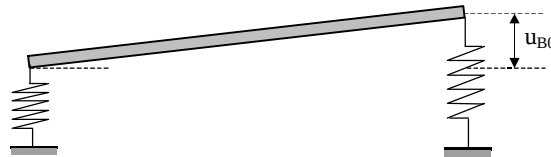
QUIZ # 1

PROBLEM 1

Consider a uniform, *rigid* bar with length $L = 10\text{m}$, width $b = 0.2\text{m}$, height $h = 0.5\text{m}$ and mass density $\rho = 2500 \text{ kg/m}^3$, as shown in the figure below. The bar is supported at the ends by identical springs of stiffness $k = 5000\text{N}$. While this is a 2-dof system (vertical and rotational motion) the vibration modes are *uncoupled* because of symmetry. Hence, this system behaves like two independent 1-dof systems.



- Find the two natural frequencies (in Hz) of this system.
- If the rightmost spring is initially extended by a vertical displacement u_{B0} , what is the free oscillation response of this system? (Hint: it will both translate and rotate).



PROBLEM 2

A 5-story building of height h and cross section $A = a^2$ is subjected to an earthquake with the response spectrum shown. If $h/a = 2$ and the building can be modeled as a 1-dof system with damping $\xi = 0.01$, estimate:

- The maximum relative displacement of the roof relative to the base.
- The maximum base shear (the horizontal force transmitted to the ground).
- If the earthquake stopped instantly at the time of maximum relative displacement (yes, this is impossible), what would be the response after this time? Sketch this response and explain the features of your sketch.
- Assume next that this building is infinitely rigid, and that the soil has a shear modulus $G = 8 \times 10^7 \text{ N/m}^2$ and poisson's ratio $\nu = 0.35$.
If to a first approximation, the foundation can be idealized as a circular foundation with equivalent radius $\pi R^2 = a^2$, what is the rocking frequency of the building?

$$\text{Rocking stiffness of circular foundation : } K_R = \frac{8 G R^3}{3 (1 - \nu)}$$

Extra Points: Obtain the relative displacement at the top of the rigid building (a 1-dof system!), caused by this same earthquake. Important: this requires for you to set up correctly the equation of motion for a rigid mass that can *rotate* about the base while following the ground *translation*)

