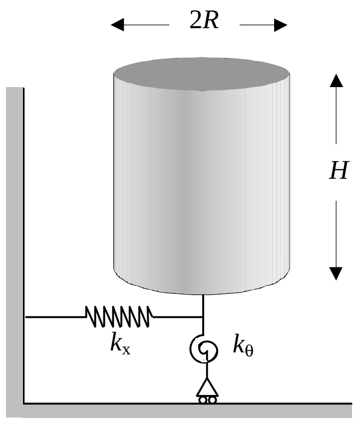
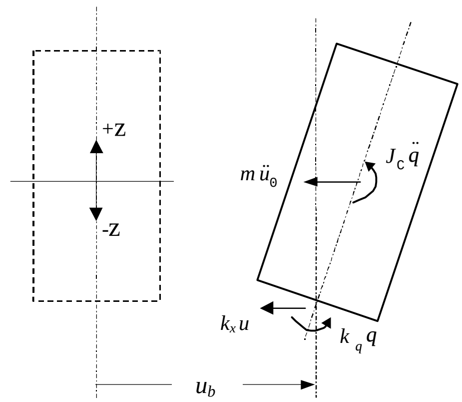
RECITATION (SEPTEMBER 5 2001)

PROBLEM 1

Obtain the equations of motion for the following systems (assume small displacements).

1.





We first draw the free body diagram and formulate the equation of motion with respect to the center of mass:

The total mass of the cylinder is $m = r\, p\, R^2 H$, and the moment of inertia around the center

$$J_c = m\left(\frac{H^2}{12}+\frac{R^2}{4}\right).$$

The displacement along the vertical axis as a function of the displacement of the center of mass u_0 and the angle of rotation,q, of the cylinder is $u(z) = u_0 + q\, z$.

Horizontal Equilibrium: $$m\,\ddot{u}_0 + K_x u_b = 0 \qquad \{1\}$$

Moment Equilibrium: $$J_c\,\ddot{q} + K_R q - K_x u_b \frac{H}{2} = 0 \qquad \{2\}$$

We substitute in equations {1} and {2}, the displacement of the base u_b, as a function of the displacement of the center of mass u_0 and the angle of rotation, q, i.e.:

$$u_b = u\left(-\frac{H}{2}\right) = u_0 - q\,\frac{H}{2} \qquad \{3\}$$

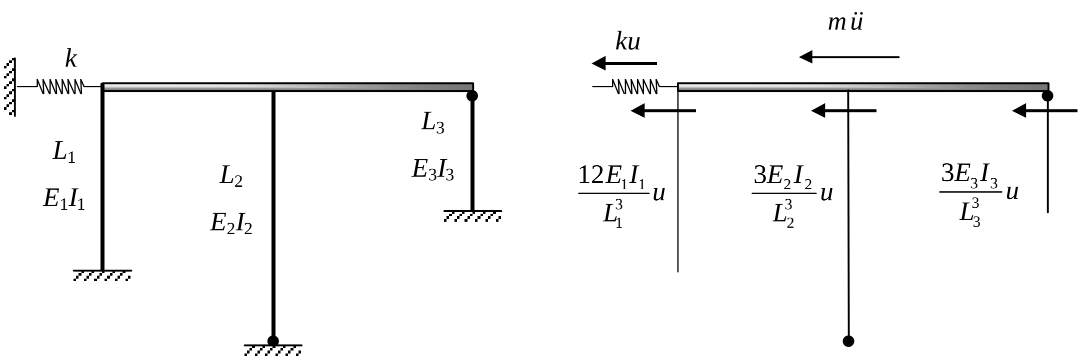
The equation of motion for the 2-dof system is now formulated as follows:

$$\begin{aligned} m\,\ddot{u}_0 + K_x u_0 - K_x \frac{H}{2} q &= 0 \\ J_c\,\ddot{q} + K_R q - K_x u_0 \frac{H}{2} + K_x q\,\frac{H^2}{4} &= 0 \end{aligned} \qquad \{4\}$$

Equation {4} can be expressed in matrix form as follows:

$$\begin{bmatrix} m & 0 \\ 0 & J_c \end{bmatrix} \begin{Bmatrix} \ddot{u}_0 \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} K_x & -K_x \dfrac{H}{2} \\ -K_x \dfrac{H}{2} & K_R + K_x \dfrac{H^2}{4} \end{bmatrix} \begin{Bmatrix} u_0 \\ q \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \{5\}$$

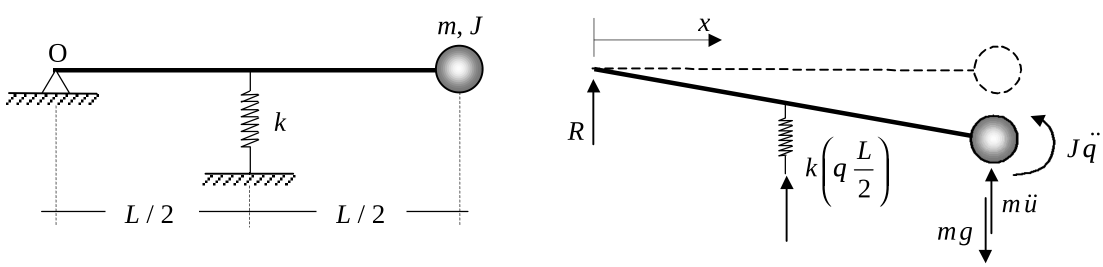
2.



We formulate the equation of motion by equilibrium of horizontal forces:

$$m\ddot{u} + \left(\frac{12E_1I_1}{L_1^3} + \frac{3E_2I_2}{L_2^3} + \frac{3E_3I_3}{L_3^3} + k \right) u = 0 \qquad \{1\}$$

3.



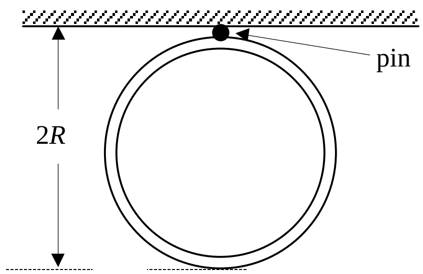
The displacement along the rigid bar is evaluated as a function of the rotation $u(x) = q\,x$.
We formulate the equation of motion by evaluating moment equilibrium around the support:

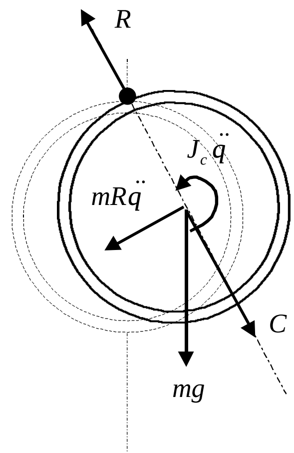
$$\Sigma M_O = 0$$

$$J\ddot{q} + m\left(\ddot{q}\,L\right)L + k\left(q\,\frac{L}{2}\right)\frac{L}{2} - m\,g\,L = 0 \qquad \{1\}$$

$$\left(J + mL^2\right)\ddot{q} + k\,\frac{L^2}{4}\,q - m\,g\,L = 0$$

4.





The rotational inertia around the center of the ring is: $J = \iint \left(x^2 + y^2\right) dm = m R^2$

We formulate the equation of motion by evaluating moment equilibrium around the pin:

$$
\begin{aligned}
&\Sigma M_{pin} = 0 \\
&J\ddot{q} + \left(m R \ddot{q}\right) R + m g \sin q \; R = 0 \qquad \{1\} \\
&2 m R \ddot{q} + m g \sin q = 0
\end{aligned}
$$

We linearize equation {1} by assuming small rotation angle $\sin q \approx q$.
Therefore we have: $2R\,\ddot{q} + g q = 0$, and length of the equivalent simple pendulum is therefore $2R$.