Key Concepts from Lectures (and Some More)

- **Material Balance**

  \[ \text{_____} + \text{_____} = \text{_____} + \text{_____} \]

  Although material balance of all types is always true, whether mass or molar basis, each can be more convenient in certain situations.

<table>
<thead>
<tr>
<th>Type of Balance</th>
<th>Mass (e.g. kg/hr)</th>
<th>Mole (e.g. mol/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total / Stream Gen</td>
<td>( = )</td>
<td>in all cases</td>
</tr>
<tr>
<td>Species / Molecule</td>
<td></td>
<td>Gen. term from</td>
</tr>
<tr>
<td>Element / Atom</td>
<td>Also useful when</td>
<td></td>
</tr>
</tbody>
</table>
<pre><code>                 |                   | there is unknown rxn|
</code></pre>

- **Steady state**

  Accumulation = ___

  A quick test: Look at the system. Turn away for a while. Look at the system again.
  If \( \text{____________} \) \( \Rightarrow \) steady state

- **Basis**

  To make calculation easier. If problem specifies flow rates, use them as basis. Some problems do not specify flow rates; the question will be e.g. “What is the ratio of moles of products to moles of reactants?” instead of “What is the molar flow rate of products?” In these cases, pick a convenient number as basis, e.g. 1000 kg/hr, 1000 mol/hr. Or you can just use some variable like ‘X’.

- **System boundary**

  You can choose whatever you want. Material balances written around any system boundary is always correct but some are more convenient. Be careful not to write redundant equations (see below).

- **Writing equations**

  A well-posed / solvable problem: \( \text{(# of variables)} \) \( \text{(# of ________________ equations).} \)
I. Which inlet and outlet streams will you need to consider if you are writing a material balance around:

1) Overall : ________________________

2) Reactor : ________________________

3) Retr & Clmn I: ________________________

II. Is there something wrong with these diagrams?

1) orange juice
   10% solid
   90% water
   mixer
   recycle stream
   25 % solid
   ACME
   Steady-State
   Juicentrator™
   orange juice concentrate
   40% solid

2) 5 kmol/hr O₂
   4 kmol/hr N₂
   4 kmol/hr H₂
   2 kmol/hr NH₃
   2 kmol/hr NO₂
   1 kmol/hr N₂
   2 kmol/hr HNO₃
Time for a problem in thin “real-life” guise...

Falstaff, Regan, and Shylock are starting a pastrami sandwich business. To maximize production, they set up a production line.

Falstaff will cut two slices of bread and a slice of pastrami and hand it over to Regan who combines them to make the sandwich. Regan hands over the sandwiches she made to Shylock who packs them for sale.

It turns out, though, that Regan cannot make sandwiches as fast as the bread and pastrami are coming. Shylock finds that sometimes Regan would hand over empty pieces of bread to him or sometimes the pastrami would fall out of the bread. In these cases, Shylock would give the loose bread and pastrami to Falstaff who would then give it back to Regan.

At the beginning, there is some piling up of bread, pastrami, and sandwiches on the table. After a while, however, they find that while all three still have their hands full, there is no more piling up and there is no bread or pastrami being thrown away. At this point, Shylock is packing 8 sandwiches every minute.

a) How many pieces of bread does Falstaff slice per minute? How many pieces of pastrami?
b) Shylock observes that, on average, one in every five sandwiches that Regan makes is “bad” and so the bread and pastrami have to be handed back to Falstaff. Knowing this, determine how many pieces of bread Falstaff gives to Regan every minute.

Food for thought:
At some point, Regan thinks to herself, “I may be able to increase our production rate if I am just more careful with my work.” So without telling Falstaff and Shylock (who are busy doing their things), she puts more care in assembling the sandwiches; now she only makes 1 mistake out of 10. What happens to the production rate? If you’re reluctant to make assumptions and need more information, what do you need to know?
Solution for 02/11/02 Recitation Practice Problem

First, convert the problem to unit operations and process streams.
- Falstaff is a mixer at the Feeding point.
- Regan is a Reactor with the reaction of Bread (B) + Pastrami (P) \( \rightarrow \) Sandwich (B\(_2P\)). The reaction is not complete (not 100% conversion) because Shylock says that he receives some B and P besides B\(_2P\).
- Shylock is the Separator that separates unconverted B and P from the desired product B\(_2P\). There is a recycle stream from Shylock that goes back to Falstaff. We also know that the stream going out from S is 8 mol/min B\(_2P\). Furthermore, there is neither loose bread nor loose pastrami leaving the system.
- No piling up on the table \( \Rightarrow \) steady state.
- Let’s label the streams. Also, put down which species exist where according to the problem statement, e.g. Falstaff only hands over B and P to Regan \( \Rightarrow \) no B\(_2P\) in stream 2.

I’ve labeled the streams. Let’s define a flowstream nomenclature: \( F_{x,i} = \text{flow of } x \text{ in stream } i \). For example, \( F_{B2P,4} = (\text{flow of } B2P \text{ in stream } 4) = 8 \text{ mol/min} \).

a) How many pieces of bread and pastrami does Falstaff slice?
In other words, what are \( F_{B,1} \) and \( F_{P,1} \)?
We have information for stream 4 and we are asked about stream 1 \( \Rightarrow \) let’s do a balance over the whole system.

Can we do a species balance? Need to deal with reaction. Doable, but an element/atomic balance seems easier. \( \text{Gen} = 0 \) for element balance (no bread is converted into pastrami and vice versa). \( \text{Steady state} \Rightarrow \text{Acc} = 0 \).

Overall balance on element B:

\[
\begin{align*}
\text{In} & + \quad \text{Gen} & = & \quad \text{Out} & + \quad \text{Acc} \\
F_{B,1} & + \quad 0 & = & (2 \times 8 \text{ mol/min}) & + \quad 0 \\
F_{B,1} & = & 16 \text{ mol/min} \\
\end{align*}
\]

2 B in each B\(_2P\)

Overall balance on element P:

\[
\begin{align*}
\text{In} & + \quad \text{Gen} & = & \quad \text{Out} & + \quad \text{Acc} \\
F_{P,1} & + \quad 0 & = & (1 \times 8 \text{ mol/min}) & + \quad 0 \\
F_{P,1} & = & 8 \text{ mol/min} \\
\end{align*}
\]

1 P in each B\(_2P\)

So Falstaff slices \textbf{16 slices of bread and 8 slices of pastrami per minute}. 

b) Shylock observes that, on average, one in every five sandwiches that Regan makes is “bad” and so the bread and pastrami have to be handed back to Falstaff. Knowing this, determine how many pieces of bread Falstaff gives to Regan every minute.

In recitation, I wasn’t clear on what I mean by “bad” here. What I mean is: 1 out of 5 sandwiches turned out to be not sandwiches but unconverted bread and pastrami → 1 out of 5 pastramis coming out of reactor is not sandwiched → 1 out of 5 P is not converted to B₂P → 20% P was not reacted → 80% of P was reacted → 80% conversion of P, since conversion is ratio of moles reacted to moles fed.

(Note that in this case, bread also has 80% conversion. As mentioned in lecture, when you have stoichiometric feed, you can just specify 1 value of conversion. If not stoichiometric e.g. 8 bread and 9 pastrami, we need to say conversion of which species.)

Doing a B₂P balance around S:

\[
\begin{align*}
\text{In} + \text{Gen} &= \text{Out} + \text{Acc} \\
F_{B₂P,3} + 0 &= 8 \text{ mol/min} + 0 \\
F_{B₂P,3} &= 8 \text{ mol/min}
\end{align*}
\]

Then:

Doing a B₂P balance around R:

\[
\begin{align*}
\text{In} + \text{Gen} &= \text{Out} + \text{Acc} \\
0 + \text{R}_{B₂P} &= 8 \text{ mol/min} + 0 \\
\text{R}_{B₂P} &= 8 \text{ mol/min}
\end{align*}
\]

\[
\text{R}_{B₂P} = \text{rate of generation of B₂P}. \quad \text{This is related to the other reaction terms through stoichiometric coefficients (νᵢ) of the reaction: } \quad 2B + P \rightarrow B₂P.
\]

\[
\frac{R_{B₂P}}{ν_{B₂P}} = \frac{R_B}{ν_B} \Rightarrow R_B = \frac{ν_B}{ν_{B₂P}} R_{B₂P} = -\frac{2}{1} R_{B₂P} \Rightarrow R_B = -2 \cdot (8 \text{ mol/min}) = -16 \text{ mol/min}
\]

Remember: νᵢ < 0 → reactant. νᵢ > 0 → product.

Doing a species B balance on R:

\[
\begin{align*}
\text{In} + \text{Gen} &= \text{Out} + \text{Acc} \\
F_{B₂} + \text{R}_{B} &= F_{B₃,2} + 0 \\
F_{B₂} &= 16 \text{ mol/min} \quad \text{(1 – 0.2) F}_{B₂} = 16 \text{ mol/min} \\
F_{B₂} &= 20 \text{ mol/min}
\end{align*}
\]

\[
\text{80% conversion of B means that 20% of B from stream 2 is not converted.}
\]

So Falstaff gives 20 slices of bread per minute to Regan.

Note: A crucial point here is the realization that F₃,2 = 20% F₂. If this confuses you, try this calculation:

\[
\text{conversion} = \frac{\text{reacted}}{\text{mole fed}} = \frac{\text{mole fed} - \text{mole out}}{\text{mole fed}} = 1 - \frac{\text{mole out}}{\text{mole fed}} \Rightarrow \text{mole out} = (1 - \text{conversion}) \cdot \text{mole fed}
\]
Another way of doing this would be to go around the recycle stream.

First we determine that \( F_{B,3} = 20\% \) \( F_{B,2} = 0.2F_{B,2} \)

Then doing balance around S will give us \( F_{B,5} = F_{B,3} \) (because no loose bread is going out of stream 4).

Then let’s do a balance of bread around Falstaff:

Doing a B element balance on F:

\[
\begin{align*}
\text{In} & + \text{Gen} = \text{Out} + \text{Acc} \\
F_{B,1} + F_{B,5} + 0 &= F_{B,2} + 0 \\
16 \text{ mol/min} + 20\% F_{B,2} &= F_{B,2} \\
16 \text{ mol/min} &= (1 - 0.2) F_{B,2}
\end{align*}
\]

Again, we have \( F_{B,2} = 20 \text{ mol/min} \). Same result as before.

Try doing the problem from different angles. You can go around the loop and check that all material balances are satisfied. This is a good practice problem for you if you are still not comfortable with manipulating system boundary and choosing what balances to write.

**Note on species and element/atomic balance**

Maybe you noticed that we treated Bread sometimes as a species, sometimes as an element. In real chemical systems, we have species like \( N_2, \ NH_3, \) etc. When we do an element balance, we should do it on N, H, etc. Our case is rather special in that B is a species that is also an element. Real chemical analogy would be monoatomic species such as Na, Xe, etc. They are chemical species that is also an element. NaOH is a different species from Na but it contains the element Na.

Note the difference:

- **Species balance:** \( \text{Gen}_i = R_i \) (for generation term, you need to consider reaction of species)
- **Element balance:** \( \text{Gen}_i = 0 \) (no generation of element)

For our case, we have to be specific. Are we doing species or element balance on B?

If we are treating it as species, we shouldn’t count the B’s in \( B_2P \) as a species because it’s not the same species.

If we are treating it as element, we should count the B’s in \( B_2P \) because it is still the same element B.

An example, rewriting what we wrote in the previous page:

Doing a species B balance on R:

\[
\begin{align*}
\text{In} & + \text{Gen} = \text{Out} + \text{Acc} \\
F_{B,2} & + R_B = F_{B,3} + 0 \\
F_{B,2} + (-16 \text{ mol/min}) &= 20\% F_{B,2} + 0 \\
(1 - 0.2) F_{B,2} &= 16 \text{ mol/min} \\
F_{B,2} &= 20 \text{ mol/min}
\end{align*}
\]

We can do also element B balance on R: (Gen = 0 for element, remember)

\[
\begin{align*}
\text{In} & + \text{Gen} = \text{Out} + \text{Acc} \\
F_{B,2} & + 0 = F_{B,3} + 2F_{B_2P,3} + 0 \\
F_{B,2} & + 0 = 20\% F_{B,2} + 2 (8 \text{ mol/min}) \\
(1 - 0.2) F_{B,2} &= 16 \text{ mol/min} \\
F_{B,2} &= 20 \text{ mol/min}
\end{align*}
\]

Same result.