Stream 4 is a superheated steam and stream 3 is a wet steam (i.e., contains some liquid). Stream 5 can either be superheated, saturated or wet. That depends on the relative amounts of streams 3 and 4.

a) To solve for \( m_1 \) and \( m_2 \) we start with a mass balance about the overall system.

\[ m_1 + m_2 = m_5 \]  

\[ \text{Energy balance} \]

\[ m_1 H_1 + m_2 H_2 = m_5 H_5 \]  

Since the process is reversible, the entropy generation is zero.

\[ m_1 S_1 + m_2 S_2 = m_5 S_5 \]
We can easily get the values of \( H_1, H_2, S_1, \) and \( S_2 \) from the steam tables. This leaves us with 5 unknowns \( m_1, m_2, m_3, H_5 \) and \( S_5 \). So we need two more equations to solve the system. We can get these two equations from the fact that upon condensation of stream \( S_1 \), 300 kJ/s is released. Therefore,

\[
\dot{m}_5 \left( H_5 - H_6 \right) = 300 \ \text{kJ/s} \tag{4}
\]

Assuming that point 5 is either a wet steam or saturated steam, condensation will occur at constant temperature (we will need to check that assumption later). Therefore,

\[
\Delta S_{\text{condensation}} = \frac{\omega_{\text{condensation}}}{T_{\text{sat}}}
\]

\[
S_5 - S_6 = \frac{H_5 - H_6}{T_{\text{sat}}} \tag{5}
\]

Now we have the complete set of equations.

From (4),

\[
H_5 = H_6 + \frac{300}{m_5} \tag{6}
\]

From (5) x (4),

\[
S_5 - S_6 = \frac{300}{m_5 T_{\text{sat}}} \Rightarrow S_6 = S_5 + \frac{300}{m_5 T_{\text{sat}}} \tag{7}
\]

Substituting (6) in (3),

\[
m_1 H_1 + m_2 H_2 = m_5 H_6 + 300 \tag{8}
\]

Substituting (8) in (3),

\[
m_1 S_1 + m_2 S_2 = m_5 S_6 + \frac{300}{T_{\text{sat}}} \tag{9}
\]

From (1),

\[
m_2 = m_5 - m_1 \tag{10}
\]

Substituting (10) in (8),

\[
m_1 H_1 + (m_5 - m_1) H_2 = m_5 H_6 + 300 \tag{11}
\]

Same for entropy,

\[
m_1 S_1 + (m_5 - m_1) S_2 = m_5 S_6 + \frac{300}{T_{\text{sat}}} \tag{12}
\]
Solving for $m_1$ in both (1) and (2). From (1)

$$(H_1 - H_2) m_1 = m_5 (H_6 - H_2) + 300$$

$$m_1 = m_5 \left( \frac{H_6 - H_2}{H_1 - H_2} \right) + \frac{300}{H_1 - H_2} \quad \text{--- (13)}$$

From (2)

$$(s_1 - s_2) m_1 = m_5 (s_6 - s_2) + \frac{300}{T_{sat}} \quad \text{--- (14)}$$

Substituting for $m_1$ from (13),

$$m_5 \left( \frac{H_6 - H_2}{H_1 - H_2} \right) + \frac{300}{H_1 - H_2} = m_5 \left( \frac{s_6 - s_2}{s_1 - s_2} \right) + \frac{300}{T_{sat} (s_1 - s_2)}$$

Solving for $m_5$,

$$m_5 = \frac{\frac{300}{T_{sat} (s_1 - s_2)} - \frac{300}{H_1 - H_2}}{\left( \frac{H_6 - H_2}{H_1 - H_2} \right) - \left( \frac{s_6 - s_2}{s_1 - s_2} \right)} \quad \text{--- (15)}$$

Getting the values from the steam tables (F.1).

Stream 1 [saturated steam @ 2700 kPa] \{use closest value from table\}

$$H_1 = 2801.7 \text{ kJ/kg} \quad S_1 = 6.2249 \text{ kJ/kg K}$$

Stream 2 [saturated steam @ 275 kPa]

$$H_2 = 2719.9 \text{ kJ/kg} \quad S_2 = 7.0261 \text{ kJ/kg K}$$

Stream 6 [saturated liquid @ 1000 kPa]

$$H_6 = 763.1 \text{ kJ/kg} \quad S_6 = 2.1393 \text{ kJ/kg K} \quad T_{sat} = 453.15 \text{ K}$$

Substituting in (15),

$$m_5 = 0.1497 \text{ kg/s}$$

Substituting in (13),

$$m_1 = 0.0864 \text{ kg/s}$$

$$m_2 = 0.0633 \text{ kg/s}$$

Therefore,
We still need to check if point 5 lies within the liquid-vapor dome.

Using eqn 6, we get

\[ H_5 = 2747.1 \text{ kJ/kg} \]

less than the saturated vapor enthalpy at 1000 kPa of 2776.3 kJ/kg.

Therefore, the above calculation is valid.

b) Since the process is now irreversible, we can not use equation (2). Equations (1) and (2) are still valid of course. We also know that the work generated by the turbine is used by the compressor. Therefore

\[ m_1 (H_3 - H_1) = m_2 (H_4 - H_2) \]

We can use the thermodynamic efficiencies given to get \( H_3 \) and \( H_4 \). For the turbine, \( \eta = 0.478 = \frac{\text{Work extractive}}{\text{Work actual}} = \frac{H_3 - H_1}{H_3' - H_1} \)

\( H_3 \) is based on an isentropic process. \( S_3' = S_1 = 6.2249 \text{ kJ/kg K} \)

Point 3' is in the liq-vap dome

mass fraction of vapor \( X_{3'} = \frac{S_{3'} - S_2}{S_v - S_2} = \frac{6.2249 - 2.1393}{4.4426} = 0.9196 \)

\[ H_3' = H_3 + X_{3'} (H_v - H_3) = 763.1 + 0.9196 (2013.1) = 2614.4 \text{ kJ/kg} \]

Therefore, \( H_3 = H_1 + \eta_{\text{turbine}} (H_3' - H_1) = 2801.7 + 0.78 (2614.4 - 2801.7) = 2655.6 \frac{\text{kJ}}{\text{kg}} \)

We do the same thing for the compressor to get \( H_4 \).

\[ \eta = 0.75 = \frac{\text{Work extractive}}{\text{Work actual}} = \frac{H_4 - H_1}{H_4' - H_1} \]

\( H_4' \) is based on an isentropic process. \( S_4' = S_2 = 7.0261 \text{ kJ/kg K} \)

Point 4' is in the superheated region. Using table F.2 @ 1000 kPa (p. 683)

\[ H_4' = 2965.2 + \left( \frac{7.0261 - 6.968}{7.0485 - 6.968} \right) (3689 - 2865.2) = 2996.8 \frac{\text{kJ}}{\text{kg}} \]
Therefore, \[ H_4 = H_1 + \frac{H_4 - H_1}{n} = 280.7 + \frac{2996.8 - 2801.7}{0.75} \]

\[ H_4 = 3061.9 \text{ kJ/kg} \]

Now the equations that we can use are:

\[ m_1 + m_2 = m_5 \]  \hspace{1cm} (1)

\[ m_1 H_1 + m_2 H_2 = m_5 H_5 \]  \hspace{1cm} (2)

\[ m_5 (H_5 - H_6) = 300 \]  \hspace{1cm} (3)

\[ -m_1 (H_3 - H_1) = m_2 (H_4 - H_2) \]  \hspace{1cm} (16)

We need to solve these 4 equations to get 4 unknowns \( m_1, m_2, m_5 \), and \( H_5 \).

\[ H_3 = H_6 + \frac{300}{m_5} \]

\[ m_1 H_1 + m_2 H_2 = m_5 H_6 + 300 \]

From (16), \[ m_2 = -m_1 \frac{(H_3 - H_1)}{(H_4 - H_2)} \]  \hspace{1cm} (17)

From (1), \[ m_1 + (-m_1) \frac{H_3 - H_1}{H_4 - H_2} = m_5 \]

Therefore,

\[ m_1 H_1 - m_1 \frac{(H_3 - H_1)}{H_4 - H_2} H_2 = m_1 H_6 \left[ 1 - \frac{(H_3 - H_1)}{(H_4 - H_2)} \right] + 300 \]

Solving for \( m_1 \),

\[ m_1 = \frac{300}{\left[ H_1 - H_6 + \frac{(H_3 - H_1)}{(H_4 - H_2)} (H_5 - H_2) \right]} \]

Substituting, we get:

\[ m_1 = 0.1043 \text{ kg/s} \]

\[ m_2 = 0.0446 \text{ kg/s} \]  \hspace{1cm} (17)
Thermodynamic Analysis

Assume \( T_0 = 300 \text{ K} \)

Using Eqn 16.2,

\[
\dot{W}_{\text{ideal}} = \Delta (H_m)_{f_5} - T_0 \Delta (S_m)_{f_5}
\]

\[
\dot{W}_{\text{ideal}} = +T_0 \left( m \dot{S}_{1} + m \dot{S}_{2} - m \dot{S}_{5} \right)
\]

To get \( S_{5} \), we need \( H_{5} \):

\[
H_{5} = H_{6} + \frac{200}{m_{5}} = 763.1 + \frac{200}{0.1043 + 0.0446} = 2777.9 \text{ kJ/kg}
\]

Point 5 is slightly superheated \( (H_{5} > H_{\text{sat, vap}} @ 1000 \text{ kPa}) \)

By interpolation, \( (\text{page } 683) \):

\[
S_{5} = 6.5828 + \frac{2777.9 - 2776.2}{28.268 - 2776.2} (6.5872 - 6.5828) = 6.5864 \text{ kJ/kg K}
\]

\[
\dot{W}_{\text{ideal}} = +300 \left[ 0.1043 \times 6.2249 + 0.0446 \times 7.0261 - 0.1489 \times 6.5864 \right] = 5.428 \text{ kJ/s}
\]

That means that we could have extracted 5.428 kJ/s from the system, if the process was ideal. Now we need to analyze each unit to calculate the work lost.

Turbine

\[
\dot{W}_{\text{lost}} = +m \dot{S}_{3} - \dot{S}_{1}
\]

To get \( S_{3} \), we use the quality \( x_{3} \):

\[
x_{3} = \frac{H_{3} - H_{\text{v}}} {H_{\text{f}} - H_{\text{v}}} = \frac{2655.6 - 763.1}{2013.1} = 0.94
\]

\[
S_{3} = S_{4} + x_{3} (S_{5} - S_{4}) = 2.1393 + 0.94 (4.4426) = 6.3153 \text{ kJ/kg K} = S_{3}
\]

\[
\dot{W}_{\text{lost, turbine}} = +0.1043 \times 300 (6.3153 - 6.2249) = 2.83 \text{ kJ/s}
\]
Compressor

\[ W_{\text{lost}} = + \dot{m}_2 T_0 (S_4 - S_2) \]

To get \( S_4 \), we interpolate based on the value of \( H_4 = 3061.8 \text{ kJ/kg} \)

\[ S_4 = 7.1009 + \left( \frac{3061.8 - 3050.8}{31.044 - 3050.8} \right) (7.1924 - 7.1009) = 7.1197 \text{ kJ/kg} \]

\[ W_{\text{lost, compressor}} = +0.0446 \times 300 (7.1197 - 7.0261) = +1.252 \text{ kJ/s} \]

Mixing Point

\[ W_{\text{lost}} = +T_0 (\dot{m}_3 S_3 - \dot{m}_1 S_1 - \dot{m}_2 S_4) \]

\[ = +300 (0.1489 \times 6.5864 - 0.1043 \times 6.3153 - 0.0446 \times 7.1197) \]

\[ W_{\text{lost, mixing}} = +1.347 \text{ kJ/s} \]

<table>
<thead>
<tr>
<th>( W )</th>
<th>kW</th>
<th>Percent of ( W_{\text{ideal}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_{\text{act (whole system)}} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( W_{\text{lost (Turbine)}} )</td>
<td>+2.83</td>
<td>52</td>
</tr>
<tr>
<td>( W_{\text{lost (compressor)}} )</td>
<td>+1.252</td>
<td>23</td>
</tr>
<tr>
<td>( W_{\text{lost (mixing)}} )</td>
<td>+1.347</td>
<td>25</td>
</tr>
<tr>
<td>( W_{\text{rev}} = W_{\text{act}} - W_{\text{lost}} )</td>
<td>-5.43</td>
<td>100</td>
</tr>
</tbody>
</table>