\[ \rho = A + Bx_1 + Cx_1^2 \]
\[ = 0.02 - 0.01x_1 + 0.005x_1^2 \]

a) Pure ①, \( x_1 = 1 \), so

\[ \rho_1 = (0.02) - (0.01)(1) + (0.005)(1)^2 \]
\[ = 0.015 \text{ mol/cm}^3 \]

\[ V_1 = \frac{1}{\rho_1} = 66.67 \text{ cm}^3/\text{mol} \]

b) Pure ②, \( x_1 = 0 \), so

\[ \rho_2 = (0.02) - (0.01)(0) + (0.005)(0)^2 \]
\[ = 0.02 \text{ mol/cm}^3 \]

\[ V_2 = \frac{1}{\rho_2} = 50 \text{ cm}^3/\text{mol} \]

\[ \rho \] from Stull, p. 323

\[ \overline{V}_1 = V + x_2 \left( \frac{dV}{dx_1} \right)_{T_1} \rho \]
\[ \overline{V}_2 = V - x_1 \left( \frac{dV}{dx_1} \right)_{T_1} \rho \]

At \( x_1 = 0.8 \),

\[ V = \frac{1}{\rho} = \frac{1}{(0.02) - (0.01)(0.8) + (0.005)(0.8)^2} = \frac{1}{0.0152} \]
\[ = 65.79 \text{ cm}^3/\text{mol} \]
\[
\frac{dV}{dx_i} = \frac{d\left(\frac{1}{\rho}\right)}{dx_i} = -\frac{1}{\rho^2} \frac{d\rho}{dx_i}
\]
\[
= -\frac{1}{\rho^2} \frac{d\left(0.02 - 0.01x_i + 0.005x_i^2\right)}{dx_i}
\]
\[
= \frac{0.01 - 0.005(2)x_i}{\rho^2}
\]
\[
= \frac{0.01 - 0.01x_i}{(0.02 - 0.01x_i + 0.005x_i^2)^2}
\]

\(\text{at } x_i = 0.8,\)
\[
\frac{dV}{dx_i} = \frac{0.01 - 0.01(0.8)}{(0.0152)^2}
\]
\[
= 8.6565 \text{ cm}^3/\text{mol}
\]

\(\text{Thus,}\)
\[
\overline{V}_1 = (65.79) + (1-0.8)(8.6565)
\]
\[
= 67.5 \text{ cm}^3/\text{mol}
\]
\[
\overline{V}_2 = (65.79) - (0.8)(8.6565)
\]
\[
= 58.9 \text{ cm}^3/\text{mol}
\]
c).
\[ V^E = V^{\text{REAL}} - V^{\text{ID}} \]

\[ V^{\text{REAL}} = x_1 \bar{V}_1 + x_2 \bar{V}_2 \]
\[ V^{\text{ID}} = x_1 V_1 + x_2 V_2 \]

\[ \text{where:} \]
\[ V = \text{volume of total solution [cm}^3/\text{mole soln.]} \]
\[ V_i = \text{volume of pure component i (not in solution)} \]
\[ \bar{V}_i = \text{partial molar volume of i/"apparent volume" (in solution)} \]

\[ \text{Thus,} \]
\[ V^{\text{REAL}} = 65.79 \text{ cm}^3/\text{mol (part b1)} \]
\[ V^{\text{ID}} = (0.8)(66.67) + (1-0.8)(50) \]
\[ = 63.33 \text{ cm}^3/\text{mol} \]

\[ V^E = 65.79 - 63.33 \]
\[ = 2.5 \text{ cm}^3/\text{mol} \]

d) See over.
We need to figure out how many moles of 1 and 2 are in the final mixture C. Thus we need to convert weight % to mole %:

- Consider 100 g of soln. C:

  in 100 g soln. \[ \Rightarrow 20 \text{ g } 1 \] = \[ \frac{20 \text{ g}}{40 \text{ g/mol}} = 0.5 \text{ mole } 1 \]

  \[ \Rightarrow 80 \text{ g } 2 \] = \[ \frac{80 \text{ g}}{60 \text{ g/mol}} = 1.33 \text{ mole } 2 \]

- Thus,

  \[ x_1 = \frac{0.5 \text{ mole }}{0.5 \text{ mole } + 1.33 \text{ mole } } = \frac{0.5}{1.83} \]

  \[ x_2 = 1 - 0.273 \]

  \[ = 0.727 \]

What is the molar density of solution C? From our empirical relationship:

\[ \rho_c = 0.02 - 0.01(x_1) + 0.005(x_1)^2 \]

\[ = 0.02 - 0.01(0.273) + 0.005(0.273)^2 \]

\[ = 0.01764 \text{ mol/cm}^3 \]

We want 1,000 cm$^3$ of solution. We thus need:

\[ N_c = V_c \cdot \rho_c \]

\[ = (1,000 \text{ cm}^3)(0.01764 \text{ mol/cm}^3) \]

\[ = 17.64 \text{ moles total in solution C} \]
How much \( \text{g} \) will \( \text{soln. A} \) give us?

Moles of \( \text{g} \) from \( \text{soln. A} = x_2 \text{A} N_A \)

\[ = (0.2)(6.01) \]

\[ = 1.203 \text{ mol} \text{ g} \]

\[ \Rightarrow \text{ but we need 12.82 mol} \text{ g} \]

Thus, we do need to add pure \( \text{g} \) (soln. B)

So,

\[ \text{mol} \text{ g} \text{ req'd} = x_2 \text{A} N_A + x_2 \text{B} N_B \]

\[ = (12.82) = (1.203) + N_B \]

Thus

\[ N_B = (12.82) - (1.203) \]

\[ = 11.62 \text{ mol} \text{ g} \text{ of soln. B} \]

Finally, we want to know what total volumes of \( \text{A} \) and \( \text{B} \) we need to add:

\[ V_A^{\text{total}} = N_A V_A \]

\[ = (6.01)(65.79) \]

\[ V_A^t = 395.4 \text{ cm}^3 \]

\[ V_B^{\text{total}} = N_B V_B = N_B V_2 \]

\[ = (11.62)(50) \]

\[ V_B^t = 581.0 \text{ cm}^3 \]

*Note: \[ V_A^t + V_B^t = 976.4 \text{ cm}^3 < 1,000 \text{ cm}^3 \text{ because of volume increase w/ mixing} \]