Before we start solving the problem, it is important to understand how the automobile air conditioners work. They use refrigeration cycles as shown in the diagram.

The cycle is divided into two sections, hot and cold. The refrigerant (point 1) is compressed to a high pressure using a compressor. Then it passes through a condenser in which it is condensed to saturated liquid at high pressure and temperature. During condensation, heat is exhausted to the atmosphere. Hence, the temperature of the condensation has to be at least 10 °F higher than that of the atmosphere so that heat can flow efficiently from the condenser to the atmosphere. Then the refrigerant passes through a valve that reduces the pressure of the refrigerant significantly to atmospheric pressure. During the process of expansion, no heat or work is transferred. This makes the expansion isenthalpic (constant enthalpy). The refrigerant passes through an evaporator at low pressure and temperature. During evaporation, heat (\(Q_c\) for cold) is transferred to the fluid (why?) at constant pressure (why?). \(Q_c\) is provided from within the car and thus the insides of the car cool down. Then the cycle continues. Let us draw this cycle on a pressure-enthalpy diagram.

Point 3 is a good place to start because we know that it is a condensed liquid (saturated) at high temperature (120 F in this problem) and high pressure. Since, the expansion occurs at constant enthalpy, the process is depicted by a vertical line on the pressure-enthalpy diagram as shown and point 4 is the intersection of this line with the P=1 atm line. Notice that during expansion, some of the saturated liquid evaporates and we know that because point 4 lies inside the liquid-vapor dome.
Then evaporation occur at constant temperature and pressure to give point 1 (saturated vapor). The fluid at point 1 is then compressed. If compression is reversible, the process will be isentropic. To reflect irreversibility inherent in the compression process, the entropy of point 2 is higher than that of point 1. Then the fluid is condensed at this high temperature and pressure back to point 3. Now we can start solving the problem.

a) Justify why replacing R-12 by R134a is reasonable based on the theory of corresponding states

The theory of corresponding states suggests that fluids would behave similarly if they have similar reduced temperatures and pressures. Since we are interested in a wide range of temperature and pressure, for the two refrigerants to behave similarly, they need to have similar critical temperatures and pressures. This will ensure that, at any T and P, the reduced temperatures and pressures will be similar.

For R-12 (from last entry in the table for saturated properties)

\[
T_c = 233.6 \, ^\circ\text{F} = 385.15 \, \text{K}
\]

\[
P_c = 596.9 \, \text{psia}
\]

For R-134a

\[
T_c = 213.9 \, ^\circ\text{F} = 374.21 \, \text{K}
\]

\[
P_c = 588.9 \, \text{psia}
\]

The % difference in \(T_c\) = \((385.15 - 374.21) / 385.15 = 2.84\) % (notice that we used absolute temperature to calculate the % difference)

The % difference in \(P_c\) = \((596.9 - 588.9) / 596.9 = 1.3\) %

The difference is within 3% for both \(T_c\) and \(P_c\). Hence based on the theory of corresponding state, their behavior at any T and P would be similar.

For Ammonia (Table B.1)

\[
T_c = 405.7 \, \text{K}
\]

\[
P_c = 112.80 \, \text{bar} = 1636 \, \text{psia}
\]

It is clear that the critical pressure is more than double that of R12, which means that ammonia would behave very differently from R12 and thus it is not a good substitute.
b) **Determine the pressure of saturated refrigerants at 120 °F**

We can use either the charts or the tables to determine the saturation pressure. Usually we use the tables for saturation properties because they are much easier. It is also correct to use the Antoine equation (or any other vapor pressure equation), but this is highly discouraged when tables and/or charts are present.

For R12,
Saturation pressure at 120 °F is 172.35 psia

For R134a
Saturation pressure at 120 °F is 186.023 psia

If a lower operating pressure is the only desired criterion for the design, clearly R12 is a better choice.

c) **Arizona’s Summer**

The design specification given thus far is the temperature of the condenser (see diagram) which is 120 °F. Since the condenser exhaust heat to the atmosphere, the environment has to be at least 10 °F less than the temperature of the condenser for heat to be exhausted efficiently. Since the temperatures in the Arizona desert can reach 120 °F (not particularly pleasant), the condenser has to be designed to operate at a higher temperature (e.g. 140 °F). So the design specification given would not be appropriate for cars driven in Arizona during the summer.

d) **Enthalpy, temperature and weight percent**

It is fairly straightforward to determine the enthalpy of R134a from the saturation table at 120 F.

\[ H_3 = 52.4 \text{ BTU/lb} \]

Since expansion occurs at constant enthalpy, we know the enthalpy of point 4.

\[ H_4 = 52.4 \text{ BTU/lb} \]

We can get the temperature of point 4 from the chart, knowing its enthalpy and pressure (1 atm). However, it is not an easy task to get an accurate figure from the chart (notice that the vertical axis has logarithmic scale). We can use our knowledge that the constant temperature lines inside the dome are horizontal lines and use the saturation tables to get the temperature of the saturated liquid (or vapor) at 1 atm. However, the saturation tables given do not go to this low pressure. We are given, instead, superheated tables. Usually the superheated tables contain some information about the saturation properties (c.f. the steam tables). So we look at the superheated table for P=1 atm (14.696 psia) to find that the first two lines are for saturated liquid and vapor. Their temperature is –14.9 °F (which would be close to our estimate from the chart).

\[ T_4 = -14.9 \text{ °F} \]

The weight percent of the liquid can also be deducted from the chart (the quality lines are the vapor weight fraction). But we will use the tables again to get a more accurate estimate. From the table we know

\[ H \text{ (saturated liquid at 1 atm)} = 7.5 \text{ BTU/lb} \]
\[ H \text{ (saturated vapor at 1 atm)} = 100.9 \text{ BTU/lb} \]

Using the equation (6.57) for enthalpy
\[ H = (1-x^v) H^l + x^v H^v \]

Which can be changed to

\[ H = x^l H^l + (1-x^l) H^v \]

Solving for \( x^l \), we get

\[
\frac{x^l}{1-x^l} = \frac{H^v - H}{H^v - H^l} = \frac{100.9 - 52.4}{100.9 - 7.5} = 0.519
\]

Therefore, the weight percent of the liquid at point 4 is 51.9%.

e) The required flow rate of R134a

The cooling load is given to the evaporator from the insides of the car. To get the mass flow rate, we use

Heat flow rate = mass flow rate * heat flow per unit mass

The heat flow per unit mass is \( Q_c \) and the heat flow rate is the cooling load given in the problem (5 BTU/hr). So if we can get \( Q_c \), we can calculate the mass flow rate using

\[
\text{mass flow rate} = \frac{\text{cooling load}}{Q_c}
\]

Since the evaporator is an open system, we have

\[ \Delta H = Q + W \]

There is no work associated with the evaporator. Therefore,

\[ \Delta H_{11} = Q_c \]

Point 1 is saturated vapor at 1 atm. Its enthalpy is known from part d

\[ H_1 = 100.9 \text{ BTU/hr} \]

The enthalpy of point 4 is also known

\[ H_4 = 52.4 \text{ BTU/hr} \]

Therefore,

\[ \Delta H_{11} = 100.9 - 52.4 = 48.5 \text{ BTU/hr} \]

Then we can calculate the mass flow rate.

\[
\text{mass flow rate} = \frac{5}{48.5} = 0.1031 \text{ lb/hr}
\]