FROM ECONOMIES OF DENSITY AND NETWORK SCALE TO
MULTIOUTPUT ECONOMIES OF SCALE AND SCOPE: A SYNTHESIS

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1. INTRODUCTION

For more than twenty years, the cost structure of transport industries in general, and the airline industry in particular, has been analyzed through the calculation of two indices: returns to density (RTD) and returns to scale (RTS), which were originally proposed by Caves et al. (1984). During this period, there have been many observations, amendments, caveats and corrections about their calculation (e.g. Gagne, 1990; Antoniou, 1991; Ying, 1992, Filippini and Maggi, 1992; Xu et al. 1994). However, both RTD and RTS have become the textbook tools to examine empirically, from a cost perspective, important issues such as merging, or the optimal shape and size of transport networks (see for instance Small, 1992; Braeutigam, 1999; Pels and Rietveld, 2000).

The objective of this article is to organize this discussion, and to offer an interpretative synthesis of a number of results on the subject that have been obtained by the authors. The main conclusion is that RTD and RTS should be replaced with three concepts: a corrected version of economies of density (Jara-Díaz and Cortés, 1996), the multi output degree of economies of scale (Basso and Jara-Díaz, 2006a), and the degree of economies of spatial scope (Basso and Jara-Díaz, 2005).

The article is organized as follows. In the next Section we present the concepts of scale and scope in general, and their application to transport industries in particular. In Section 3 we explain what has been the usual way to analyze transport cost structure in the literature; we show here what we think are the shortcomings of RTD and RTS, and propose methodologies to empirically estimate a corrected version of economies of density, economies of scale, and economies of spatial scope. In the fourth Section, we synthesize the analytical results and show, with (rudimentary) empirical evidence, how the proposed methodologies affect conclusions regarding the technology and the expected structure of transport industries. Section 5 concludes.

2. SCALE AND SCOPE IN TRANSPORT

The output of a transport firm is a vector of flows between many origins-destination pairs, of the form $y_{ijyt}$, disaggregated by the type of cargo $k$ and period $t$ (Jara-Díaz, 1982). If we keep only the spatial dimension of output, transport product is a vector of components $y_{ij}$. In order to produce a flow vector $Y$, the firm has to take a number of decisions: number and capacity of vehicles (fleet size), design of the ways (location, capacity), design of terminals (location, loading and unloading capacities), frequencies, and so on. Some decisions involve choices about the characteristic of inputs, while
others are related to their use, that is, to the way in which inputs are combined in order to produce the flow vector; we shall call the latter type of decisions “operation rules”. Since transport takes place on a network, the firm also has also to choose a service structure—the generic way in which vehicles visit the nodes in order to produce the flows—and a link sequence. Together, these two endogenous decisions define a route structure, which is to be chosen based on exogenous information, namely the OD structure of demand (defined by vector $Y$), and the physical network (Jara-Díaz and Basso, 2003). It is important to note that, in the end, the route structure decision is a consequence of the spatial dimension of transport output.

To illustrate these concepts, consider a three nodes OD system as in Figure 1.a, together with a physical network as in Figure 1.b.

![Figure 1. OD structure and physical network](image1)

For a given vector $\{y_{ij}\}$, the best combination of inputs and operation rules will depend on many factors. Three possible service structures are shown in Figure 2 (Jara-Díaz, 2000). Structure (a) corresponds to a general cyclical system (Gálvez, 1978), structure (b) to three simple cyclical systems (point-to-point service), and structure (c) to hub-and-spoke, where a distribution node—the hub—has been created (this structure is very pervasive in the airlines industry; the hub may coincide or not with an origin or a destination).

![Figure 2. Service structures](image2)
Regarding the allocation of vehicles to fleets—one of the components of a service structure—in case (a) there can only be one fleet (and therefore only one frequency), while in cases (b) and (c) there may be up to three different fleets. If the service structure chosen is as in Figure 2.a, a possible route structure is the one shown in Figure 3.

![Figure 3. A route structure](image)

Conceptually, the transformation of inputs $X$ into outputs $Y$—in this case flows—may be represented through a transformation function $F(X, Y) \geq 0$ (see Jara-Díaz and Basso, 2003), where equality represents an efficient use of inputs. The multioutput degree of economies of scale, $S$, is defined as the maximal equiproportional expansion of $Y$, $\lambda^S Y$, that is possible after an equiproportional expansion of $X$ to $\lambda X$ (Panzar and Willig, 1977). Analytically,

$$F(\lambda X, \lambda^S Y) = 0$$  \hspace{1cm} (1)

A value of $S$ larger, equal or smaller than 1 implies increasing, constant or decreasing returns to scale respectively. Hence, what is relevant in scale analysis is the optimal combination of inputs when all components of the output vector increase by the same proportion. Under some simple regularity conditions, $S$ can be calculated from the cost function $C(w, Y)$, which represents the minimum expenditure necessary to produce $Y$ at input prices $w$. In particular, increasing returns to scale imply that an equiproportional expansion of $Y$ will induce a less than proportional increase in cost. In general, and omitting input prices for notational simplicity, $S$ can be calculated from $C$ as,

$$S = \frac{C(Y)}{\sum_i \frac{\partial C}{\partial y_i}} = \frac{1}{\sum_i \eta_i}$$  \hspace{1cm} (2)

where $\eta_i$ is the elasticity of $C$ with respect to the $i$-th output. Applying this to the case of transport, the degree of economies of scale will depend on the operational re-organization that the firm may achieve, after proportional increases of the flow vector. For example, if the flows are small and similar,
the OD structure of Figure 1.a may be well served with small vehicles and a service structure as in Figure 2.a. If flows were larger, service structures such as (b) or (c) may become more attractive. Hence, not only the number and size of inputs is relevant, but also the spatial re-design of service. What is important to note here is that the firm chooses endogenously its route structure, which is indeed not assumed as fixed in equation (2).

On the other hand, economies of scope exist if

\[ SC_A = SC_B = \frac{C(Y^A) + C(Y^B) - C(Y^D)}{C(Y^D)} \]  

(3)

is positive, where \( D \) is the set of all outputs, \( A \cup B = D \) and \( A \cap B = \emptyset \) (i.e. \( A \) and \( B \) are an orthogonal partition of \( D \)). \( Y^A \) is vector \( Y^D \) but with \( y_i = 0, \forall i \in A \subseteq D \); \( Y^B \) is defined analogously. Therefore, a negative value for \( SC_A \) indicates that it is cheaper to have a second firm producing \( Y^B \), rather than to expand the production line of a firm already producing \( Y^A \). If \( SC_A \) is positive, then it is cheaper that a single firm produces everything (\( Y^D \)). It is easy to verify that \( SC \) should lie in the [-1;1] interval.

Since the output in transport is a vector of OD flows, an expansion of the line of production necessarily implies serving new OD pairs. Therefore, in transport, economies of spatial scope are analyzed in a context in which the size of the network—understood as the OD structure—changes. This does not happen with \( S \). Hence, \( SC \) enables to examine whether it is cost convenient that a firm \( A \), who serves \( PS_A \) nodes and potentially \( PS_A \cdot (PS_A^D - 1) \) OD flows, expands its network to \( PS_D \) nodes—serving \( PS_D \cdot (PS_D^D - 1) - PS_A \cdot (PS_A^D - 1) \) new flows—, or if it is more cost convenient that other firm does it. If firm \( D \) produces on the OD structure of Figure 1.a, a possible partition is the one show in Figure 4. Note that analyzing this type of economies of (spatial) scope is equivalent to analyze an increase in one node of firm \( A \)'s network.

\[
[A] \quad Y^A = \{y_{12}; y_{21}; 0; 0; 0\} \\
[B] \quad Y^B = \{0; 0; y_{13}; y_{23}; y_{21}\} \\
[D] \quad Y^D = \{y_{12}; y_{21}; y_{13}; y_{23}; y_{21}; y_{22}\}
\]

![Figure 4. Variable network size and spatial scope](https://example.com/image.png)

For synthesis, and emphasizing the spatial dimension of output in the transport case, with \( S \) one analyzes the behaviour of costs after an equiproportional expansion of the OD flows keeping the number of OD pairs constant, while with \( SC \) one analyzes the behaviour of cost when new OD flows are added.

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3. RTD, RTS, AND EMPIRICAL METHODS FOR SCALE AND SCOPE

The large size that the output vector $Y$ achieves in practice precludes its direct use in empirical work. It is a necessity, then, to estimate cost functions using aggregate output descriptions, $\tilde{Y} = \{\tilde{y}_h\}$, which represents outputs and attributes such as ton-kilometers, seat-kilometers, average distance or load factor. When a network size variable, $N$, is included in the estimation, empirical studies of transport industries distinguish between two concepts of ‘scale’: returns to density (RTD) and returns to scale (RTS). In the former it is assumed that the network is fixed when output increases; it is said that traffic density increases. In the latter, though, both output and network size would increase, keeping traffic density unchanged. RTD is calculated as the inverse of the sum of a subset of the cost-output elasticities. This subset varies from study to study, which has become a source of ambiguity. In RTS, the elasticity of the network size is also included in the calculation. Several empirical studies on the airline network (where the number of points served, $PS$, is usually the network size variable) have reported the existence of increasing returns to density ($RTD>1$) and constant returns to scale ($RTS \approx 1$). This would indicate that there would be cost advantages if the density of traffic is increased, but there would not be such advantages if firms operated larger networks. However, the observed behaviour in the industry has been different: after the deregulation, in the US first and then in the rest of the world, the concentration of the industry and the size of the networks have increased through mergers, acquisitions and alliances. These efforts from the part of firms to increase their network size—in seeming contradiction with the constant returns to scale—provoked a re-examination of the methods used to calculate economies of scale (e.g. Gagné, 1990; Ying, 1992; Xu et al., 1994 and Oum and Zhang, 1997).

Jara-Díaz and Cortés (1996, hereafter JDC) proposed a different approach to study economies of scale in transport. They noted that behind the aggregates included in the vector $\tilde{Y} = \{\tilde{y}_h\}$, lies the real output of a transport firm, that is, the vector $Y = \{y_{ij}^k\}$ of flows of type $k$ between origins $i$ and destinations $j$ in periods $t$. JDC noted that the inability of using $Y$ in the empirical work does not mean that its definition should be abandoned when using an estimated cost functions to make economic inferences. If the estimated function represents well the real multioutput cost function, then the characteristics of the latter should be obtainable from the estimated parameters of the former. Let us take the case of economies of scale. Since economies of scale analyze the behaviour of costs when the output vector increases equiproportionally, a correct calculation of economies of scale in transport would be related to an increase in the same proportion of all the flows in $Y$. This may be analyzed from an estimated cost function $\tilde{C}(\tilde{Y}, N)$ if one examines the behaviour of aggregates $\tilde{y}_h$ when $Y$ varies. If the aggregates can be actually described as functions of $Y$, i.e. $\tilde{y}_h = \tilde{y}_h(Y)$, then $\dot{C}(Y) = \dot{C}(\tilde{Y}(Y), N)$ can be considered as an approximation of the cost function in terms of $Y$. Calculating from $\dot{C}(Y) = \dot{C}(\tilde{Y}(Y), N)$ the elasticities of cost with respect to the components of $Y$,
JDC obtained a method to calculate the degree of economies of scale. First, disaggregate marginal costs can be calculated as:

$$\frac{\partial \hat{C}}{\partial y_i} = \sum_{j=1}^{n} \frac{\partial \hat{C}}{\partial \hat{y}_j} \frac{\partial \hat{y}_j}{\partial y_i}.$$  \hspace{1cm} (4)

The corresponding cost elasticities with respect to $y_i$ are

$$\hat{\eta}_i = \frac{\hat{C} y_i}{C} = \frac{y_i}{C} \sum_{j=1}^{n} \frac{\partial \hat{C}}{\partial \hat{y}_j} \frac{\partial \hat{y}_j}{\partial y_i} = \sum_{j=1}^{n} \frac{\partial \hat{y}_j}{\partial y_i} \frac{\partial \hat{C}}{\partial \hat{y}_j} \frac{\hat{y}_j}{C} = \sum_{j=1}^{n} \epsilon_{ji} \hat{\eta}_j,$$  \hspace{1cm} (5)

where $\epsilon_{ji}$ is the elasticity of aggregate output $\hat{y}_j$ with respect to $y_i$, and $\hat{\eta}_j$ is the elasticity of $\hat{C}$ with respect to $\hat{y}_j$, i.e.

$$\epsilon_{ji} = \frac{\partial \hat{y}_j}{\partial y_i} \frac{y_i}{\hat{y}_j} \quad \text{and} \quad \hat{\eta}_j = \frac{\partial \hat{C}}{\partial \hat{y}_j} \frac{\hat{y}_j}{C}.$$  \hspace{1cm} (6)

In this way, the correct calculation for an estimator of $S$, $\hat{S}$, is

$$\hat{S} = \left[ \sum_i \eta_i \right]^{-1} = \left[ \sum_j \alpha_j \hat{\eta}_j \right]^{-1} \quad \text{where} \quad \alpha_j = \sum_i \epsilon_{ji}.$$  \hspace{1cm} (7)

Note that $\alpha_j$ is the degree of homogeneity of the $j$-th aggregate with respect to the disaggregated flows, and that its calculation avoids the discussion regarding which aggregate should be considered in the calculation of $S$. $\hat{\eta}_j$ is the elasticity which is obtained directly from the estimated cost function.

Since the number of OD pairs do not change when flows increase, JDC argued that the elasticity of the network size should never be included in a scale calculation. Recall that this is also imposed in the calculation of RTD. Does this mean that $RTD$ is actually $S$, as suggested by Panzar (1989)? Not exactly; although the main idea behind $RTD$ is to examine the behavior of costs when there is an output increase but the network size does not change, Basso and Jara-Díaz (2006a) identified a second implicit condition behind its calculation: the route structure also remains unchanged. This condition is required because the idea of estimating the degree of economies of density is to analyze whether “the average costs of a direct connection decreases with proportionate increases in both flows on that connection” (Hendricks et al., 1995). Hence, a fixed route structure is needed to ensure that only the existing links handle the new traffic. If the route structure changes, some new links may be added while others may disappear. This condition, however, was not even mentioned in the strict calculation of $\hat{S}$ synthesized above. Along these lines, Basso and Jara-Díaz (2006a) proposed to distinguish $RTD$ from
S, assuming the route structure fixed in the former, but variable in the latter. Obviously, this distinction induces differences in the application of equation (7), particularly in the calculation of the \( \alpha_j \). For example, in RTD, the \( \alpha_j \) of the average distance will always be zero as flows grow by the same proportion holding the route structure fixed; it could be different from zero in S if the minimum cost occurs for a different rout structure after flows grow. We consider this distinction to be useful and relevant. Economies of density will be useful to know if, for example, there are economies of vehicle size, that is, if larger flows in non-stop routes imply decreasing average costs in that route because of larger vehicles. Hub-and-spoke networks would be strongly influenced by the existence of economies of density. On the other hand, economies of scale S, are important because, when traffic increases significantly, it may not be efficient to further increase the size of the vehicles, while a frequency increase may be expensive because of congestion. With a reconfiguration of the route structure however, it may happen that the increases in flows may be handled without increasing costs too much; for example, through point-to-point service in certain OD pairs (phenomenon that has been observed; see Swan, 2003).

As explained above, RTS is aimed at analyzing the behaviour of costs when both traffic and network size increase by the same proportion. As in RTD, the proportional increase applies to the vector of aggregates \( \tilde{Y} = \{\tilde{y}_i\} \) (or a sub-vector), but in this case the network size variable \( N \) also increases. Although this makes RTS look like the definition of scale, it is not the case because of two interrelated structural reasons. First, as shown by Basso and Jara-Díaz (2006b), this procedure, performed on the aggregates, imposes analytical conditions on the OD flow vector which seem to be indefensible. Second, increasing \( N \) implies the variation of the number of OD pairs, that is, a variation on the dimension of \( Y \), which is something that should be examined with a scope analysis.

As an example of the strange implicit conditions imposed by RTS on \( Y \), consider the cost function \( \tilde{C} = \tilde{C}(PK, ALT, PS) \), where \( PK \) is passenger-kilometres, \( ALT \) is average length of trip, and \( PS \) is the number of points served. If only the expansion of \( PK \) and \( PS \) were considered (only two elasticities in the calculation of RTS), then it is imposed that (Basso and Jara-Díaz, 2006b)

\[
\frac{y'}{y} = \frac{1}{2} \cdot \frac{PS - 1}{PS},
\]

where \( y' \) is the average of the new flows served, \( y \) is the average of the flows served originally, and \( PS \) is evaluated at its starting value. Hence, (8) shows that RTS implicitly examines the addition of flows that are, on average, between a quarter and a half the original ones, depending on the original number of points served. Note that the very fact that the condition depends on the value of \( PS \) is an undesirable property, because comparisons between firms, who are likely to have different number of points served, is a useless exercise.
If both \( PK \) and \( ALT \) were considered in the calculation of \( RTS \) (plus the elasticity of \( PS \), which is always considered by definition), the conditions on the flows would be even stronger. Denoting with an asterisk the value of the aggregates after the network expansion, it holds that \( PK^* = \lambda PK \) and \( ALT^* = \lambda ALT \) (equiproportional expansion of both “products”). However, since \( ALT^* = PK/P \) where \( P \) is the total number of passengers, then \( PK^*/P^* = \lambda PK/P \), which leads to \( P^* = P \). That is, the total number of passengers, before and after the expansion of the number of points served, is the same. Hence, calculation of \( RTS \) assumes that the new flows to be served after the network expansion will be zero. And this is independent of the network, the number of points served and the route structure: a new node is added, but nothing arrives to or departs from it.

As stated above, the main problem with \( RTS \) is that it was designed as a scale index, but it examines a problem that should be dealt with as a scope problem: network size. The empirical problem is that a direct calculation of \( SC \) using Equation (3) is seldom feasible (an example is Jara-Díaz, 1988). However, the approach proposed by JDC delivers a way to deal with the problem: since most aggregates \( \tilde{y}_h \) are implicit functions of \( Y \), even though the (disaggregate) output vectors \( Y^A \), \( Y^B \) and \( Y^D \) might be unknown, \( SC \) might be calculated correctly if the corresponding aggregate vectors. \( \tilde{Y}(Y^A) \), \( \tilde{Y}(Y^B) \) and \( \tilde{Y}(Y^D) \) were known, and a cost function \( \tilde{C}(\tilde{Y}, N) \) was available (Jara-Díaz, Cortés and Ponce, 2001). Analytically, and considering \( PS \) as the network size variable, scope could be calculated as

\[
SC_A = SC_B = \frac{\tilde{C}(\tilde{Y}(Y^A), PS^A) + \tilde{C}(\tilde{Y}(Y^B), PS^B) - \tilde{C}(\tilde{Y}(Y^D), PS^D)}{\tilde{C}(\tilde{Y}(Y^D), PS^D)}
\]  

(9)

Note that the arguments in \( \tilde{C}(\tilde{Y}, PS) \) in Equation (9) are likely never evaluated at zero, as opposed to what happens by definition with \( C(\cdot) \) in (3). This occurs because aggregates (such as total passengers or ton-kilometres) do not go to zero when only some OD flows are zero, as is the case with \( Y^A \) or \( Y^B \) in Figure 4. The problem is then reduced to study the behaviour of the aggregates under different orthogonal partitions of \( Y \), when possible.

It is important to explain that the calculation of equation (9) can be seen from different perspectives. For example, if one knows \( \tilde{Y}(Y^A, PS^A) \) and the functions \( \tilde{y}_h \equiv \tilde{y}_h(Y) \), the problem becomes identifying \( Y^D \), \( Y^B \) (which must be orthogonal to \( Y^A \)), \( PS^D \) and \( PS^B \), in order to generate \( \tilde{Y}(Y^B) \), \( \tilde{C}(\tilde{Y}(Y^B), PS^B) \), \( \tilde{Y}(Y^D) \) and \( \tilde{C}(\tilde{Y}(Y^D), PS^D) \). Analytically, the challenge is to find out a system of equations that allows these calculations.

The previous discussion provides a way to face the problem of calculating \( SC \), but it does not solve everything. Analyses of several cases—particularly the one presented by Basso and Jara-Díaz (2005)—have shown the necessity to
assume, in occasions, conditions on the new flows that appear after the network expansion; for example, to say something explicit about how the non-nil flows in $Y^b$ compare to those in $Y^a$ in Figure 4. This is not something new, because in $RTS$ there are related (but implicit) conditions through the constant density imposition, as we have shown. When these sorts of conditions are needed, we have deemed reasonable to impose that the new flows will be, in average, similar to the existing ones. This is, precisely, what some authors (e.g. Braeutigam, 1999) suggested it was imposed by $RTS$.

For synthesis, Table 1 shows which indicator should be calculated with the methods presented in this article, and under what conditions. We believe that these deliver correct inferences about the technological characteristics of transport industries from cost functions estimated with aggregate product.

<table>
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<th>Table 1. Conditions for the calculations of cost structure indices in transport</th>
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<td>Fixed route structure</td>
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<td>Fixed network size</td>
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<td>Variable network size</td>
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4. EMPIRICAL EVIDENCE AND POLICY CONCLUSIONS.

We established in the previous Section that proper calculations of $RTD$ and $S$ require estimation of the $\alpha_j$ coefficients. In our analysis of the US airline industry (Basso and Jara-Díaz, 2006a), for the average length of trip, we used simple log-linear equations to estimate $\alpha_{ALT}$ from annual information of five airlines in the period 1980-1989. The coefficients obtained where used together with the cost functions reported by Liu and Lynk (1999), given the important overlap on the data used in both cases. We obtained $RTD=1.161$ and $S=1.378$. This means modest returns to density but clear returns to scale (in the $S$ sense), which reveals the importance of identifying the opportunities that carries have by re-designing route structures when trying to accommodate increasing volumes of traffic. Methodologically, these results reinforce the necessity of distinguishing between concepts such as network size and route structure—and hence between economies of density and economies of scale on a fixed-size network—when studying transport industries.

Regarding the replacement of $RTS$ by $SC$, we applied our approach to the study reported by Gillen et al. (1990), which is based on Gillen et al. (1985). We chose this article for several reasons. First, a declared objective is to extend the knowledge about the cost structure of airlines when there is significant variation in size, and clarify the issue of economies of scale for

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small firms. Second, the estimated cost function includes aggregates and attributes that allow for a relatively direct application of the method we proposed. Third, the econometric work was carefully made, which makes the quantitative results reliable for analysis. Finally, their calculation of $RTD$ is a correct estimation of economies of density, according to the JDC and Basso and Jara-Díaz (2006a) approaches.

The article uses data on six Canadian airlines, regional and transcontinental, in the period 1964-1980. The authors report a long-run translog cost function, where the variables are expressed as deviations around their means. Our analysis focused on three airlines, two regional and one transcontinental (Air Canada). The reported $RTS$ values for the three firms are 0.88, 0.99 and 1.15 en 1980. However, all values of $SC$ calculated with our approach happened to be positive (and below one, as it should be theoretically). In addition, the largest firm showed the largest value for $SC$. The $RTS$ values do not contribute to explain satisfactorily the observed process of consolidation of Canadian regional airlines, since they do not indicate cost advantages for larger networks, except in one case. On the other hand, our results show that airlines with small network size have larger values of $SC$, independently of their density and $RTD>1$, meaning that they have larger cost advantages if they expand their networks. While the presence of increasing returns to density stimulate merging of firms that operate the same or heavily overlapped networks (parallel merging), the presence of increasing returns to spatial scope stimulate merging of firms that operate networks with minimum overlapping. And the latter seems to have been the case, because a series of statements established in the 1960s, particularly 1969’s regional carrier policy, confined the regional carriers’ networks to their respective geographical areas with minimum overlap, showing that the potential for parallel merging was limited. This makes our finding of increasing returns to spatial scope a relevant cost explanation for the consolidation observed afterwards, as it would be a means to enlarge the network size in order to take advantages of these economies.4

5. SYNTHESIS AND CONCLUSIONS

Returns to density, $RTD$, and returns to scale, $RTS$, are the indices currently accepted to analyze, from a cost perspective, important aspects of transport industry structure such as merging and network size. Both concepts, however, are applied in a context in which the definition of output is highly aggregated, as opposed to the detailed output description that can be made in theory (Jara-Díaz, 1982). The direct application of multioutput scale properties to this aggregate description of output provokes several ambiguities which many authors have pointed out,5 but has not prevented the use of both indices as standard textbook measures.

After showing that the aggregates may be expressed as functions of the many flows that a transport firm produces between many origin-destination pairs, we have argued analytically that: (i) a corrected version of what today is known as economies of density, $RTD$, is close to what has been defined to be the
multioutput degree of economies of scale, but it has an extra assumption. (ii) The precise calculation of the multioutput degree of economies of scale, $S$, requires to relax the imposition of a fixed route structure, but not the condition of a fixed-size network. This leads to the inclusion of some elasticities in the calculation, such as the average length of trip, which have only been considered before in the context of variable network size. (iii) RTS is inadequate to analyze optimal shape and network size and should be replaced by the calculation of the degree of economies of spatial scope.

We have shown that the calculation of (corrected) $RTD$ and $S$ require the estimation of the degree of homogeneity of each aggregate with respect to the disaggregated flows, considering a fixed route structure in the former, and a variable route structure in the latter. Calculation of $SC$ requires the identification of relations that allow finding out the value achieved by the aggregates when evaluated at an orthogonal partition of the (disaggregated) output vector.

Finally, we showed with rudimentary empirical evidence how the conclusions about the technological structure of transport firms, and the consequent policy conclusions, are affected by the proposed methodologies.

REFERENCES


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NOTES

1 For example, Caves, Christenesen and Treheway (1984), Kirby (1986), Gillen, Oum and Treheway (1990), Kumbhakar (1992), Keeler and Formby (1994) and Baltagi, Griffin and Rich (1995)

2 It is important to explain here that what the analysis of cost functions seek to do, is to determine the number of firms that constitute a feasible and efficient industry configuration (Baumol et al., 1982). This does not mean that it is logical to think that only efficient configurations can exist in reality. What these studies attempt to do, instead, is to determine the concentration levels that are required according to productive efficiency considerations. Now, even though there may be important reasons why firms may stay away from an efficient configuration (e.g. strategic interaction between firms or transaction costs), productive efficiency is indeed an important economic force that makes these industry configurations particularly likely. Thus, there may be strategic reasons why a monopoly may subsist in an industry that is structurally competitive, but the argument does not hold the other way around (see Panzar, 1989).

3 American Airlines, Continental, Delta, United and Northwest

4 It is important to note that, since we did not have access to variance-covariance matrices, we were not able to calculate confidence statistics for our estimations of SC.

5 Other important references to these problems are Daughety (1985) and specially Panzar (1989).