The Atmosphere:  
Part 5: Large-scale motions

- Composition / Structure
- Radiative transfer
- Vertical and **latitudinal heat transport**
- **Atmospheric circulation**
- Climate modeling

*Suggested further reading:*

Calculated rad-con equilibrium $T$ vs. observed $T$

pole-to-equator temperature contrast too big in equilibrium state (especially in winter)
Zonally averaged net radiation

Diurnally-averaged radiation

Observed radiative budget

Implied energy transport: requires fluid motions to effect the implied heat transport
Basic dynamical relationships

Equation of motion

\[ \rho \left[ \frac{du}{dt} + 2\Omega \times u \right] = -\nabla p \quad (+ \text{friction}) \]

Shallow atmosphere:

\[
\begin{align*}
\frac{du}{dt} - f v &= -\frac{1}{\rho} \frac{\partial p}{\partial x} \\
\frac{dv}{dt} + fu &= -\frac{1}{\rho} \frac{\partial p}{\partial y} \\
f &= 2\Omega \sin \phi
\end{align*}
\]

\[
\begin{align*}
\frac{du}{dt} - f v &= -g \frac{\partial z}{\partial x} \\
\frac{dv}{dt} + fu &= -g \frac{\partial z}{\partial y}
\end{align*}
\]

\[
\begin{align*}
\left( \frac{\partial p}{\partial z} \right)_{x,y} &= -g \rho \\
\frac{1}{\rho} \left( \frac{\partial p}{\partial x} \right)_z &= g \left( \frac{\partial z}{\partial x} \right)_p \\
\frac{1}{\rho} \left( \frac{\partial p}{\partial y} \right)_z &= g \left( \frac{\partial z}{\partial y} \right)_p
\end{align*}
\]
**Geostrophic balance**

\[
\begin{align*}
\frac{du}{dt} - fv &= -g \frac{\partial z}{\partial x} \\
\frac{dv}{dt} + fu &= -g \frac{\partial z}{\partial y}
\end{align*}
\]

\[Ro = \frac{U}{fL} \ll 1\]

\[
\begin{align*}
u &= -g \frac{\partial z}{\partial y} \\
v &= g \frac{\partial z}{\partial x}
\end{align*}
\]

\[
\begin{align*}
\left( \frac{\partial p}{\partial z} \right)_{x,y} &= -g \rho \\
\left( \frac{\partial z}{\partial p} \right)_{x,y} &= -\frac{1}{g \rho} = -\frac{R}{g \rho} T
\end{align*}
\]

\[
\frac{\partial^2 z}{\partial p \partial x} = -\frac{R}{g \rho} \frac{\partial T}{\partial x}
\]

**thermal wind shear balance**

\[
\begin{align*}
\frac{\partial u}{\partial p} &= \frac{R}{f \rho} \frac{\partial T}{\partial y} \\
\frac{\partial v}{\partial p} &= -\frac{R}{f \rho} \frac{\partial T}{\partial x}
\end{align*}
\]
Rotating vs. nonrotating fluids
Rotating vs. nonrotating fluids

\[ f > 0 \]

\[ f = 0 \]

\[ f < 0 \]
Atmospheric energetics: where does the energy of atmospheric motions come from?

- Flattening density/temperature surfaces always reduces potential energy

- **Available potential energy** inherent in density/temperature gradients

- In order to *generate* available potential energy, on average must heat where hot and cool where cold:
  \[
  < J T > < 0
  \]

- In order to *release* available potential energy (and generate motion), on average, warm air must rise, cold air sink:
  \[
  < w T > > 0
  \]