The Atmosphere:
Part 6: The Hadley Circulation

- Composition / Structure
- Radiative transfer
- Vertical and **latitudinal heat transport**
- **Atmospheric circulation**
- Climate modeling

*Suggested further reading:*

James, *Introduction to Circulating Atmospheres* (Cambridge, 1994)

Lindzen, *Dynamics in Atmospheric Physics* (Cambridge, 1990)
Calculated rad-con equilibrium $T$ vs. observed $T$

pole-to-equator temperature contrast too big in equilibrium state (especially in winter)
Zonally averaged net radiation

Diurnally-averaged radiation

Observed radiative budget

Implied energy transport: requires fluid motions to effect the implied heat transport
Roles of atmosphere and ocean

Trenberth & Caron (2001)
Rotating vs. nonrotating fluids

\[ f > 0 \]

\[ f = 0 \]

\[ f < 0 \]
Hypothetical 2D atmosphere relaxed toward RCE

2D: no zonal variations
\[ \frac{\partial}{\partial x} \equiv 0 \]

Annual mean forcing — symmetric about equator
A 2D atmosphere forced toward radiative-convective equilibrium

\[ T_e(\varphi, p) \]

\[ \rho \frac{dQ}{dt} = \rho \left( c_p \frac{dT}{dt} + g \frac{dz}{dt} \right) = J \]

\( J = \text{diabatic heating rate per unit volume} \)

\[ \rightarrow \frac{dT}{dt} + \Gamma_w = \frac{J}{\rho c_p} \approx -\frac{1}{\tau} [T - T_e(\varphi, z)] \]

Figure 6-30. Time-dependent “radiatively determined” temperature \( T \), for 15 January 1982 from the calculation of Fels and Schwarzkopf (1985). The surface temperatures are prescribed at their seasonally-varying observed values. Cloudiness, and ozone below 35 km, are prescribed at annual-mean values, as in Fels et al. (1980); ozone above 35 km is allowed to “float”, in response to temperature variations, towards a crude photochemical equilibrium. Details of the water vapor prescription are relatively standard and are described in Fels and Schwarzkopf (1985), [From Manimann and Umscheid, 1984].
Hypothetical 2D atmosphere relaxed toward RCE

\[ \frac{dT}{dt} + \Gamma w = -\frac{1}{\tau} [T - T_e(\varphi, z)] \]

Above the frictional boundary layer,

\[ \frac{du}{dt} - fv = -g \frac{\partial z}{\partial x} = 0 \]

\[ \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - fv = 0 \]

Absolute angular momentum per unit mass

\[ m = ur + \Omega r^2 \]
\[ = ua \cos \varphi + \Omega a^2 \cos^2 \varphi \]

\[ \frac{dm}{dt} = \frac{\partial m}{\partial t} + \mathbf{u} \cdot \nabla m = 0 \]
Angular momentum constraint

\[ m = ua \cos \varphi + \Omega a^2 \cos^2 \varphi \]

Above the frictional boundary layer,

\[ \frac{dm}{dt} = \frac{\partial m}{\partial t} + \mathbf{u} \cdot \nabla m = 0 \]

In steady state,

\[ \mathbf{u} \cdot \nabla m = 0 \]

**Either**

1) \( v=w=0 \)

\[ \frac{dT}{dt} + \Gamma w = -\frac{1}{\tau} [T - T_e(\varphi,z)] \]

\[ v \frac{\partial T}{\partial y} + w \left( \frac{\partial T}{\partial z} + \Gamma \right) = -\frac{1}{\tau} [T - T_e(\varphi,z)] \]

\[ T = T_e(\varphi,z) \]

**Or**

2) \( v, w \neq 0 \) but \( m \) constant along streamlines

\[ u = \Omega a \frac{\sin^2 \varphi}{\cos \varphi} \quad \text{(if} \ u=0 \ \text{at equator)} \]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\varphi & 0 & 15 & 30 & 45 & 60 \\
\hline
\text{U(ms}^{-1}) & 0 & 32 & 134 & 327 & 695 \\
\hline
\end{array}
\]
2) \( v \neq 0 \)

\[ \mathbf{u} \cdot \nabla m = 0 \]

\[ \rightarrow \] solution (2) no good at high latitudes

1) \( v = 0 \); \( T = T_e(\phi, z) \)

Near equator, \( T_e \sim A - B\phi^2 \)

\[ \rightarrow \frac{\partial u}{\partial p} = \frac{R}{fp} \frac{\partial T}{\partial y} = \frac{R}{ap} \frac{1}{2\Omega \sin \phi} \frac{\partial T}{\partial \phi} \sim -\frac{RB}{\Omega ap} \]

\[ \rightarrow \] \( u \) finite (and positive) in upper levels at equator;

\[ \rightarrow \] angular momentum maximum there; not allowed

\[ \rightarrow \] solution (1) no good at equator
FIGURE 7.19. Zonal-mean cross sections of the mass stream function in $10^{10}$ kg s$^{-1}$ for annual, DJF, and JJA mean conditions. Vertical profiles of the hemispheric and global mean values are shown on the right.
In upper troposphere,

\[ u_{ut} = \Omega a \frac{\sin^2 \varphi}{\cos \varphi} \]
Observed Hadley cell

Subtropical jets

\[ v, w \]

\[ u \]
In upper troposphere,  

\[ u_{ut} = \Omega a \frac{\sin^2 \varphi}{\cos \varphi} \]

Near equator,  

\[ u_{ut} \approx \Omega a \varphi^2 \]

But in lower troposphere,  

\[ u_{lt} \approx 0 \]  
(because of friction).

\[
\begin{align*}
\frac{\partial u}{\partial p} &= \frac{R}{f p} \frac{\partial T}{\partial y} = \frac{R}{afp} \frac{\partial T}{\partial \varphi} \\
\rightarrow \quad \frac{\partial T}{\partial \varphi} &= \frac{afp}{R} \frac{\partial u}{\partial p} \approx \frac{2\Omega a}{R} \varphi \frac{\partial u}{\partial \ln p} \\
\int_{lt}^{ut} \frac{\partial T}{\partial \varphi} \; d\ln p &\approx \frac{2\Omega a}{R} \varphi (u_{ut} - u_{lt}) \\
&\approx \frac{2\Omega a}{R} \varphi u_{ut} \approx \frac{2\Omega^3 a^3}{R} \varphi^3 \\
\int_{lt}^{ut} T \; d\ln p &\approx \text{const.} + \frac{\Omega^3 a^3}{2R} \varphi^4 
\end{align*}
\]
Observed Hadley cell

\(v, w\)

**FIGURE 7.10.** Zonal-mean cross sections of the mass stream function in \(10^{12} \text{ kg s}^{-1}\) for annual, DJF, and JJA mean conditions. Vertical profiles of the hemispheric and global mean values are shown on the right.

**FIGURE 7.5.** Zonal-mean cross sections of the temperature for annual-mean, DJF, and JJA conditions in °C. Vertical profiles of the hemispheric and global mean temperatures are shown on the right.
In upper troposphere,

\[ u_{ut} = \Omega a \frac{\sin^2 \varphi}{\cos \varphi} \]

Near equator,

\[ u_{ut} \approx \Omega a \varphi^2 \]

But in lower troposphere,

\[ u_{lt} \approx 0 \]

(because of friction).

\[ v \frac{\partial T}{\partial y} + w \left( \frac{\partial T}{\partial z} + \Gamma \right) = -\frac{1}{\tau} [T - T_e(\varphi, z)] \]
The symmetric Hadley circulation
JJA circulation over oceans

(“ITCZ” = Intertropical Convergence Zone)
Monsoon circulations
(in presence of land)
Satellite-measure (TRMM) rainfall, Jan/Jul 2003
Heat (energy) transport by the Hadley cell

Poleward mass flux (kg/s) = $M_{ut}$

Equatorward mass flux (kg/s) = $M_{lt}$

Mass balance: $M_{ut} = M_{lt} = M$

Moist static energy (per unit mass)

$$E = c_p T + gz + Lq$$

Net poleward energy flux

$$F = M_{ut}E_{ut} - M_{lt}E_{lt}$$

$$= M(E_{ut} - E_{lt})$$

But

(i) Convection guarantees

$$E_{ut} = E_{lt}$$ at equator

(ii) Hadley cell makes $T$ flat across the cell

→ weak poleward energy transport
Roles of atmosphere and ocean

Trenberth & Caron (2001)