1. The pressure field at depth $z = 1000\text{ m}$ in the ocean is found to follow the rule,

$$p = p \sin \left( \pi x / 10^4 \right) \cos \left( \pi y / 10^4 \right).$$

The origin of $y$ is taken to be $30^\circ \text{N}$ and $x,y$ are measured in meters. What are the northward and eastward components of geostrophic velocity at $x = 10^4 / 2$, $y = 0$? If the fluid density is approximated as uniform, $\rho = \rho_0 = 1.03 \times 10^3 \text{kg/m}^3$, how much water (mass) is moving northward between the seafloor and 1000m, between $x = 0$ and $x = 10^4 / 2$? For a numerical answer, let $p_0 = 238 \text{N/m}^2$. (Hint: Treat the seafloor as flat, and note that the pressure gradient is independent of depth if the density is uniform.)

2. A ship measures the temperature and salinity in the ocean at $x = 0$, and $x = 50\text{km}$ at a latitude of $30^\circ \text{N}$. When converted to density, the two profiles are found to be closely approximated as,

$$\rho(x = 0, z) = 1.03 \times 10^3 \text{kg/m}^3 \left( 1 - z / (2 \times 10^4) \right),$$

$$\rho(x = 50\text{km}, z) = 1.03 \times 10^3 \text{kg/m}^3 \left[ 1 - (z + 1 \times 10^{-7} z^2) / (2 \times 10^4) \right]$$

where $z$ is in meters. Compute and plot the northward velocity as a function of $z$ for $0 \leq z \leq 3000\text{m}$ under the assumption that $z = -1500\text{m}$ is a level of no motion. What is different at $10^\circ \text{N}$? Take gravity, $g = 10\text{m/s}^2$.

3. A uniform wind blows towards the north such that the vector windstress on the ocean is $\tau = \tau_0 (0, 1)$. Using the equations

$$-fv = A \frac{\partial^2 u}{\partial z^2},$$

$$fu = A \frac{\partial^2 v}{\partial z^2},$$

which govern the Ekman layer, find $u,v$ as a function of $z$. (Hint: multiply the second equation by $i$ and add to the first equation. Solve this equation for the complex quantity $u + iv$.)

(In Problems 1 and 2, the numerical result is less important than telling me what you are doing.)