1. (a) Write a code, in any software system you like, to compute the standard real form discrete Fourier series coefficients as in Eqs. (6.30, 6.31) of the notes. Employ the code on any sequence $x_k$ you choose to make up and show that the synthesized Fourier series reconstructs your original one.

(b) Apply your code to $y_k = 4 \sin (2 \pi t/10); t = k \Delta t, \Delta t = 0.5, 0 \leq t \leq 100$. Show that you can deduce from your Fourier coefficients that the amplitude was 4, that the period was 10, and that it was a sinusoid. (I won’t read computer code!)

2. (a) Using any software of your choice (that is, you don’t need to write your own), compute the fast Fourier transform (FFT) of the sinusoid in 1(b) at the Fourier series frequencies. (You can change the number of data points if you wish.) Show that you can recover the amplitude, period and phase, that you can, with an inverse transform, recover the original sinusoid.

(b) Using the FFT algorithm, compute the values of the Fourier transform four times more densely than in (a).

3. Let $x_n = [1, -1, 1, -1, 1, \ldots], c_m = [1, -1/3]$. Using the z-transform, convolve $c, x$. Let $d(z) = 1/(1 - z/3)$. What happens when you convolve $c, d$? Let

$$\hat{y}(z) = \frac{\hat{x}(z)}{\hat{c}(z)}.$$ 

Find $y_n$.

4. You have the difference equation

$$y_{n+1} - 0.2y_n + y_{n-1} = p_n$$ 

where $p_n = \exp (-n \varepsilon) \cos (n/2), n = 0, 1, 2, \ldots, 10$. $\varepsilon = 1, y_0 = 0, y_1 = -1$. Find $y_n$. Suppose $n$ is permitted to go to infinity, and $\varepsilon = 0$. Is the solution stable? Now change the problem to $y_0 = 0, y_{10} = 1$. What is $y_n, n = 0, 1, \ldots, 10$.

5. (a) Generate, however you please, 512 random numbers, $x_p$. Using the software of problem 2, compute and plot as a function of frequency the magnitude of the Fourier coefficients $H_k = \sqrt{a_k^2 + b_k^2}$, and their phase $\phi_k = \arctan (b_k/a_k)$. Make histogram plots of $H_k, H_k^2$ and $\phi_k$ and describe the result.

(b) Define $c_p = [1, -0.9]$. Convolve $c_p$ with your random numbers, $y_r = c \ast x$, and repeat the calculations in (a) describing how things have changed. (You can change the length of time series a bit if you want.)