Taxing externalities with measurable pollution

Atmosphere:

\[ A = \sum_i a_i x_i^0 \]  

(1)

The only route for externalities is through \( A \). Note \( A \) is the same for all consumers. Note linearity is not important. This could be done with a vector of different atmospheres, e.g., for different locations.

Pareto optimality assuming linear technology with fixed producer prices \( p \):

\[
\begin{align*}
\text{Max} & \quad \sum_h \alpha_h u^h [x_0^h, x^h, A] \\
\text{s.t.} & \quad \sum_h (p_0 x_0^h + p x^h) = R
\end{align*}
\]  

(2)

First order conditions with respect to \( x_i^h, x_0^h \):

\[
\alpha_h \frac{\partial u^h}{\partial x_i} = \lambda p_i \quad h = 1, 2, ..., H; \ i = 1, 2, ..., N
\]  

(3)

\[
\alpha_h \frac{\partial u^h}{\partial x_0} + a_h \sum_k \alpha^k \frac{\partial u^k}{\partial A} = \lambda p_0
\]  

(4)

Substituting from (3) in (4)

\[
\frac{\partial u^h / \partial x_0^h}{\partial u^h / \partial x_i^h} = \frac{p_0}{p_i} - a_h \sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x_i^k}
\]  

(5)
I. If we can measure the pollution contribution, \( a^h x_0^h \), we can decentralize the PO by pricing pollution, although prices might need to vary by person.

The consumer problem becomes:

\[
\begin{align*}
\text{Max} & \quad u^h [x_0^h, x^h, A] \\
\text{s.t.} & \quad p_0 x_0 + p_x x^h + t^h a^h x_0 = I^h
\end{align*}
\]  

First order conditions

\[
\frac{\partial u^h / \partial x_0^h + a^h \partial u^h / \partial A}{\partial u^h / \partial x_1^h} = \frac{p_0 + t^h a^h}{p_1}
\]  

This will support the PO provided

\[
\frac{t^h}{p_1} = \frac{\partial u^h / \partial A}{\partial u^h / \partial x_1^h} - \sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x_1^k}
\]

If individuals ignore their own feedback to the atmosphere, individual choice now has FOC:

\[
\frac{\partial u^h / \partial x_0^h}{\partial u^h / \partial x_1^h} = \frac{p_0 + t^h a^h}{p_1}
\]

This allows support for the PO with uniform taxes

\[
\frac{t}{p_1} = -\sum_k \frac{\partial u^k / \partial A}{\partial u^k / \partial x_1^k}
\]

Note this extends to a vector of (local) atmospheres and more than one externality generating good, provided pricing distinguishes each atmosphere.
II. Alternatively, assuming consumers ignore the feedback on self through $A$, decentralization can be approached by taxing good zero.

$$\text{Max } u^h \left[ x^h_0, x^h, A \right]$$

$$\text{s.t. } \left( p_0 + t^h \right) x^h_0 + p x^h = I^h$$

First order condition:

$$\frac{\partial u^h}{\partial x^h_0} = \frac{p_0 + t^h}{p_1}$$

$$t^h = d^h p_1 \sum_k \frac{\partial u^k}{\partial A} \frac{\partial \tilde{x}^k}{\partial x^k_1}$$

This relies on the lack of choice in how the good is consumed, with different choices resulting in different levels of pollution.