In the U.S. criminal justice system, the state government imprisons criminals and determines sentence length \( l \) while local communities devote resources to generate arrests. Suppose a state is composed of two local communities \((1,2)\) who each devote resources to police work, which determines a local arrest rate \((a_{1,2})\). The state finances prisons, but the two local communities must pay for local police to arrest criminals. The cost to the local community per arrest is \( p \). Hence with local income of \( (y_{1,2})\), each community’s budget constraint is \( y_i = x_i + p a_i \) \((i=1,2)\), where \( x_i \) is income devoted to all “other” activities.

There be a common utility function for residents of each community: \( u_i = x_i / C \) where \( x_i \) is again income left over after devoting resources to law enforcement (arrests), and \( C \) is the stock of all state criminals (out of prison) who can commit crimes in both communities (i.e. there is a common pool of criminals who commit crimes uniformly across towns).

Suppose that the number of criminals on the street is determined by the normal inflow/outflow steady state condition. The state population of potential criminals is \( K \), and the inflow is the sum of the two town’s arrest rates times total criminals on the street.

1). Determine the formula for street-walking criminals as a function of the arrest rates and sentence length.

2). Taking (state determined) sentencing as given derive the formula for the arrest rate that each local community chooses as a function of what the other town does. Are town arrest rates strategic compliments or substitutes?

3). If the state increases the average length of sentences, how does this impact the local choice of an arrest rate? Explain your answer intuitively.

4). Determine the symmetric Nash equilibrium across the two towns for expenditure and the number of criminals on the state’s streets (assume \( y_1 = y_2 \)).

5). Do you think that the Nash Equilibrium in (4) is efficient from the state-wide point of view? Why?