EXERCISE 5: TAX COMPETITION BETWEEN REGIONAL ECONOMIES

Consider a two-region economy with a fixed national stock of capital $K$, which is competitively allocated between regions ($k_1$, $k_2$). The labor force of each region is fixed ($L_1$, $L_2$). Region 2 residents are contemplating a tax ($T$) on each unit of capital, hoping to raise money even as they risk the flight of capital to the other region. Regional wages ($w_1$, $w_2$) and outputs ($q_1$, $q_2$) as well as the national return to capital ($r$) are all determined as capital is re-allocated between regions in reaction to the tax. Regional output is a common, freely traded good ($q_1$, $q_2$), which has a unit price.

The solution to this 2-region economy (the seven variables: $k_1$, $k_2$, $w_1$, $w_2$, $r$, $q_1$, $q_2$) is based on the solution to the following seven equations.

\[
\begin{align*}
q_1 &= f(L_1, k_1) ; & q_2 &= f(L_2, k_2) \\
\frac{\partial f}{\partial L_1} &= w_1 ; & \frac{\partial f}{\partial L_2} &= w_2 \\
\frac{\partial f}{\partial k_1} &= r ; & \frac{\partial f}{\partial k_2} &= r + T \\
k_1 + k_2 &= K
\end{align*}
\]

When $q_i=L_i^{\alpha}k_i^{1-\alpha}$ ($i=1,2$), then the system of 7 equations can be reduced to a single equation below, which must be solved implicitly for the national return to capital ($r$):

\[
K = L_2 \left( \frac{1-\alpha}{r+T} \right)^{\alpha} + L_1 \left( \frac{1-\alpha}{r} \right)^{\alpha}
\]

(6)

Solve numerically, this system of equations when $\alpha=.7$, $K=100$, $L_1=90$, $L_2=10$, and when there is no tax in region 2. Then consider two alternative solutions to the system:

a). Region 2 imposes a tax of .15 per unit of capital.

b). Region 2 imposes a subsidy per unit of capital (-.15).

In each case get the solution and compare the following: region 2's wages, output, and capital income if locally owned [$r_k_1$], as well as capital income if it is a share of national capital income [$rL_2/(L_1+L_2)$]. Is the region better off? By what measures is this true or not true?