Differential PSK

QAM, PSK: receiver must have an exact replica of the carrier waveform used at transmitter

ex. BPSK, carrier at receiver has phase offset \( \theta \)

\[ d(t) = +1, \quad u(t) = +g(t), \quad t \in [0, T] \]

delay through the transmission channel:

\[ r(t) = g(t - 2T) = g(t - 2T) \cos[\omega d - \omega_c t] \]

\( u(t) \)

\( u(t-T), \quad T << T \)

\[ u(t-T) \equiv u(t) \]

\( \tau \rightarrow \omega_c T = \theta \) non-negligible

\[ \cos \omega_c (t - T) = \cos [\omega_c t + \theta] \]

Even if receiver uses an estimate of \( \tau \) to re-align signals, a small error in \( \tau \) will result in large offset \( \theta \).
Carrier phase offset causes loss of signal during demodulation

\[ r(t) = u(t) \cos(\omega t + \theta) \]

\[ n(t) = r(t) \cdot 2 \cos(\omega t + \hat{\theta}) \bigg|_{LP} = u(t) \cdot \left[ \cos(\theta - \hat{\theta}) + \cos(2\omega t + \theta + \hat{\theta}) \right] \bigg|_{LP} \]

this component rejected by LP

\[ P_n = P_u \cdot \cos^2(\theta - \hat{\theta}) \leq P_u \]
In practice, carrier synchronization is performed by a Phase-Locked Loop (PLL). PLL is a filtering method that extracts carrier from the received signal. If this is deemed too complicated, differentially coherent demodulation is an alternative:

\[ r(t) \]

\[ u(t) \times u(t-T) \]

\[ r(t) = u(t) \cos[\omega_c t + \Theta] \]

\[ r(t-T) = u(t-T) \cos[\omega_c t + \Theta - \frac{2\pi T}{T}] = u(t-T) \cos[\omega_c t + \Theta] \]

For \( t \in [nT, nT+T) \):

\[ u(t) = d(n) \cdot A \]

\[ u(t-T) = d(n-1) \cdot A \]

\[ u(t) \cdot u(t-T) \approx d(n) \cdot d(n-1) \]
There is no phase offset now, but we observe $d(n) \cdot d(n-1)$ instead of $d(n)$.

To avoid this, we use **differential encoding at transmitter**.

$$b(n) \in \{ \pm 1 \}$$

Original bit sequence

$(0, 1 \rightarrow \pm 1)$

$$d(n) = b(n) \cdot d(n-1) \Rightarrow d(n) \cdot d(n-1) = b(n)$$

multiply $\pm 1$

or add modulo 2
FSK Frequency Shift Keying

Binary:
\[ s_1(t) = A \cos \omega c_1 t, \quad t \in [0,T] \]
\[ s_2(t) = A \cos \omega c_2 t, \quad t \in [0,T] \]

Noncoherent detection is possible — receiver does not need to know the carrier phase.

\[ r(t) = \begin{cases} 
A \cos (\omega c_1 t + \theta) & \text{if "0"} \\
A \cos (\omega c_2 t + \theta) & \text{if "1"} 
\end{cases} \]

Only one detector will produce an output.

Energy detectors or envelope detectors
M - FSK

\[
\begin{align*}
00 & \rightarrow s_1(t) \\
01 & \rightarrow s_2(t) \\
10 & \rightarrow s_3(t) \\
11 & \rightarrow s_4(t) \\
\end{align*}
\]

\[
s_m(t) = A \cdot \cos \omega_{cm} t, \quad t \in (0, T) \quad m = 1, 2, \ldots, M
\]

\[
f_{cm} = f_{c1} + (m-1) \Delta f
\]

\[
T = kT_b
\]

\[\text{If } k = 2 \Rightarrow M = 2^k = 4\]

During a signaling interval, one of \( M \) possible tones is transmitted:

\[
\begin{align*}
#2 \\
\uparrow \quad \uparrow \Delta t \\
f_{c1} \quad f_{c2} \quad \ldots \quad f_{cm} \\
\downarrow \quad \downarrow \\
\text{B} = M \Delta f
\end{align*}
\]

Exact calculation of FSK signal spectrum is not straightforward.

Detection: noncoherent with \( M \) filters and \( M \) energy detectors.

Also, FSK signals can be demodulated coherently (if all the carriers are known).

Q: What \( \Delta f \) to use?
Coherent detection of FSK signals

Suppose \( s_2(t) \) was transmitted during \( t \in [0, T] \): \( r(t) = s_2(t) \)

The first integrator output is

\[
\int_0^T r(t) 2 \cos \omega_c t \, dt =
\]

\[
= A \int_0^T 2 \cos \omega_c t \cos \omega c t \, dt
\]

\[
= A \int_0^T \cos (\omega_c - \omega_c) t \, dt + \int_0^T \cos (\omega_c + \omega_c) t \, dt
\]

\[
= A T \sin \Delta \omega T \Delta \omega T
\]

minimal freq. separation

if \( \Delta \omega T = \pi \), i.e.

\[
\Delta f = \frac{1}{2T}
\]

there is no cross-talk

\( s_1(t) \) and \( s_2(t) \) are "orthogonal"

\[
\text{cross-talk between carriers #1 & #2}
\]
In general, a set of $M$ signals $s_1(t), s_2(t), \ldots, s_M(t)$ is orthogonal if

$$\int_{-\infty}^{\infty} s_i(t)s_j(t) \, dt = \begin{cases} E_s, & i=j \\ 0, & i \neq j \end{cases} \quad \text{no "cross-talk"}$$

M-FSK: minimal frequency separation between each two carriers is $\Delta f = \frac{1}{2T}$ for $\{s_m(t)\}$ to be orthogonal.

Bandwidth occupancy: $B = M \Delta f = \frac{M}{2T}$ (roughly)

Bit rate: $R_b = \frac{1}{T_b} = \frac{\log M}{T}$

Bandwidth efficiency: $\frac{R_b}{B} = 2 \left( \frac{\log M}{M} \right)$ decreases with increasing $M$

Average energy per bit

$$E_s = \frac{1}{2} E_g = \log M \cdot E_b$$

(different than QAM, PSK)
PPM: Pulse Position Modulation

\[ u_1(t) \quad \rightarrow \quad g(t) \quad \rightarrow \quad t \quad \{1000\} \]

\[ u_2(t) \quad \rightarrow \quad t \quad \{0100\} \]

\[ u_3(t) \quad \rightarrow \quad t \quad \{0010\} \]

\[ u_4(t) \quad \rightarrow \quad t \quad \{0001\} \]

Bandwidth: \( B \approx \frac{1}{\Delta t} = \frac{M}{T} \)

Bit rate: \( R_b = \frac{kdM}{T} \)

\[ \Rightarrow \frac{R_b}{B} \approx \frac{kdM}{M} \quad \text{not efficient} \]

Energy: \( E_3 = \frac{1}{2} E_g = kdM \cdot E_6 \)

The geometric representation can do with M-FSK as well

\( s_m(t) = u_m(t) \cos \omega_c t \)

\( \{s_m(t)\} : \text{orthogonal (no overlap in time)} \)