Correction to Class Notes
March 17, 2002

In class today, the solutions to the homogeneous linear equation

\[ x'' + p^2 x = 0 \]

were given as

\[ z_1(t) = \cos pt, \quad z_2(t) = \sin pt, \]

which is fine for \( p^2 \neq 0 \), but Your Humble Instructor has tried to take pains to have all results be valid for \( p = 0 \).

At one point, YHI said that the combination

\[ z_1 z_2' - z_1' z_2 = 1, \]

which is demonstrably untrue for \( p \neq 1 \). You can do the math (Craig make funny) to show that this quantity is in general equal to \( p \), if it is recognized that the cosines and sines of a purely imaginary argument are hyperbolic functions.

This would not be a VLD (Very Large Deal) except for the fact that we at one point divided by this quantity. In what we did today, we could take the limit at \( p \to 0 \), but we can do better. Here goes:

Instead of the above, let

\[ z_1 = \cos pt \]
\[ z_2 = \begin{cases} \frac{\sin pt}{p}, & p \neq 0 \\ t, & p = 0. \end{cases} \]

Then, \( z_2(t) \) is a continuous and real function of \( p \), even for \( p^2 < 0 \), and

\[ z_1 z_2' - z_1' z_2 = 1 \]

for all \( p \), which is amazingly great, if you’ll forgive the superlative in a math subject.