Advanced Nonlinear Dynamics and Chaos

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• Lectures: Tuesday, Thursday, 11:00am-12:30pm, Room 1-242

• Office hours: Tuesdays, 3-4:30pm, Rm. 3-352

• Homeworks: - Typically every week, out on Thursday, due in a week
  - Late homework accepted if prior arrangement is made

• Report: - Written report on a research article as part of the final grade.

• Textbook: None required. Recommended books on reserve in Baker library:
  1. Guckenheimer, J., and Holmes, P., *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*
  2. Chicone, C., *Ordinary Differential Equations with Applications*
Course outline

• Normally hyperbolic invariant manifolds
  • Introduction to manifolds
  • Existence and persistence of invariant manifolds
  • Geometric singular perturbation theory

• Global bifurcations
  • Higher-dimensional Melnikov methods
  • Shilnikov orbits
  • Homoclinic bifurcations

• The internal structure of chaos
  • Symbolic dynamics
  • Bernoulli shift map
  • Subshifts of finite type
  • Higher-dimensional chaos
• Hamiltonian dynamical systems
  • Canonical and noncanonical Hamiltonian systems
  • Symplectic geometry
  • Conservation properties, phase space geometry

• Integrable and near-integrable systems
  • Liouville-Arnold theory: existence of invariant tori
  • KAM-theory: persistence of invariant tori
  • Arnold diffusion

• Introduction to infinite-dimensional dynamics
  • Attractors, inertial manifolds
  • PDEs as infinite-dimensional Hamiltonian systems
  • Chaos in infinite dimensions
Motivational example I: Energy-efficient trajectory of a spacecraft along an unstable manifold (Caltech-JPL)

A **halo orbit** around the L1 equilibrium point in the circular restricted three-body problem. (plot by GAIO of Michael Dellnitz and Oliver Junge, Institute of Mathematics, University of Padeborn)
Motivational example II: Unsteady fluid flow separation on no-slip surfaces

Flow separation: particles following the boundary are suddenly ejected into the flow

Separation on the roof of a passenger car
Gillieron & Chometon [1999]

O. Grunberg [2003]
Motivational example III: Mixing of diffusive substances

T=2 periods

T=20 periods

T=50 periods

T=50.5 periods

In function space:

Liu & H. [2003]

Rothstein, Henry, & Gollub [1999]