Problem 1:

In Problem 6 of Problem Set #2, we designed a P.D. controller to achieve a point-to-point task with a planar two-link manipulator.

For impedance control of the manipulator in this problem, we proposed a control law for the joint torque $\tau$ that can achieve in the task space a restoring force $F$ corresponding to the virtual programmable cartesian spring $K_p$ and damper $K_d$.

With gravity compensation, the control law was formulated in terms of the end-effector cartesian displacement error $\tilde{x} = x - x_d$ as:

$$\tau = g(q) - JT(K_p\tilde{x} + K_d\dot{\tilde{x}})$$

Consider now the case of impedance control of a planar four-link manipulator for achieving a point-to-point task.

In addition to the virtual cartesian spring and damper at the end-effector, let us say we also have a virtual cartesian spring and damper at joint-2 (the joint adjacent to the “shoulder” of the manipulator).

Propose a control law similar to the one shown above for the torque $\tau$ for the case of this four-link manipulator.

- The control law must be written in terms of the end-effector cartesian displacement error $\tilde{x}_e$ and the joint-2 cartesian displacement error $\tilde{x}_2$.

- Neglect the gravity compensation term $g(q)$. (You can find this term easily, we have done a similar exercise as part of Problem 3(b) in Problem Set #2.)
• No simulations are required (i.e. just propose the control law: NO coding is required in this problem!)

[Hint: Your answer should contain two different (why?) $J^T$ matrices respectively pre-multiplying the restoring force vectors corresponding to the mechanical impedances at the end-effector and the joint-2. Explicitly derive expressions for the Jacobian matrices in terms of joint angles and link lengths of the manipulator.]

Problem 2:

Consider the dynamic equations in the vertical plane for a two degree-of-freedom planar manipulator with two rotational joints

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$$

where

$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \quad H = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix}, \quad C = \begin{bmatrix} -h\dot{\theta}_2 & -h\dot{\theta}_1 - h\dot{\theta}_2 \\ h\dot{\theta}_1 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

$$H_{11} = m_1l_1^2 + I_1 + m_2(l_1^2 + l_2^2 + 2l_1l_2cos\theta_2) + I_2$$

$$H_{22} = m_2l_2^2 + I_2$$

$$H_{12} = m_2l_1l_2cos\theta_2 + m_2l_2^2 + I_2$$

$$h = m_2l_1l_2sin\theta_2$$

$$G_1 = m_1l_1gcos\theta_1 + m_2l_2gcos(\theta_1 + \theta_2) + m_2l_1gcos\theta_1$$

$$G_2 = m_2l_2gcos(\theta_1 + \theta_2)$$

Design and simulate an adaptive controller without any initial knowledge of the constant parameters. The desired trajectory is

$$\theta_{d1} = 1 - e^{-t} \quad \theta_{d2} = 2(1 - e^{-t})$$

(In the simulation, you can choose the real values of the parameters and the initial conditions by yourself.)
Problem 3:

Consider the dynamic equations of a robot manipulator

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \tau$$

where the matrix $\mathbf{D}$ is constant symmetric positive definite. Consider an adaptive P.D. controller for such a robot with the control law

$$\tau = \mathbf{Y}\hat{\mathbf{a}} - \mathbf{K}_d\dot{\mathbf{q}} - \mathbf{K}_p\mathbf{q}$$

and the adaptation law

$$\dot{\mathbf{a}} = -\mathbf{P}\mathbf{Y}^T\mathbf{s}$$

where

$$\mathbf{Y}\mathbf{a} = \mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}_r + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \mathbf{D}\dot{\mathbf{q}}_r + \mathbf{g}(\mathbf{q})$$

$$\mathbf{s} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_r = \dot{\mathbf{q}} + \lambda\mathbf{q}$$

$$\mathbf{q}_d = \mathbf{q} - \mathbf{q}_d$$

The adaptation and controller gain matrices $\mathbf{P}$, $\mathbf{K}_d$ and $\mathbf{K}_p$ are all symmetric positive definite.

Show that this controller will force the tracking error $\mathbf{q}$ to converge to zero.

[Hint: You may want to use the Lyapunov function candidate]

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{H}\mathbf{s} + \frac{1}{2}\hat{\mathbf{a}}^T\mathbf{P}^{-1}\hat{\mathbf{a}} + \frac{1}{2}\mathbf{q}^T(\mathbf{K}_p + \lambda\mathbf{K}_d)\mathbf{q}$$