Flexure Error due to fabrication

There are two primary sources of error in waterjet cutting:
1. Dimensional variation (i.e. thickness of flexure arms). We will assume a worst case of +/-.005”.
2. Taper caused by process parameters (namely cutting speed and torch height), wear of the nozzle, and inherent properties of the process. We will assume a worst case of 2 degrees (included angle).

The most significant effect that this variation has on the crossfeed flexure is on the cross section of the flexure arms. Both dimensional variation and taper will cause thicker and/or thinner cross sections, changing the stiffness of the flexure. Because the height term in cross sectional moment of inertia is cubed, the effect on stiffness is quite significant. Table 1 shows the upper and lower bounds for stiffness based on variation of the flexure thickness by +/-.005”.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>X-sec Inertia (in^4)</th>
<th>Stiffness (lb/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>h MAX</td>
<td>0.06</td>
<td>5.20633E-06</td>
</tr>
<tr>
<td>h nom</td>
<td>0.045</td>
<td>3.79688E-06</td>
</tr>
<tr>
<td>h MIN</td>
<td>0.04</td>
<td>2.66667E-06</td>
</tr>
</tbody>
</table>

The effect from taper is similar, however it causes the flexure to be more triangular shaped in cross section. With a 1 degree taper on each side of the arm, the thickness at the bottom of the feature will be nearly .020” thinner than nominal. This will cause a significant change in the stiffness of the member. Taper can be relatively well controlled by using a fresh nozzle and optimizing the cutting speed through trial and error.

Because the effects on each arm may vary, the overall effect on the stiffness of the flexure is hard to estimate. Each arm will vary by some amount, some being thicker than others, which may cancel out some of the effect. The thickness of each arm will (probably) follow a normal distribution, as will the effective stiffness of the whole flexure.

Flexure Deflection Analysis

The deflection due to cutting forces felt by the tool may be expressed in the form of a linear elastic spring,

\[ F = kx, \]

where
Therefore, the deflection force-displacement relationship may be re-written as

\[ F = \left( \frac{3EI}{L^3} \right) x, \]

where an effective spring constant \( k \) for each “cantilever” within the flexure may be described as

\[ k = \frac{3EI}{L^3}. \]

The flexure behaves as a system of cantilevers in series and parallel.

The following is a schematic of the cantilevers which make up the flexure, where each horizontally oriented rectangle represents a spring of spring constant \( k \):

![Diagram of cantilevers](image)

For springs in parallel, spring constants add in the same fashion that resistors in series add:

\[ k_{\text{parallel}} = k_1 + k_2 \]

Springs in series have an effective constant which is like that of resistors in parallel:

\[ k_{\text{series}} = \frac{k_1 k_2}{k_1 + k_2} \]

In the model above, the effective spring constant is equal to the spring constant of four “systems” in series.

\[ k_{\text{eff}} = 4k_{\text{system}} \]

where

\[ k_{\text{system}} = \frac{(k + k)(k + k)}{k + k + k + k} = k \]

and therefore

\[ k_{\text{eff}} = 4k = \frac{12EI}{L^3}. \]

Given that
\[ I = \frac{bh^3}{12}, \]

we now have

\[ k_{\text{eff}} = \frac{Ebh^3}{L^3} \]

The given data is:

\[ E = 69 \text{ GPa (Aluminum)} \]
\[ b = .5^\prime\prime \text{ (max possible for waterjetting)} \]
\[ h = .0625^\prime\prime \text{ (from Solidworks model)} \]
\[ L = 1.5^\prime\prime \text{ (from Solidworks model)} \]

we end up with:

\[ k_{\text{eff}} = 64260 \frac{N}{m} \]

Recall that

\[ x = \frac{F}{k_{\text{eff}}} \]

Therefore, for a force of 900 N,

\[ x = 14mm \]

**Flexure Errors – Thermal**

Worst Case Thermal Expansion:

\[ \alpha_{\text{alu}} = 23 \cdot 10^{-6} \frac{\text{K}}{\text{K}} \quad \Delta T = 30\text{K} \]

\[ \bar{\varepsilon} = \alpha_{\text{alu}} \Delta T \]

\[ L_1 = 3.94^\prime\prime \quad \delta_1 = \varepsilon \cdot L_1 \quad \delta_1 = 6.905 \times 10^{-5} \text{ m} \]

\[ L_2 = 2.5^\prime\prime \quad \delta_2 = \varepsilon \cdot L_2 \quad \delta_2 = 4.381 \times 10^{-5} \text{ m} \]

\[ L_3 = 0.5^\prime\prime \quad \delta_3 = \varepsilon \cdot L_3 \quad \delta_3 = 8.763 \times 10^{-6} \text{ m} \]
Manufacturing/Assembly Errors

- Misalignment of carriage rail that bushing slides over. The more the carriage moves the worse this becomes as the deviation grows. This is probably the most severe cause of error during assembly.
- Jamming of bushing when pressed into flexure leading to misalignment with rail. A misalignment of 7 degrees results in two point contact within the bushing.
- Misalignment of flexure when bolted to carriage, causing misalignment of bushing on rail. A misalignment of 7 degrees results in two point contact inside the bushing.
- Bolting the flexure onto the carriage too tightly might result in deformation of the flexure and the bushing might no longer sit concentric with the rail. This might result in a surface contact inside the bushing.

All of these result in forces applied between bushing and rail, resulting in friction and wear, possibly resulting in binding or fatigue. Applied force might cause failure of flexure or rail.