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Measurement of intraocular distances by backscattering spectral interferometry

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Abstract

The diffraction tomography theorem is adapted to one-dimensional length measurement. The resulting spectral interferometry technique is described and the first length measurements using this technique on a model eye and on a human eye in vivo are presented.

1. Introduction

In 1969, Wolf [1] presented a solution to the optical inverse scattering problem [2], which can be used for the determination of the structure of weakly scattering objects. He showed that within the accuracy of the first Born approximation the three-dimensional distribution of the scattering potential of the object can be computationally reconstructed from the distribution of amplitude and phase of the light scattered by the object. This theorem, sometimes called the Fourier diffraction theorem, relates the Fourier transform of the measured scattering data with the Fourier transform of the object structure [3]. The first experiments using the amplitude and phase of the scattered field in the Fresnel zone confirmed the soundness of this concept [4].

It has also been shown [5] that if the scattered field is measured in the far field of a weakly scattering object, one Fourier transform of Wolf's original theory can be omitted and the scattering potential can be obtained by a single Fourier transform of the measured scattered field data. The first optical reconstructions of the three-dimensional scattering potential of microscopic particles have been obtained by using this technique.

A rather basic question in this imaging process is to what extent the scattered field data can be measured. One wavelength and one direction of illumination only yield data on the surface of the Ewald sphere. The same problem arises in X-ray, neutron and electron diffraction [6,7]. No true three-dimensional reconstruction is possible with such a limited Fourier data set. As mentioned already by Wolf [1] and discussed in detail by Dändliker and Weiss [8], additional Fourier data can be obtained by multi-wavelength and multi-directional illumination. Multi-directional illumination can be realized if a diffraction grating is used in front of the object [5]. Multi-wavelength illumination can be achieved with tunable lasers or broadband light sources like diode lasers and superluminescent diodes.

In this paper we show how the original formalism of Wolf can be modified for the one-dimensional problem of length measurement. In this case it suffices to measure the intensity of the scattered light at various wavelengths and only one direction of illumination. The technique described below can also be considered as a

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generalization of the old channelled spectra technique [9]. In the channelled spectra technique a white light source (with rather low spatial *and* low temporal coherence) is used to measure the thickness of thin films using interferences (channels) in the spectrum of the light remitted from the object [10–13]. This technique is more or less limited to thin films with parallel interfaces. With the technique described below, however, large distances even between interfaces of different geometrical forms are measured using backscattered light. Hence the light used must be spatially coherent in order to provide interference between the various reflected light waves. As this technique relies on the spectral measurement of the resulting light intensity we use the term "spectral interferometry".

2. Scattering approximation

We illuminate an object by a monochromatic (wavenumber k) Gaussian laser beam, see Fig. 1. Let the object be positioned at the beam waist and the object depth T be of the order of magnitude of the corresponding Rayleigh length. Then we can assume the object being illuminated by an approximately plane monochromatic wavefront $E^{(i)}$.

$$E^{(i)}(\boldsymbol{r}, \boldsymbol{k}^{(i)}, t) = A^{(i)} \exp(\mathrm{i}\boldsymbol{k}^{(i)} \cdot \boldsymbol{r} - \mathrm{i}\omega t) , \tag{2.1}$$

where $\boldsymbol{k}^{(i)}$ is the wavevector of the illuminating wave and $|\boldsymbol{k}^{(i)}| = k = 2\pi/\lambda$ is the wave-number. Here we use a simplified classical description of the light beam. We ignore any field quantization and treat the electric field E of the light as a scalar, i.e. we also ignore polarization effects. Let $E^{(s)}(\boldsymbol{r}, \boldsymbol{k}^{(s)}, t)$ be the scattered wave. The sum of the two waves $E^{(i)}(\boldsymbol{r}, \boldsymbol{k}^{(i)}, t) + E^{(s)}(\boldsymbol{r}, \boldsymbol{k}^{(s)}, t)$ satisfies the Helmholtz equation. In case of weakly scattering objects the scattered field can be obtained by the first Born approximation [1] as a volume integral extended over the illuminated object volume $V(\boldsymbol{r}')$:

$$E^{(s)}(\boldsymbol{r}, \boldsymbol{k}^{(s)}, t) = -\frac{1}{4\pi} \int\limits_{V(\boldsymbol{r}')} F(\boldsymbol{r}', \boldsymbol{k}^{(i)}) E^{i}(\boldsymbol{r}', \boldsymbol{k}^{(i)}, t) \times G(|\boldsymbol{r} - \boldsymbol{r}'|) \mathrm{d}^3 \boldsymbol{r}' , \tag{2.2}$$

with the Green's function

$$G(|\boldsymbol{r} - \boldsymbol{r}'|) = \frac{\exp(\mathrm{i}k^{(s)} |\boldsymbol{r} - \boldsymbol{r}'|)}{|\boldsymbol{r} - \boldsymbol{r}'|} . \tag{2.3}$$

Eq. (2.2) can be considered a quantification of Huygens' principle: Green's function represents the secondary wavelets which combine to form the scattered light. The scattering potential $F(\boldsymbol{r}, \boldsymbol{k}^{(i)}) = -k^2[n^2(\boldsymbol{r}, \boldsymbol{k}^{(i)}) - 1]$ determines the relative amplitude of these wavelets.

Now we shall confine the treatment to backscattering. We choose the origin 0 of the coordinate system x, y, z at the axis of the illuminating beam at the back surface of the object as indicated in Fig. 1. The scattered light field is detected at point $P(\boldsymbol{r})$ on the z-axis a distance D outside of the object. If D is much larger than the depth T of the object structure the denominator $|\boldsymbol{r} - \boldsymbol{r}'|$ of the Green's function G can be approximated by D because the scattering potential F is zero outside the region occupied by the object. Furthermore the exponent of G is

$$k^{(s)} |\boldsymbol{r} - \boldsymbol{r}'| = \boldsymbol{k}^{(s)} \cdot (\boldsymbol{r} - \boldsymbol{r}') . \tag{2.4}$$

In addition we neglect any dispersion of the refractive index of the object, i.e. we assume the scattering potential to be independent of the wavenumber. This approximation simplifies the subsequent treatment. But it should be stressed, however, that a more general treatment would have to include dispersion as well. With these approximations we obtain

$$\begin{aligned} E^{(s)}(\boldsymbol{r}, \boldsymbol{k}^{(s)}, t) &= -\frac{1}{4\pi D} \int\limits_{V(\boldsymbol{r}')} F(\boldsymbol{r}') A^{(i)} \exp(\mathrm{i}\boldsymbol{k}^{(i)} \cdot \boldsymbol{r}' - \mathrm{i}\omega t) \\ &\quad \times \exp[\mathrm{i}\boldsymbol{k}^{(s)} \cdot (\boldsymbol{r} - \boldsymbol{r}')] \mathrm{d}^3 \boldsymbol{r}' \\ &= -\frac{A^{(i)}}{4\pi D} \exp(\mathrm{i}\boldsymbol{k}^{(s)} \cdot \boldsymbol{r} - \mathrm{i}\omega t) \\ &\quad \times \int\limits_{V(\boldsymbol{r}')} F(\boldsymbol{r}') \exp(-\mathrm{i}\boldsymbol{K} \cdot \boldsymbol{r}') \mathrm{d}^3 \boldsymbol{r}' , \end{aligned} \tag{2.5}$$

where the amplitude $A^{(i)}$ of the illuminating wave has been assumed constant within the object. $\boldsymbol{K} = \boldsymbol{k}^{(s)} - \boldsymbol{k}^{(i)}$ is the scattering vector.

Finally we replace the integrations over x' and y' by a constant factor W chosen proportional to the cross section of the beam waist of the illuminating beam. This can be done if the Fresnel number $d^2/\lambda D$ of the illuminating beam is smaller than 1 and provided the scattering potential F is constant in the x' and y' direction within the beam. Then we have

$$E^{(s)}(\boldsymbol{r}, \boldsymbol{k}^{(s)}, t) = -\frac{A^{(i)}W}{4\pi D}\exp(\mathrm{i}\boldsymbol{k}^{(s)}\cdot\boldsymbol{r} - \mathrm{i}\omega t)$$

$$\times\int_0^T F(z')\exp(-\mathrm{i}Kz')\,\mathrm{d}z', \tag{2.6}$$

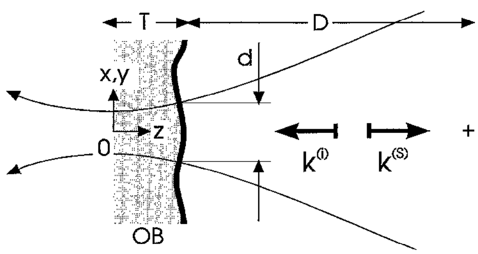

Fig. 1. The object is illuminated along the negative z-axis. The backscattered light wave is detected at point P. $\boldsymbol{k}^{(i)}$ = wavevector of illuminating light, $\boldsymbol{k}^{(s)}$ = wavevector of scattered light. OB = object, P = detector position.

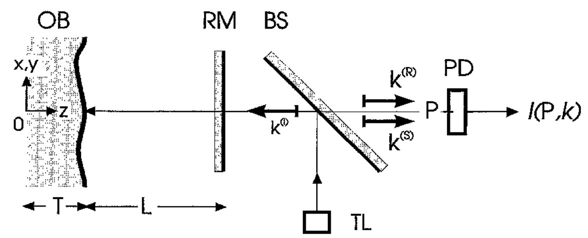

Fig. 2. Tunable laser configuration to measure the scattered field at a range of k-values. The object is illuminated via the beam splitter BS. $\boldsymbol{k}^{(i)}$ = wavevector of illuminating light, $\boldsymbol{k}^{(s)}$ = wavevector of scattered light, $\boldsymbol{k}^{(R)}$ = wavevector of reference light. OB = object, RM = reference mirror, BS = beam splitter, PD = photodetector, TL = tunable laser.

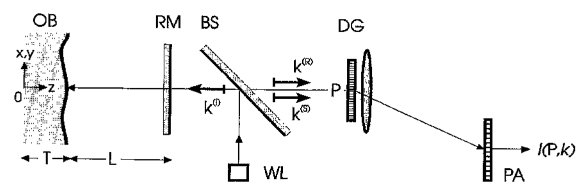

Fig. 3. "White" light source configuration to measure the scattered field at a range of k-values. $\boldsymbol{k}^{(i)}$ = wavevector of illuminating light, $\boldsymbol{k}^{(S)}$ = wavevector of scattered light, $\boldsymbol{k}^{(R)}$ = wavevector of reference light. The object OB is illuminated via the beam splitter BS. RM = reference mirror, BS = beam splitter, DG = diffraction grating, PA = photodetector array, WL = "white" light source.

i.e. we can replace the three-dimensional Fourier transform of Eq. (2.5) by a one-dimensional Fourier transform. The scattered light has an amplitude which is proportional to the (one-dimensional) Fourier transform of the scattering potential $F(z)$ of the object potential (from here on we replace z' by z):

$$E^{(s)}(\boldsymbol{r}, \boldsymbol{k}^{(s)}, t) = -\frac{A^{(i)}W}{4\pi D}\exp(\mathrm{i}\boldsymbol{k}^{(s)}\cdot\boldsymbol{r} - \mathrm{i}\omega t)$$

$$\times\mathrm{FT}\{F(z)\}. \tag{2.7}$$

At P the backscattered light wave is:

$$E^{(s)}(\mathrm{P}, k, t) = A^{(s)}(\mathrm{P}, k)\exp[\mathrm{i}\phi^{(s)}(\mathrm{P}, k)]$$

$$= -\frac{A^{(i)}W}{4\pi D}\exp(\mathrm{i}kD - \mathrm{i}\omega t)\mathrm{FT}\{F(z)\}, \tag{2.8}$$

i.e. proportional to the Fourier transform of the scattering potential. $F(z)$ can be obtained by an inverse Fourier transform of $E^{(s)}(\mathrm{P}, k)$. Obviously this is only possible if the amplitude and phase of the scattered field $E^{(s)}(\mathrm{P}, k)$ are known for at least a limited range of k-values. Though we only have access to the intensity of the scattered light it is clear that we have to use multi-wavelength illumination.

3. Measurement of scattered field data

As shown above the scattered light wave has to be measured for a range of wavenumbers k. Two basic techniques can be used. Either a tunable laser can be used as indicated in Fig. 2, or a "white" light source together with a spectrometer as indicated in Fig. 3. With the tunable laser technique the wavelength has to be tuned over a range of wavenumbers and the spectral light intensity remitted from the object is detected by a photodetector. If a "white" light source, i.e. a light source emitting spatially coherent light with large spectral bandwidth like a multi-mode laser diode or a superluminescent diode is used, a spectrometer is needed to display the intensity of the remitted light at the various wavelengths (or wavenumbers).

The wavenumber dependent intensity spectrum $I(\mathrm{P}, k)$ of the backscattered light is according to Eq. (2.8)–besides a constant C–equal to the square of the Fourier transform of the scattering potential of the object:

$$I(\mathrm{P}, k) = |E^{(s)}(\mathrm{P}, k)|^2 = C|\mathrm{FT}\{F(z)\}|^2. \tag{3.1}$$

Taking the inverse Fourier transform of $I(\mathrm{P}, k)$ yields the auto-correlation function (ACF) of the scattering potential [14]:

$$\mathrm{FT}^{-1}\{I(\mathrm{P}, k)\} = C\langle F^*(z)F(z+Z)\rangle = C\ \mathrm{ACF_F}(Z)\ . \quad (3.2)$$

Hence we obtain the ACF of the object scattering potential and not the object scattering potential F itself. Only with very simple object structures the ACF can be deciphered and the true scattering potential of the object can be obtained.

There are two possibilities to obtain the scattering potential of the object. Firstly, the scattering potential of the object can be obtained if an additional singular light remitting interface (reference mirror RM in Figs. 2 and 3) can be positioned at a distance L from the object (at $z=z_\mathrm{I}$). In this case the scattering potential can be described as a sum of the actual object $F_0(z)$ plus a delta-like potential (with amplitude reflectivity R):

$$F(z) = F_0(z) + R\delta(z-z_\mathrm{I})\ . \quad (3.3)$$

Then the auto-correlation yields four terms:

$$\begin{aligned}&\langle F_0^*(z)F_0(z+Z)\rangle + \langle F_0^*(z)R\delta(z+Z-z_\mathrm{I})\rangle \\ &+ \langle R\delta^*(z-z_\mathrm{I})F_0(z+Z) \\ &+ \langle R^2\delta^*(z-z_\mathrm{I})\,\delta(z-z_\mathrm{I}+Z)\rangle \\ &= \mathrm{ACF_F}(Z) + RF_0^*(z_\mathrm{I}-Z) + RF_0(z_\mathrm{I}+Z) \\ &+ R^2\delta(Z)\ . \end{aligned} \quad (3.4)$$

Here the third term yields – besides the constant factor R – a true reconstruction of the object structure, centered at $Z=-z_\mathrm{I}$. A dominating light remitting interface can be realized by a reference mirror, as indicated by RM in Figs. 2 and 3 in front of the object. Any overlap between the four terms of the ACF is avoided by choosing the distance L between the interface and the object larger than the object depth T: $L>T$. But it must be kept in mind, that extending the depth of the object structure from T to $T+L$ increases the frequency of the Fourier transform and demands for an increased resolution in the k-space.

Secondly, the true scattering potential can also be obtained if the object itself contains one interface (at $z=z_\mathrm{I}$) with a relatively large reflectivity R acting as a reference mirror. Then the scattering potential of the object $F_0(z)$ can be represented as a sum of the residual scattering potential F_R plus a delta-like scattering potential:

$$F_0(z) = F_\mathrm{R}(z) + R\delta(z-z_\mathrm{I})\ . \quad (3.5)$$

Three of the ACF-terms will dominate, namely those containing R:

$$\begin{aligned}&\langle F_\mathrm{R}^*(z)R\delta(z+Z-z_\mathrm{I})\rangle + \langle R\delta^*(z-z_\mathrm{I})F_\mathrm{R}(z+Z)\rangle \\ &+ \langle R^2\delta^*(z-z_\mathrm{I})\,\delta(z-z_\mathrm{I}+Z)\rangle \\ &= RF_\mathrm{R}^*(z_\mathrm{I}-Z) + RF_\mathrm{R}(z_\mathrm{I}+Z) + R^2\delta(Z)\ . \end{aligned} \quad (3.6)$$

In this case we obtain the complex conjugate object scattering potential reversed in the Z-coordinate, the true object scattering potential with the origin at $Z=-z_\mathrm{I}$ and a large peak at $Z=0$. In general the two reconstructed scattering potentials will overlap. Only if the strongly reflecting interface is at the surface of the object two separated object scattering potentials will be obtained. (An equivalent configuration is achieved if a strongly reflecting interface is positioned close to the object.)

4. Experiments

The "white" light source technique described above has been used to measure the distances within an eye model and the corneal thickness of a human eye in vivo. The optical scheme used in these preliminary experiments was that of Fig. 3. Because of the simple structure of the object used we have omitted the reference mirror (RM). A multi-mode laser SHARP LT 023 MDO, $\lambda = 780$ nm, $\Delta\lambda = 3$ nm (FWHM) was used as light source. The intensity spectrum $I(\mathrm{P}, k)$ was displayed on a photodiode array. Mod. 6700, COHU, 320×288 pixels (240×240 pixels were used), with the help of a holographic diffraction grating (1800 lines/mm, Carl Zeiss). The Fourier transform was performed by a computer (386-SX) on-line. Because the laser emission spectrum showed a distinct mode structure the spectrum of the empty interferometer was subtracted from the measured spectrum before the Fourier transform. In both experiments the beam waist diameter was approximately 1 mm and $D\cong10$ mm, i.e. the Fresnel number was of the order of 10^2 (!). Hence we basically would have to perform the three-dimensional Fourier transform of Eq. (2.5). But the light remitted

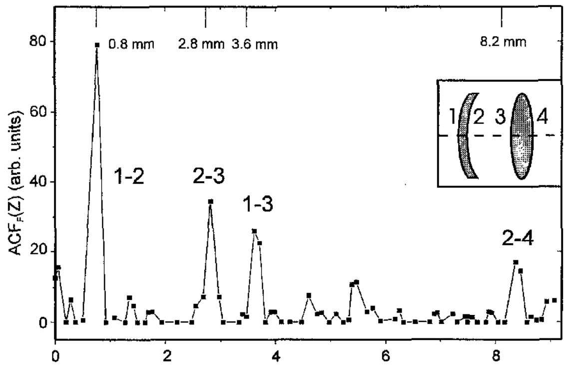

Fig. 4. ACF of the scattering potential $F(z)$ of the anterior part of the model eye with interfaces 1, 2, 3 and 4 (see insert) along the beam illuminating the eye at the optical axis. Optical distance = geometrical distance times refractive index. The ACF peaks correspond to various distances within the object and can be used to derive additional distances. E.g. the distance between the peaks 2-3 and 2-4 yields the optical lens thickness.

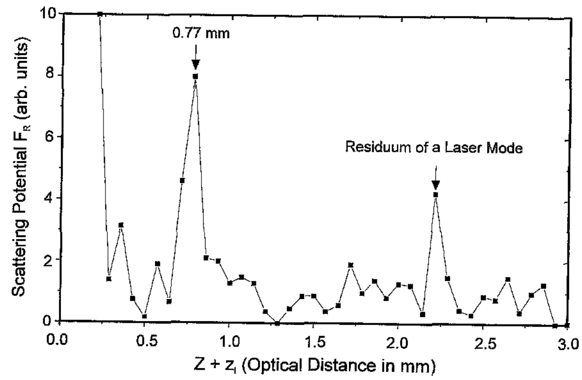

Fig. 5. Scattering potential of a human eye *in vivo*. The main peak at $z = 0.77$ mm indicates the optical corneal thickness. Here too the subtraction of the spectra was used to eliminate the laser mode peak at 1.1 mm. Because of the large intensity differences of the two spectra in this case a rather large peak was left over at the origin and a peak corresponding to a higher laser mode.

from the object is regularly reflected at the interfaces. Those parts of the reflected waves which are detected at $\boldsymbol{P}$ are reflected at a more or less diffraction limited area within the illuminating beam on these interfaces. Hence the one-dimensional approximation of Eq. (2.7) can be used.

In a first experiment a model eye with a convex spherical window (geometrical thickness 0.5 mm; refractive index $n = 1.52$) representing the cornea and a biconvex lens (geometrical axial thickness 3.5 mm, refractive index $n = 1.52$) representing the crystalline lens at a distance of 2.8 mm was used as object. As the reflectivities of the four interfaces were approximated equal and no additional singular interface was used we obtained the ACF of the scattering potential of the object under measurement according to Eq. (3.2). Fig. 4 shows the auto-correlation of the scattering potential $F(z)$ along the axis of the model eye. Note that an auto-correlation has its maximum at the origin, the weak signal at $Z = 0$ is caused by the subtraction of the two spectra before the Fourier transform. As expected the distances measured with the spectral interferometer agree with the a priori distances, at least within ± 0.1 mm (up to round-off errors). Compared to the dual beam low coherence interferometry technique [19] this precision is about one order of magnitude worse. This is due to the low resolution photodiode array (only 240 pixels in one direction) used in this preliminary experiment and can substantially be improved by using a high resolution photodiode array. Furthermore to obtain high precision conversion of optical lengths to geometrical lengths in case of dispersive substances the corresponding group indices would have to be used [19].

In a second experiment we measured the corneal thickness of a human eye at the vision axis *in vivo* (the optical thickness was known from a dual beam partial coherence interferometry measurement [15] to be equal to 0.77 mm). Here the relatively strong reflecting anterior surface of the cornea acted as the reference mirror. The reconstructed residual scattering potential F_R (according to Eq. (3.6)) is shown in Fig. 5. In this experiment no larger distances than the corneal thickness could be measured. This is partly due to the weak intensity of the light waves remitted at the corresponding interfaces in the eye, partly due to the divergence of the reflected waves and partly due to the limited etendue of the spectrometer used in this experiment.

5. Conclusion

It has been shown that a modification of the original formalism developed by Wolf [1] yields the basic physics for spectral interferometric length measurements. Spatially coherent "white" light sources can be used to measure distances by backscattering spectral interferometry. The advantage of this technique as compared e.g. to partial coherence interferometry [15–19] is that no moving parts are needed. As this technique can be very fast and yields the distribution of the object scattering potential along the illuminating light beam it can also be used in partial coherence tomography [20].

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