Doctoral Qualifying Examination  
Department of Mechanical Engineering  
Manufacturing Written  
January 2009

*Note: Don't panic if some of this is new to you. We seek to examine how well you are able to analyze material which is on par with that covered in 2.008 or 2.810, but which you might not have seen before. The idea is that you demonstrate your intuition and reasoning in a different, though relatively simple, context. Also, please be concise -- a few bullet points are sufficient. No essays please!*  

1. Machining

a) The machinist at your shop is cutting a step in a stock of some metal using an end-mill on a vertical CNC milling machine as shown below (in top and isometric views). He says that the cutting forces are too high, causing unacceptable chatter. Keeping the material removal rate (MRR) constant, and keeping the plunging depth constant, but by changing the speeds, feeds and depth of cut, is it possible to reduce the cutting forces to an arbitrarily small value? How would you do so? Assume that the whole process can be thought of as orthogonal cutting.

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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{orthogonal_cutting.png}
\caption{Orthogonal Cutting Diagram}
\end{figure}
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b) In orthogonal cutting, how would you expect the temperature in the vicinity of a cut to change as you make the following changes to the cutting parameters individually (keeping all else the same):

1) Machine a stock material with higher thermal conductivity.
2) Machine a stock material with higher specific energy (of cutting).
3) Machine at a greater cutting speed.
4) Machine a stock material with higher specific heat.

c) Consider two physical measures in orthogonal cutting: the temperature of the cutting zone and the cutting speed.

1) If you keep the cutting speed constant and increase the temperature, do you
expect cutting force to increase or decrease?
2) If you keep the temperature constant (with some sort of a control system which provides or takes away heat rapidly) do you expect cutting forces to increase or decrease when the cutting speed increases?

2. Forging

We reproduce below the notes of Professor Zabaras of Cornell on Open Die Forging. Unfortunately, some aspects of the notes were smudged during scanning. Students are asking for help with the notes. Please answer the 4 questions in the balloons. The original notes are at http://mpdc.mae.cornell.edu/Courses/MAE212/Lecture10.pdf.
Direct Compression in Plane Strain

Let us consider the workpiece at a given height $h$. Let assume that the average yield stress of the material at this stage is $Y$.

Take $\sigma_x, \sigma_y = -p, \sigma_z = \frac{p_x}{2}$ as the principal stresses. The expression for $\sigma_z$ is the result of the plane strain conditions and $p$ is the pressure at the die/workpiece interface. Assume little effect of friction on principal stresses.

For plane strain conditions, the criterion becomes:

$$\sigma_x - \sigma_y = \frac{2Y}{\sqrt{3}} \quad \Rightarrow \quad \sigma_z = \frac{2Y}{\sqrt{3}} - p$$

(1)

The force equilibrium equation for the slab in the $x$ direction is the following:

$$\left(\sigma_x + \sigma_z\right) h w - \sigma_z h w - 2 \mu p \, dx \, w = 0$$

(2)

or after simplification

$$h \, d\sigma_x = 2 \mu \, p \, dx$$

(3)
Justify this step in the derivation. d) What is the point of this statement?

But from $\sigma_x = -dp$ (note that $Y$ is constant for a given $h$). Substitution of the above equation in equation (3) results in the following:

$$\frac{dp}{dx} = -\frac{2\mu}{h} \sigma_x$$

(4)

We need some boundary conditions to integrate the above equation: At $x = \frac{b}{2}$ we know that $\sigma_x = 0$ (free surface). Using the von-Mises criterion at the free surface results in the following:

$$p\left(\frac{b}{2}\right) = \frac{2Y}{\sqrt{3}} - \sigma_x\left(\frac{b}{2}\right) = \frac{2Y}{\sqrt{3}}$$

(5)

Using equation (5) and integrating equation (4) from position $x = 0$ to position $x = \frac{b}{2}$ results in the following:

$$p\left(\frac{b}{2}\right) = \frac{2}{\sqrt{3}} \exp\left[\frac{2\mu}{h} \left(\frac{b}{2} - x\right)\right], \quad 0 \leq x \leq \frac{b}{2}$$

(6)

which governs from $x = 0$ to $x = \frac{b}{2}$. The maximum value for $p$ occurs at the centerline where

$$\left(\frac{p}{\frac{2Y}{\sqrt{3}}}\right)_{\text{max}} = \exp\left(\frac{\mu b}{h}\right)$$

(7)

Also of great interest is $p_{\text{avg}}$, i.e., the average or mean pressure at the tool-workpiece interface (for a given height $h$). For simplicity, let $a = \frac{b}{2}$ and $c = \frac{2b}{h}$ in the following derivation, and note that $e$ is the base of natural logarithms.

$$p_{\text{avg}} = \frac{1}{a} \int_0^a \int_0^b \frac{2Y}{\sqrt{3}} e^{\frac{2Y}{\sqrt{3}} \sigma_x} \sigma_x e^{\frac{2Y}{\sqrt{3}} x} \sigma_x dx = \frac{2Y e^{\frac{2Y}{\sqrt{3}}}}{\sqrt{3} a} \int_0^a e^{\frac{2Y}{\sqrt{3}} x} dx$$

(8)

so

$$p_{\text{avg}} = \frac{2Y e^{\frac{2Y}{\sqrt{3}}}}{\sqrt{3} a} \left[ -e^{-\frac{2Y}{\sqrt{3}}} \right]_{x=0}$$

(9)

Thus,

$$p_{\text{avg}} = \frac{2Y}{\sqrt{3}} \left( e^{\frac{2Y}{\sqrt{3}}} - 1 \right)$$

(10)

Now

$$\exp\left(\frac{\mu b}{h}\right) - 1 = 1 + \frac{\mu b}{h} + \left(\frac{\mu b}{h}\right)^2 + \ldots - 1$$

(11)

so

$$p_{\text{avg}} = \frac{2Y}{\sqrt{3}} \left( 1 + \frac{\mu b}{h} + \ldots \right) \approx \frac{2Y}{\sqrt{3}} \left( 1 + \frac{1}{2!} \frac{\mu b}{h} \right) \text{ for small } \frac{\mu b}{h}$$

(12)