EXPERIMENT 10
Compton Scattering

EQUIPMENT NEEDED FROM EG&G ORTEC

- 1–10 mCi sealed ^{137}Cs source
- Two 113 Scintillation Preamplifiers
- Two 266 Photomultiplier Tube Bases
- Bin and Power Supply
- Two 556 High Voltage Power Supplies
- 480 Pulser
- Two 575A Amplifiers
- 551 Timing Single-Channel Analyzer
- 427A Delay Amplifier
- 905-3 NaI(Tl) Detector and Photomultiplier
- 905-23 Plastic Scintillator 1/2 x 4 in., mounted to photomultiplier tube
- Source Kit SK-1G
- AlRd-1 aluminum scattering rod (see Appendix)
- ACE-2K MCA System including suitable IBM PC (other EG&G ORTEC MCAs may be used)
- Oscilloscope
- 309 Complete Compton Scattering Apparatus
- ORC-10 Cable Set

Purpose

In this experiment the techniques for studying the effects of Compton scattering will be studied. The source will be ^{137}Cs, and the scattering will be caused by its gamma rays striking an aluminum rod and an organic scintillator.

Introduction

The collision of a gamma ray with a free electron is explained by the Compton interaction. The kinematic equations that describe this interaction are exactly the same equations as for two billiard balls colliding with each other, except that the balls are of different size. Figure 10.1 shows the interaction.

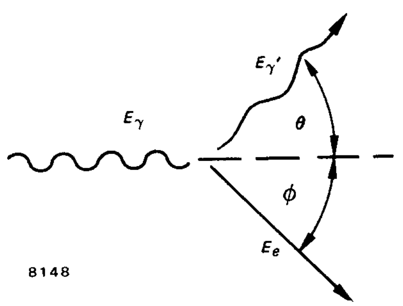

Fig. 10.1. Scattering Caused by Compton Interaction.

In Fig. 10.1 a gamma of energy, E_{γ}, scatters from an electron with an energy $E_{\gamma'}$. (For convenience, all energies are expressed in MeV.) The energy that the electron gains in the collision is E_e. In Fig. 10.1, θ and ϕ are the scattering angles for γ' and the electron respectively. The laws of conservation of energy and momentum for the interaction are as follows:

Conservation of energy,

$$E_{\gamma} = E_{\gamma'} + E_e \, . \qquad (1)$$

Conservation of momentum,

$$\text{x direction } \frac{hf}{c} = \frac{hf'}{c} \,(\cos\theta) + mv\,(\cos\phi) \, . \qquad (2)$$

Conservation of momentum,

$$\text{y direction } 0 = \frac{hf'}{c} \,(\sin\theta) - mv\,(\sin\phi) . \qquad (3)$$

In the above equations, $E_{\gamma} = hf$, $E_{\gamma'} = hf'$, $E_e = mc^2 - m_0c^2$, $m = m_0/(1 - v^2/c^2)^{-1/2}$ when m_0 = rest mass of the electron, and v = the velocity of the recoil electron.

Solving Eqs. (1), (2), and (3) for $E_{\gamma'}$ results in the following:

$$E_{\gamma'} = \frac{E_{\gamma}}{1 + \dfrac{E_{\gamma}}{m_0c^2}\,(1 - \cos\theta)} \, . \qquad (4)$$

Note that Eq. (4) is easy to use if all energies are expressed in MeV. From Experiments 3 and 7, m_0c^2 is equal to 0.511 MeV. In this experiment E_{γ} is the energy of the source (0.662 MeV for ^{137}Cs), and θ is the measured laboratory angle.

Figure 10.2 shows the geometry that will be used for the experiments outlined for Compton scattering. Experiments

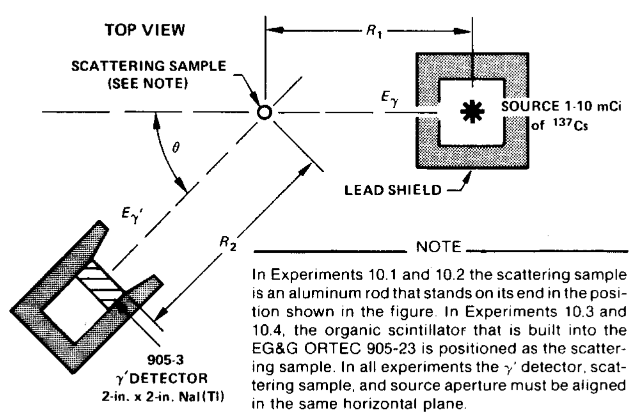

Fig. 10.2. Arrangement of Source, Sample, and Detector for Experiment 10 Using an EG&G ORTEC 309 Compton Scattering Apparatus.

10.1 and 10.2 are simple scattering experiments using an aluminum scattering sample. In Experiments 10.3 and 10.4 the aluminum sample is replaced with an organic scintillator coupled to a phototube, and a coincidence is required between the pulse in the organic scintillator and a pulse in the NaI(Tl) crystal.

EXPERIMENT 10.1

Simple Compton Scattering (Energy Determination)

Procedure

1. Using $E_\gamma = 0.662$ MeV for ^{137}Cs in Eq. (4), calculate the values for $E_{\gamma'}$ and enter them in Table 10.1 for the angles that are to be used in the experiment.

2. Set up the electronics as shown in Fig. 10.3. Calibrate the MCA so that the ^{137}Cs line is in approximately channel 800. This procedure was outlined in Experiment 3.

3. Plot the energy vs channel number for your calibration. This calibration will be used to determine the $E_{\gamma'}$ (Measured) values in Table 10.1.

4. Set $\theta = 20°$ (Fig. 10.2) and accumulate for a period of time long enough to determine the position of the photopeak. (Note: if you also plan to do Experiment 10.2, the sum under each photopeak should be at least 1000 counts.) From your calibration curve fill in $E_{\gamma'}$ (Measured) in Table 10.1. Continue for the other values in the table. Figure 10.4 shows spectra at 20° and 120°.

EXERCISES

a. Plot $E_{\gamma'}$ (Calculated) vs θ on linear graph paper. Put the experimental points with the estimated error on the curve. Do your experimental values agree with the theory?

b. For ^{137}Cs, $E_\gamma = 0.662$ MeV, and since $m_0c^2 = 0.511$, Eq. (4) becomes:

$$E_{\gamma'} = \frac{E_\gamma}{1 + 1.956E_\gamma}\ (1 - \cos\theta). \qquad (5)$$

This can be written:

$$\frac{1}{E_{\gamma'}} = 1.51 + 1.956\ (1 - \cos\theta). \qquad (6)$$

Therefore, a plot of $1/E_{\gamma'}$ vs $(1 - \cos\theta)$ should be a straight line with intercept 1.51 and a slope equal to 1.956. Table 10.2 shows θ, $1/E_{\gamma'}$, and $(1 - \cos\theta)$ for ^{137}Cs.

Make a plot of $1/E_{\gamma'}$ vs $(1 - \cos\theta)$ and put your experimental points from Table 10.1 on the graph. Figure 10.5 shows a typical graph of this function and the experimental data points.

Table 10.1

θ (deg)	$E_{\gamma'}$ (Calculated)	$E_{\gamma'}$ (Measured)
20		
40		
60		
80		
100		
120		
140		
160		

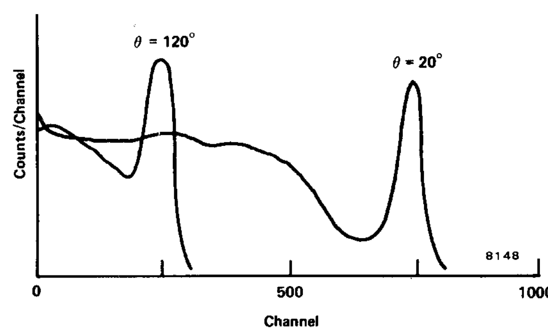

Fig. 10.4. Overlapped Spectra Obtained at Two Scattering Angles.

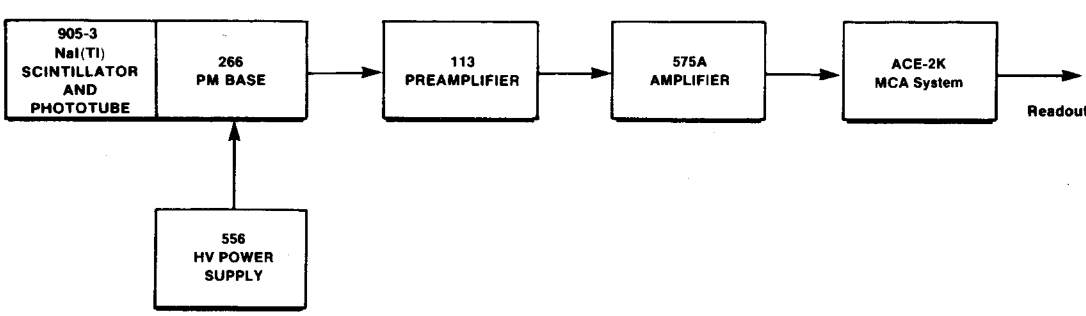

Fig. 10.3. Instrument Interconnections for Experiments 10.1 and 10.2.

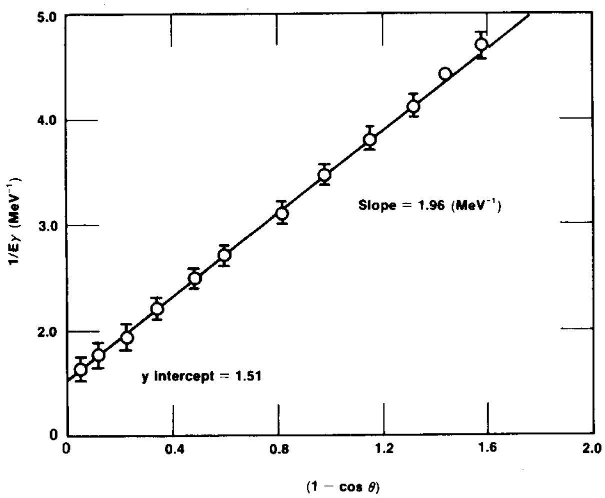

Fig. 10.5. $1/E_{\gamma'}$ vs $(1 - \cos\theta)$ for ^{137}Cs.

Table 10.2

Angle (θ)	$1/E_{\gamma'}$ (MeV^{-1})	$1 - \cos\theta$
0	1.51	0
10	1.54	0.015
20	1.63	0.060
30	1.77	0.133
40	1.97	0.234
50	2.20	0.357
60	2.49	0.500
70	2.79	0.658
80	3.12	0.826
90	3.46	1.00
100	3.80	1.17
110	4.13	1.34
120	4.44	1.50
130	4.72	1.64

EXPERIMENT 10.2

Simple Compton Scattering (Cross-Section Determination)

The differential cross section for Compton scattering was first proposed by Klein and Nishina. This formulation is discussed in ref. 1. The expression has the following form:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{theory}} = \frac{r_0^2}{2}\left\{\frac{1+\cos^2\theta}{[(1+\alpha(1-\cos\theta)]^2}\right\} \tag{7}$$

$$\times \left\{1 + \frac{\alpha^2(1-\cos\theta)^2}{[1+\cos^2\theta]\;[1+\alpha(1-\cos\theta)]}\right\},\left(\frac{\text{cm}^2}{\text{sr}}\right),$$

where

$r_0 = 2.82 \times 10^{-13}$ cm (classical electron radius),

$$\alpha = \frac{E_\gamma}{m_0c^2} = \frac{0.662\text{ MeV}}{0.511\text{ MeV}} = 1.29 \text{ for } ^{137}\text{Cs},$$

$d\Omega$ = the measured solid angle in steradians.

In this experiment we will verify Eq. (7) from the experimental measurements.

Procedure

The procedure here is the same as that for Experiment 10.1 except that for each run the sum under the photopeak should be at least 1000 counts.

1. Solve Eq. (7) for the values of θ used in Table 10.1. (A computer is quite valuable at this point, although not absolutely necessary.)

EXERCISE

a. Plot $(d\sigma/d\Omega)_{\text{theory}}$ vs θ on linear graph paper.

2. Find the measured differential cross section by solving the following expression:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{measured}} = \frac{\Sigma_{\gamma'}}{N\Delta\Omega I}, \tag{8}$$

where

$\Sigma_{\gamma'}$ = sum under the photopeak divided by the intrinsic peak efficiency (see Experiment 3),

N = number of electrons in the scattering sample

$$= \frac{\text{(volume) (density) (atomic no.) (Avogadro's no.)}}{\text{atomic weight}},$$

$\Delta\Omega$ = solid angle in steradians of detector

$$= \frac{\text{area of detector (cm}^2)}{[R_2 \text{ (cm)}]^2},$$

I = the number of incident γ's per cm^2 per s at the scattering sample; this number can be calculated if the activity of the source is known.

EXERCISE

b. Solve Eq. (8) for the measured values. Put the measured values with their estimated errors on the theoretical curve for the Klein-Nishina formula.

EXPERIMENT 10.3

Compton Scattering (Coincidence Method)

Procedure

1. Set up the electronics as shown in Fig. 10.6. The 905-23 is an organic scintillation detector that will be used as the scatterer and should be placed in sample position as shown in Fig. 10.2. It will also provide a coincident enable signal to the MCA to permit the energy analysis of the simultaneous scattered signal from the 905-3 NaI(Tl) detector.

2. For the 905-3 circuit, adjust its 556 high voltage and then adjust the gain of its 575A Amplifier so that the ^{137}Cs photopeak is near the top of the analyzer range. Set the MCA Gate toggle switch at Off during this adjustment.

3. For the 905-23 circuit, adjust its 556 high voltage and then adjust the gain of its 575A Amplifier so that the Compton edge of the ^{137}Cs signal is ~6 V in amplitude at the 575A output, measured with an oscilloscope.

4. Set the 551 Timing SCA Lower-Level dial at 50/1000 or as low as possible without counting noise. Set the Window or Upper-Level dial at 10 V. Adjust the Delay to 0.1 μs.

5. Turn on the 480 Pulser and adjust its attenuated output so that the amplitude at the 575A output in the 905-23 circuit is ~5 V.

6. Set the MCA Gate toggle switch at Coinc (coincidence mode). The analyzer should now store the pulser output pulses. Turn off the 480. The experimental arrangement is now ideal for measuring $E_{\gamma'}$ vs θ, because the coincidence requirement from the 905-23 organic scintillator eliminates virtually all the undesired background.

EXERCISE

Repeat all the measurements made in Experiment 10.1 with this coincidence technique. Plot the experimental and theoretical values as in Experiment 10.1.

EXPERIMENT 10.4

Compton Scattering (Electron Recoil Energy)

Procedure

1. Set up the electronics as shown in Fig. 10.7. The equipment is the same as was used for Experiment 10.3. The 905-23 plastic scintillator is used as the scatterer, set in the sample position on the EG&G ORTEC 309 table, and also provides the source used for energy measurements. The 905-3 will be used to provide the coincident enable signal to the MCA to permit energy analysis of the recoil electrons from the 905-23.

2. Adjust the gain of the 575A Amplifier in the 905-3 circuit so that its output for the 0.662-MeV line of the ^{137}Cs is ~6 V, measured with an oscilloscope. Set the 551 Timing SCA controls as in Experiment 10.3.

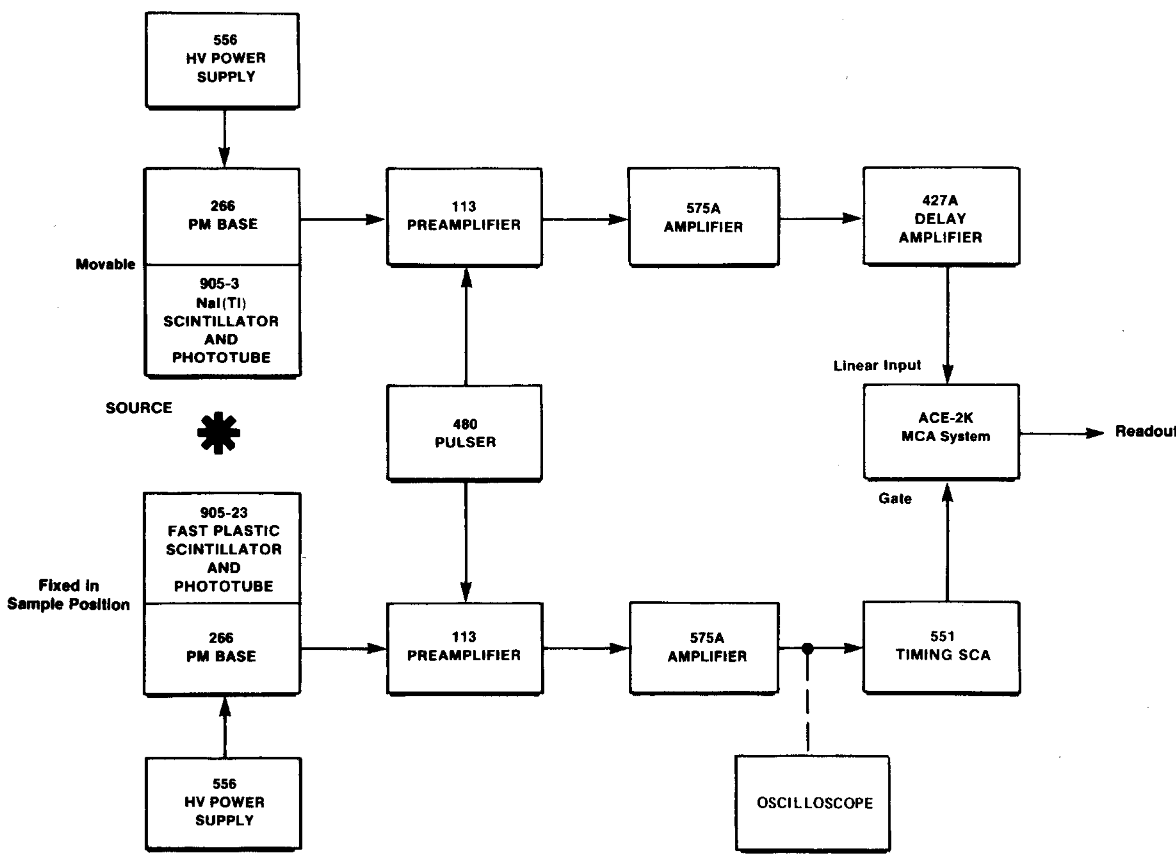

Fig. 10.6. Instrument Interconnections for Experiment 10.3.

3. For the 905-23 circuit, adjust the gain of the 575A Amplifier so that the Compton edge from the 0.662-MeV line of ^{137}Cs is stored in the upper channels in the analyzer.

4. Use Eqs. (1) and (4) to calculate the values for E_e in Table 10.3 and fill in this column of the table.

Table 10.3

θ (deg)	E_e (Calculated)	E_e (Measured)
0		
20		
40		
60		
80		
100		
120		
140		
160		

5. Set the pulse generator for the value of E_e at 0°. Adjust the Calibrate control and the Attenuator switches to place the pulser pulses in about the same analyzer channel as the Compton edge. The pulser is now approximately calibrated.

6. Store pulses from the pulser for simulated energy levels of 0.100 MeV, 0.200 MeV, 0.300 MeV, 0.400 MeV, and 0.500 MeV. Read the data out of the analyzer and plot the calibration curve. Turn off the pulser.

7. Set the 905-3 detector at 20° and store a coincidence spectrum for the 905-23 output for a period of time long enough to determine the position of the recoil electron energy; record the value in Table 10.3.

8. Repeat the measurements for the other angles in Table 10.3. Figure 10.8 shows a typical pair of spectra taken at θ = 160° and 40°.

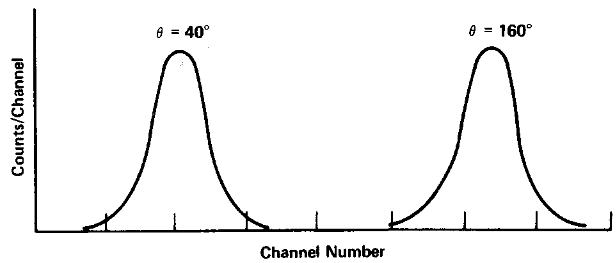

Fig. 10.8. Typical Recoil Electron Spectra Taken at Two Different Angles.

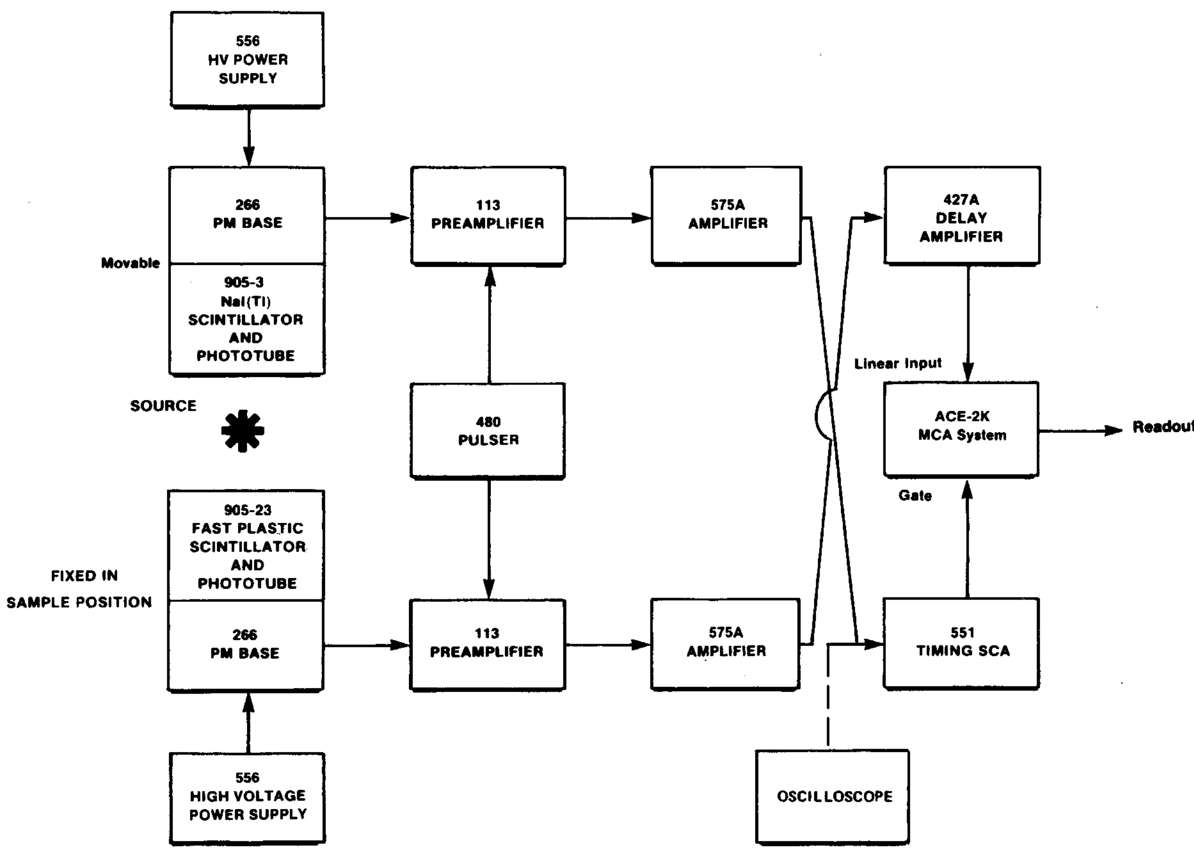

Fig. 10.7. Instrument Interconnections for Experiment 10.4.

EXERCISE

Plot E_c (Calculated) vs θ from Table 10.3. From the calibration curve and the data taken in steps **6** and **7**, put the experimental points on the curve with the estimated error. How do your values agree?

References

1. A. C. Melissinos, *Experiments in Modern Physics,* Academic Press, New York (1966).

2. G. F. Knoll, *Radiation Detection and Measurement,* John Wiley and Sons, New York (1979).

3. P. Quittner, *Gamma Ray Spectroscopy,* Halsted Press, New York (1972).

4. W. Mann and S. Garfinkel, *Radioactivity and its Measurement,* Van Nostrand-Reinhold, New York (1966).

5. K. Siegbahn, Ed., *Alpha-, Beta-, and Gamma-Ray Spectroscopy,* **1,** North Holland Publishing Co., Amsterdam (1965).

6. C. M. Lederer and V. S. Shirley, Eds., Table of Isotopes, 7th Edition, John Wiley and Sons, New York (1978).

7. R. L. Heath, "Scintillation Spectrometry," *Gamma-Ray Spectrum Catalog,* 2nd Edition **1** and **2**, IDO 16880, August (1964). Available from Clearinghouse for Federal Scientific and Technical Information, Springfield, Virginia.

8. G. Marion and P. C. Young, *Tables of Nuclear Reaction Graphs,* John Wiley, New York (1968).