3.22 Mechanical Behavior of materials

PS3

Due: March, 2, 2004 (Tuesday) before class (10:00am)

1. A sample of a material subjected to a compressive stress $\sigma_z$ is confined so that it cannot deform in the y-direction, but deformation is permitted in the x-direction, as shown below. Assume that the material is isotropic and exhibits linear-elastic behavior. Determine the following in terms of $\sigma_z$ and the elastic constants of the material:

   (a) The stress that develop in the y-direction.
   (b) The strain in the z-direction.
   (c) The strain in the x-direction.
   (d) The stiffness $E' = \frac{\sigma_z}{\varepsilon_z}$ in the z-direction. Is this apparent modulus equal to the elastic modulus $E$ from a uniaxial test on the material? Why or why not?

2. An orthotropic material has the compliance matrix:

$$
S_{ij} = 
\begin{pmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\
S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\
S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & S_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}
\end{pmatrix}
$$

The reciprocal relation states that:

$$
\nu_{12}E_2 = \nu_{21}E_1, \text{ where } \nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}
$$

   (a) Prove the reciprocal relation.
   (b) A fiber composite is transversely isotropic and has the elastic moduli:

   $E_1 = E_2 = 5 \text{ GPa}$ \quad $E_3 = 25 \text{ GPa}$
\[ \nu_{12} = 0.25 \quad \nu_{13} = 0.33 \]
\[ G_{13} = 8 \text{ GPa} \]

Calculate the strain tensor for the stress state of  \( \boldsymbol{\sigma}_y = \begin{pmatrix} 5 & 0 & 4 \\ 0 & 5 & 3 \\ 4 & 3 & 5 \end{pmatrix} \) MPa.

3. (a) How and why would you expect the elastic modulus to vary with melting temperature? How would you explain that, although aluminum (Al) and magnesium (Mg) have very similar melting temperatures (\( T_m \sim 650 ^\circ \text{C} \)), they have very dissimilar elastic moduli (\( E_{\text{Al}} = 70 \text{ GPa}, E_{\text{Mg}} = 45 \text{ GPa} \))?

(b) Pure Al has an elastic modulus of 70 GPa. An Al alloy commonly used in aero/astro engineering applications is 7075 T6 Al. The designation “7075” indicates the chemical composition of the alloy: 1.6% Cu, 2.5%Mg, 5.6% Zn, 0.23 Cr, balanced Al. The designation “T6” indicates that the heat treatment of the alloy: tempered at 650 \( ^\circ \text{C} \).

Despite the addition of so many alloying elements, the elastic modulus of 7075 T6 Al is not much different from that of the pure Al: \( E_{7075 \text{ T6}}=70.7 \text{GPa} \). Explain why alloying has little effect on \( E \).

(c) Cobalt is magnetostrictive and contracts upon exposure to a magnetic field. What is the effect of a magnetic field on the elastic modulus of Cobalt?

4. (a) As temperature increases, explain how the elastic modulus changes for crystalline materials and rubbers. Relate each type of behavior to fundamental properties of material.

(b) Speculate on the relation between the modulus of elasticity of a group of crystalline solids and their respective melting points. Also relate the modulus of elasticity to the respective coefficients of thermal expansion.

5. Using the figures below for cases of isostress and isostrain, derive the upper and lower bounds for Young’s modulus for a composite material that were given in class.