1. The creep behaviour of polyethylene is given by the creep compliance data in the table below.

### Creep compliance of Polyethylene

<table>
<thead>
<tr>
<th>t (hours)</th>
<th>J(t) (psi⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.600 x10⁻⁴</td>
</tr>
<tr>
<td>100</td>
<td>0.700 x10⁻⁴</td>
</tr>
<tr>
<td>200</td>
<td>0.720 x10⁻⁴</td>
</tr>
<tr>
<td>300</td>
<td>0.730 x10⁻⁴</td>
</tr>
<tr>
<td>400</td>
<td>0.740 x10⁻⁴</td>
</tr>
<tr>
<td>500</td>
<td>0.750 x10⁻⁴</td>
</tr>
<tr>
<td>600</td>
<td>0.760 x10⁻⁴</td>
</tr>
<tr>
<td>700</td>
<td>0.765 x10⁻⁴</td>
</tr>
<tr>
<td>800</td>
<td>0.770 x10⁻⁴</td>
</tr>
</tbody>
</table>

Another, similar specimen of polyethylene is subjected to the following stress history.

### Stress History for the Polyethylene Specimen

<table>
<thead>
<tr>
<th>Stress (psi)</th>
<th>Duration (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>unloaded</td>
<td></td>
</tr>
</tbody>
</table>

Calculate the strain in the specimen at times t = 0, 300, 500 and 800 hours.
2. A thin walled cylindrical pressure vessel has a radius, \( r \), of 0.5m and a wall thickness, \( t \), of 0.01m. The pressure vessel is made of a steel with a yield stress, \( \sigma_y \), of 320 MPa. The longitudinal stress in a thin-walled cylindrical pressure vessel is \( \sigma_1 = pr/2t \) and the hoop stress is \( \sigma_2 = pr/t \). The stress in the radial direction is zero. (Denote the longitudinal direction as \( x_1 \), the hoop direction as \( x_2 \) and the radial direction as \( x_3 \).)

(a) What pressure is required to produce yielding in the wall of the pressure vessel?

(b) What is the equivalent plastic strain increment (\( d\varepsilon_{eq} \)) in terms of the increment of normal plastic strain in the \( x_2 \) direction (\( d\varepsilon_{22}^p \))?

3. Describe how a metal can be hardened by each of the following techniques. Derive an equation that gives the hardening in each case.

(a) solid solution hardening

(b) precipitation hardening

(c) dispersion hardening
Solutions

1.

\[ \varepsilon(0) = \Delta \sigma_1 J(0) = 50 \times 10^{-4} \]

\[ \varepsilon(300) = \Delta \sigma_1 J(300) + \Delta \sigma_2 J(300-100) + \Delta \sigma_3 J(300-200) \]

\[ = (50) J(300) + (-20) J(300-100) + (20) J(300-200) \]

\[ = (50) J(300) + (-20) J(200) + (20) J(100) \]

\[ = 36.1 \times 10^{-4} \]

\[ \varepsilon(500) = \Delta \sigma_1 J(500) + \Delta \sigma_2 J(500-100) + \Delta \sigma_3 J(500-200) + \Delta \sigma_4 J(500-400) \]

\[ = (50) J(500) + (-20) J(500-100) + (20) J(500-200) + (-50) J(500-400) \]

\[ = (50) J(500) + (-20) J(400) + (20) J(300) + (-50) J(100) \]

\[ = 2.3 \times 10^{-4} \]

\[ \varepsilon(800) = \Delta \sigma_1 J(800-t_1) + \Delta \sigma_2 J(800-t_2) + \Delta \sigma_3 J(800-t_3) + \Delta \sigma_4 J(800-t_4) \]

\[ + \Delta \sigma_5 J(800-t_5) + \Delta \sigma_6 J(800-t_6) \]

\[ = (50) J(800-0) + (-20) J(800-100) + (20) J(800-200) + (-50) J(800-400) \]

\[ + (40) J(800-600) + (-40) J(800-700) \]

\[ = (50) J(800) + (-20) J(700) + (20) J(600) + (-50) J(400) \]

\[ + (40) J(200) + (-40) J(100) \]

\[ = 2.2 \times 10^{-4} \]
2.

a)

\[ \sigma_1 = \frac{Pr_1}{2t}; \sigma_2 = \frac{Pr_2}{t}; \sigma_3 = 0 \]

\[ \sigma_{eq}^2 = \frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \]

\[ = \frac{3}{4} \left( \frac{Pr}{t} \right)^2 = \sigma_y^2 \]

\[ \rightarrow P = \frac{2}{\sqrt{3}} \frac{\sigma_y t}{r} = 7.4 \text{MPa} \]

b)

\[ \frac{d\varepsilon_{22}^p}{\sigma_2 - \frac{1}{2}(\sigma_1 + \sigma_3)} = \frac{d\varepsilon_{eq}^p}{\sigma_{eq}} \]

\[ \rightarrow d\varepsilon_{eq}^p = \frac{\sigma_{eq}}{\sigma_2 - \frac{1}{2}(\sigma_1 + \sigma_3)} d\varepsilon_{22}^p = \frac{\sqrt{3}}{2} \frac{\sigma_2}{\sigma_2 - \frac{1}{2} \sigma_1} d\varepsilon_{22}^p = \frac{2\sqrt{3}}{3} d\varepsilon_{22}^p \]
MICROSTRUCTURAL STRENGTHENING MECHANISMS IN METALS

- Metals - intrinsically soft - non local bonds - low energy per bond
- Have to be strengthened
- Introduce obstacles to dislocation motion
  - Solid solution hardening
  - Precipitation hardening
  - Dispersion hardening

SOLID SOLUTION HARDENING

- Single element dissolved in primary metal
  - e.g. Brass: Zn in Cu
  - Bronze: Sn in Cu
  - Stainless steel: Ni, Cr in Fe

- Elastic interaction bet’n solute & primary metal
  - Geometric misfit
    - Dilute sol’n - individual atoms
    - Concentrated soln - clusters of atoms dissolve
      - Short range order of solute
  - Modulus misfit

- If interaction energy bet’n dislocation & solute is $U_s$

- Stresses req’d to move dislocation (work done)

$$T_0 \cdot b \cdot L = U_s$$

$$T_0 = \frac{U_s}{b \cdot L}$$

$$T_0 \cdot \frac{U_s \cdot \sqrt{c}}{b^2} = 0$$

$T_0 \cdot b \cdot L = \frac{U_s \cdot \sqrt{c}}{b^2}$

$= 0$ strength $\uparrow$ with $\sqrt{c}$ $T_0 \cdot b \cdot L = \frac{U_s \cdot \sqrt{c}}{b^2}$
Precipitation Hardening

- DUCAL - HARD Al Alloy - PPT of CuAl₂ in Al
- Mild Steel - Iron Carbide (Fe₃C) PPT
- Nimonic - Ni₃Al in Ni

- PPT are weak obstacles to dislocation motion
- Dislocations can cut through their lattice
- Work done in doing so raises \( \sigma \)

\[ L = \text{spacing bet'n PPT} \]

Dislocation moves through entire diameter of PPT (2r)

\[ \text{Work done by dislocation:} \]

\[ \tau_p \cdot \delta L \cdot 2r \]

Energy req'd to cut through PPT lattice per unit surface area = \( \Gamma \)

Energy to cut through PPT lattice

\[ \approx \Gamma \cdot (2r) \cdot (6) \cdot 2 \quad (2 \text{ surfaces}) \]

\[ \Rightarrow \text{Shear stress req'd to cut through PPT} \]

\[ \tau_p = \frac{4rb \cdot \Gamma}{2bLr} = \frac{2\Gamma}{L} \]

\[ \Rightarrow \text{More closely spaced PPT, higher yield strength} \]

\[ \Rightarrow \text{High strength Al alloys, e.g. Nimonic (Ni₃Al) \( \approx L \approx 10^{-7} \text{m)\} } \]
• DISPERSION - COMPOUND INTRODUCED INTO A CRYSTAL 
  e.g. Al₂O₃ ADDED TO Al - SINTERED Al - POWDER METALLUlgY

• TYPICALLY CHOOSE CERAMICS THAT ARE COVALENTLY BONDED 
  & ARE INTRINSICALLY HARD & STRONG. - D DISLOCATIONS 
  CAN'T CUT THROUGH THEM

• DISLOCATION BOWS OUT BETWEEN DISPERSION PARTICLES 
  (PINNED)

[Diagram of dispersion particle and dislocation]

• MAXIMUM RESISTANCE TO SHEAR FROM LINE TENSION WHEN 
  θ = 90°

  Force Equil. : \( 2T = \sigma_d b L \)

  \[ T = \frac{G b^2}{2} \]

  \[ \sigma_d = \frac{2T}{bl} = \frac{G b^2}{bl} = \frac{Gb}{L} \]

  **Example to raise yield strength to** \( \frac{\sigma_d}{G} = 10^{-3} \Rightarrow b = 10^{-3} \)

  \( b \approx 10^{-9} \) to \( 10^{-10} \) m \( \Rightarrow L \ll 1 \mu \text{m} \) to \( 0.1 \mu \)

  = D dispersions have to be very fine to get small enough space
  to be USEFUL.