A portion of the atoms in a (111) plane in a fcc crystal. Unit slip of atoms in the B plane occurs with displacement \((a/2) [\overline{1}01]\) (the solid arrow). Instead of slip occurring by unit displacement it can take place in two segmental steps (\(X\) and \(Y\)), as shown. The displacement \(X\) results in atoms in plane B temporarily occupying a C stacking sequence in the fcc lattice.
Theoretical Yield Strength

\[ \frac{d\tau}{d\phi} = \frac{d\tau}{dx} \cdot \frac{dx}{d\phi} = \alpha T_{\text{max}} \frac{2\pi}{a} \cos \frac{2\pi x}{a} \]

\[ G = \left. \frac{d\tau}{d\phi} \right|_{\phi \rightarrow 0} = T_{\text{max}} \cdot \frac{2\pi}{10} \]

\[ T_{\text{max}} = \frac{G}{2\pi} \approx \frac{G}{10} \approx 10 \text{ GPa} \]
Strain Invariants and Dislocation Core Structure of Simulated B2 NiAl

- Relaxed lattice of the stoichiometric pure edge dislocation
- Contours of $\varepsilon_{xx}^2 + \varepsilon_{xx}^2 + 2\varepsilon_{xy}^2$
- From http://www.sv.vt.edu/class/Student_Proj/class94/ternes/ternes.html
Effect of obstacle spacing

\[ \delta = \frac{b}{2\pi r} \]

\[ U = \frac{1}{2} \dot{r}^2 = \frac{G\dot{b}^2}{2} \approx \frac{G\dot{b}^2}{8\pi^2 r^2} \sim G\dot{b}^2 \]

\[ 2T \sin \frac{d\theta}{2} = 2G\dot{b}r d\theta \]

\[ \tau = \frac{G\dot{b}}{L} \sim \frac{G\dot{b}}{L} \]

\[ \tau_y = \tau_0 + AG\dot{b}\sqrt{\rho} \]

\[ \mathrm{d}W = \frac{1}{m^2} \]

Stresses around an edge dislocation

Maple solution from Airy stress function

\[
\Phi := \frac{1}{2} y \ln (x^2 + y^2)
\]

Obtain stresses by differentiation

\[
\sigma_x := \frac{y (3 x^2 + y^2)}{(x^2 + y^2)^2}
\]

\[
\sigma_y := \frac{y (-x^2 + y^2)}{(x^2 + y^2)^2}
\]

\[
\tau_{xy} := -\frac{x (-x^2 + y^2)}{(x^2 + y^2)^2}
\]

Plot of shear stress:

\[
\text{plot3d}(\tau_{xy}, x=-1..1, y=-1..1);
\]
Work Hardening

Dislocation jogs – hard obstacles

Dislocation multiplication

Increase in yield stress

□, polycrystalline Cu; ○, single-crystal Cu—one slip system; ◊, single-crystal Cu—two slip systems; △, single-crystal Cu—six slip systems. (After H. Weidensch, J. Metals, 16, 425, 1964.)

Resolved shear stress (MN/m²) vs. Dislocation density, p (m⁻²)

Slope = ½
Precipitation Hardening

Al 7075-T6

$\Delta$ temper: solutionize, aging

1.6% Cu, 2.5 Mg, 0.2 Mn

$\sigma_f = 82$ ksi, $\sigma_y = 72$ ksi, $\epsilon_f = 11\%$

\[ \text{T, } ^\circ\text{C} \]

\[ \alpha + \Theta (\text{CuAl}_2) \]

\[ \text{wt % Cu} \]

\[ \text{VHN} \]

log (t, days)

optimal coherent particle size & distribution
Grain Boundary Strengthening

Hall-Patch: \( \sigma_y = \sigma_0 + k \gamma d^{-1/2} \)

Want \( d \leq 5 \mu m \) for substantial strengthening (small obstacle spacing)