A pressure vessel is constructed with a diameter of $d=10''$ and a length of $L=6'$. The vessel is to be capable of withstanding an internal pressure of $p=1000$ psi, and the wall thickness is such as to keep the nominal hoop stress under 2500 psi. However, the vessel bursts at an internal pressure of only 500 psi, and a micrographic investigation reveals the fracture to have been initiated by an internal crack 0.1" in length. Calculate the fracture toughness ($K_{ic}$) of the material.

Eq. 23.5 for $K_{ic}$:

$$ eq1:=\sigma_f=K_{ic}/(\alpha\sqrt{\pi a}); $$

Since the vessel was sized to give 2500 psi maximum stress at 1000 psi internal pressure and failed at half that pressure, the hoop stress at failure was $2500/2=1250$ psi. Solving the above equation for $K_{ic}$ at this stress along with $\alpha=1$ and $2a=0.1$;

$$ \text{Digits:=4;} 'K_{ic}'=\text{solve(subs(\{\sigma_f=2500/2,\alpha=1,a=0.05\},eq1),K_{ic})}; $$

$$ K_{ic} = 495.6 $$

The units here for $K_{ic}$ are psi√in; this converts to 0.545 MPa√m. Comparison with the values in Table 23.2 show this to be very low, comparable with wood or glass.