8.02 ESG Independent Study

Unit 3: Electric Potential

Using electric potential has two advantages. Its definition in terms of "work per unit charge" has obvious applications when dealing with the dynamics of charged objects and the relations between mechanical and electric energy (this aspect is reserved until unit four). Unit three investigates the electric potential as a scalar field related to the electrostatic vector field, and hence the charge distribution. Dealing with potential is often easier than dealing with the vector fields. If this treatment seems to lean a bit too much on mathematics, it will help to keep in mind that electric potential is electrostatic potential energy per unit charge.

Objectives: After completing this unit, you should be able to calculate

1. the electric potential at a point in space from the electric fields,
2. the electric potential at a point in space from the charge distribution,
3. the electrical field at a point in space from the electric potential.

Suggested procedure:

1. Read chapter 23 in UP11. Suggested problems include 16, 21, 44, 59, 79, 91 (this is somewhat involved, and it needs your 8.01 knowledge; please keep at it until you get the answers, which are, I believe, correct.). Or,
2. Read Purcell chapter 2, sections 1-6 and 12. The remainder of the chapter is mostly mathematical, and should be studied if you're interested in mathematical physics. If you've already had 18.02 or an equivalent vector calculus course, much of those sections should be familiar. Suggested problems from this chapter include #s 1, 4, 7, 8, 10, 11 (difficult, by so much more enjoyable when it's done), 12 and 15.
3. Do the attached problems.
4. Take a unit test.

Unit 3 — Supplementary Problems

1. For a point charge Q, the electric potential at a distance r away is given by

$$\phi = \frac{Q}{4\pi\epsilon_0}\frac{1}{r} \qquad (\phi = Q/r \text{ in Gaussian units})$$

where $r = \sqrt{x^2 + y^2 + z^2}$, $\phi = 0$ at infinity.

Find $\vec{E}$ from

$$\vec{E} = \hat{\imath}\frac{\partial \phi}{\partial x} + \hat{\jmath}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z},$$

and show explicitly that this reproduces Coulomb's law.

2. This problem deals with the electric field and potential of a finite line of charge.

(a) By integrating

$$\phi = \int d\phi = \int \frac{dq}{4\pi\epsilon_0 r} \qquad (= \textstyle\int dq/r \text{ in Gaussian units}),$$

find the potential at a point in the X–Y plane which is at a perpendicular distance $\sqrt{x^2 + y^2}$ from a finite line charge on the z-axis between $z = -L/2$ and $z = L/2$, with linear charge density λ $(= Q/L)$.

(b) The above assumes $\phi = 0$ at infinity. Re-express ϕ so that $\phi = 0$ at the point $(R, 0)$ in the X–Y plane.

(c) Let $L \to \infty$, keeping λ a constant.

(d) Find $\vec{E}$ from

$$\vec{E} = \hat{\imath}\frac{\partial \phi}{\partial x} + \hat{\jmath}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z},$$

and show explicitly that this reproduces the result from Gauss's law.