

## 8.02 ESG Independent Study

### Unit 15: Maxwell's Equations

Before finishing the C&MS sequence of units for 8.02 with a unit on electromagnetic waves, we need to do some mathematical groundwork and let you in on a little secret: Ampère's Law as stated in UP11 chapter 28 and in Purcell chapter 6 is a fraud. Something is missing.

In the integral form (in MKS units),  $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 i$ , the integral is around a closed contour  $C$ , and  $i$  is the current *through* that contour;  $i$  must be the current crossing a surface which is bounded by  $C$ . However, there are at least dozens of surfaces bounded by a given contour, and if  $i$  is not the same through each of them, Ampère's Law gives inconsistent results, and is hardly worthy of being called a "law".

In differential form (in Gaussian units),  $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J}$ . Taking the divergence of both sides and recalling that the divergence of a curl is necessarily zero shows that Ampère's Law *cannot* be valid unless  $\vec{\nabla} \cdot \vec{J} = 0$ . But if the charge density at any point is changing,  $\vec{J}$  *must* have a divergence, so Ampère's "Law" is violated. This unit will show how we can save Ampère's Law and restore it to status worthy of being put on a tee shirt.

University Physics introduces the "displacement current" into Ampère's Law, just as James Maxwell did in 1865. Although not really a current, the displacement current does play a very important physical role in Ampère's Law.

Purcell shows how Ampère's Law *must* be corrected in order to be mathematically consistent with Gauss' Law and the equation of continuity; it's of no use to express physical relations mathematically if the math is inconsistent.

**Objectives:** After completing this unit, you should be able to show how inclusion of changing electric flux in Ampère's Law yields consistent results, appreciate why a changing electric field induces a magnetic field, and express various qualitative E&M phenomena quantitatively via the appropriate Maxwell's equation(s).

#### Suggested Procedure:

1. Read section 29.7 in UP11. Section 9–1 of Purcell is recommended; if the different units or the use of differential vector operators is confusing, relate Maxwell's equations as you know them to figures 9.2, 9.3, and 9.4. Suggested problems include 36, 37 and the two problems attached to this unit.

or,

2. Read Chapter nine in Purcell, section 1–3 (the material in the remainder of the chapter will be covered in unit 16). Suggested problems include (pp. 343–345) #s 8, 10, and the two problems attached to this unit.
3. Take a unit test

**Supplementary Problem 1:** Suppose a point charge is instantaneously introduced (“magically appears” in the vernacular) in an infinite region of conductivity  $\sigma$  (recall that  $\vec{J} = \sigma \vec{E}$ ). Find the magnetic field at all points. Pick a specific contour and show explicitly that Ampère’s Law is valid.

**Supplementary Problem 2:** The figures on the next page represent a side view of a charging capacitor; the white arrows represent conduction current and the black arrows represent the electric field. The figure shows a realistic configuration for  $\vec{E}$ , but for the purpose of this problem, assume that the capacitor plates are circles of radius  $R$ , and that  $\vec{E}$  is uniform between the plates but vanishes outside the plates. We wish to find the magnetic field at point  $P$  by using the integral form of Ampère’s Law. We will do this by considering the contour  $C$  used in  $\oint_C \vec{B} \cdot d\vec{l}$  to be a circle of radius  $r < R$  coaxial with the capacitor axis, but we will use four different surfaces bounded by the contour. The surfaces are:

$S_a$ : the disc bounded by  $C$ .

$S_b$ : the cylinder shown, open at the end determined by  $C$ .

$S_c$ : the partially open cylinder shown: a closed cylinder with a removed disc of radius  $r < r_0 < R$ .

$S_d$ : same as  $S_c$ , but with  $r < R < r_0$ .

For  $S_b$  and  $S_c$ , you will need to find the current which passes through the curved part of the surfaces; this will depend on  $r$  or  $r_0$ .

Your answers should of course agree for the four surfaces.