

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
ESG Physics

8.02 with Kai

Spring 2003

Problem Set 3 Solution

Problem 1: 25.14

An insulating rod having linear charge density and linear mass density $\mu = 0.100 \text{ kg/m}$ is released from rest in a uniform electric field $E = 100 \text{ V/m}$ directed perpendicular to the rod (Fig. P25.14).

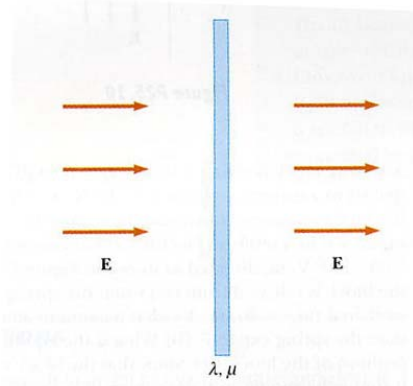


Fig. P25.14

- (a) Determine the speed of the rod after it has traveled 2.00 m.
- (b) How does your answer to part (a) change if the electric field is not perpendicular to the rod? Explain.

Solution:

(a) When the rod is moving along the electric field, it is moving from a high potential point to a low potential point, so the rod is going to lose potential energy. And by conservation of energy, the lost in potential energy is the same as the gain in kinetic energy. Thus

$$PE_{lost} = KE_{gain} \quad (1.1)$$

If the rod travels a distance s along the electric field, the potential difference from the starting point to the end point would be

$$\Delta V = Es \quad (1.2)$$

Suppose the rod has its length l , then the charge of the rod is given by

$$Q_{rod} = \lambda l \quad (1.3)$$

Therefore

$$PE_{lost} = Q_{rod} (\Delta V) = \lambda l E s \quad (1.4)$$

If the rod has its mass density μ , then it has a mass

$$m_{rod} = \mu l \quad (1.5)$$

Thus, according to equation (1.1), we have

$$\lambda l E s = \frac{1}{2} (\mu l) v^2 \quad (1.6)$$

which gives

$$v = \sqrt{\frac{2\lambda E s}{\mu}} \quad (1.7)$$

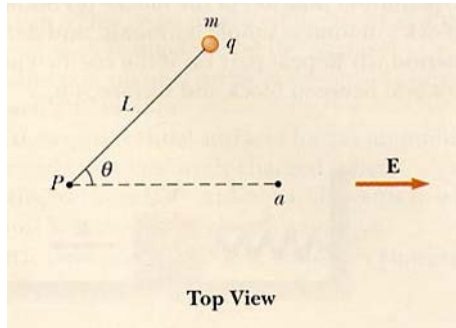
Substituting the value that we have, we obtain

$$\boxed{v = 0.400 \text{ m/s}} \quad (1.8)$$

(b) The speed would be the same as part (a), since the electric field is a conservative field, that the work done by the field (the potential energy) does not depend on the path that it takes.

Problem 2: 25.15

A particle having charge $q = +2.00 \mu\text{C}$ and mass $m = 0.0100 \text{ kg}$ is connected to a string that is $L = 1.50 \text{ m}$ long and is tied to the pivot point P in Figure P25.15.

**Fig. P25.15**

The particle, string, and pivot point all lie on a horizontal table. The particle is released from rest when the string makes an angle $\theta = 60.0^\circ$ with a uniform electric field of magnitude $E = 300 \text{ V/m}$. Determine the speed of the particle when the string is parallel to the electric field (point a in Fig. 25.15).

Solution:

The potential difference between the initial point and the end point would just be the product of the electric field and the “perpendicular distance” between the two points, therefore,

$$\Delta V = -E(L - L \cos \theta) \quad (2.1)$$

Thus, having the charge q , the charge would lose a potential energy of magnitude

$$PE_{\text{loss}} = q\Delta V = qEL(1 - \cos \theta) \quad (2.2)$$

And by conservation of energy, this would be the gain in kinetic energy, thus

$$\frac{1}{2}mv^2 = qEL(1 - \cos \theta) \quad (2.3)$$

which would gives

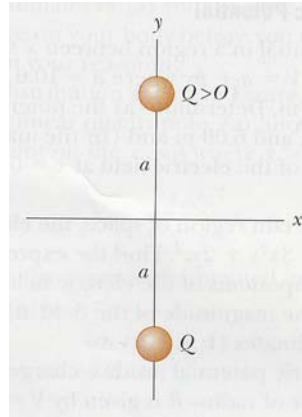
$$v = \sqrt{\frac{2qEL(1 - \cos \theta)}{m}} \quad (2.4)$$

and the answer should be

$$\boxed{0.300 \text{ m/s}} \quad (2.5)$$

Problem 3: 25.30

Two point charges of equal magnitude are located along the y axis equal distances above and below the x axis, as shown in Figure P25.30.

**Fig. P25.30**

- (a) Plot a graph of the potential at points along the x axis over the interval $-3a < x < 3a$. You should plot the potential in units of $\frac{k_e Q}{a}$.
- (b) Let the charge located at $-a$ be negative and plot the potential along the y axis over the interval $-4a < y < 4a$.

Solution:

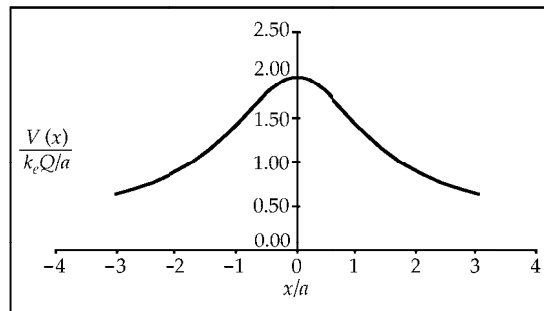
For any point in x axis, its potential would be

$$\varphi(x) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(Q)}{\sqrt{x^2 + a^2}} + \frac{k_e(Q)}{\sqrt{x^2 + (-a)^2}} \quad (3.1)$$

which gives

$$\frac{\varphi(x)}{\left(\frac{k_e Q}{a}\right)} = \frac{2}{\sqrt{\left(\frac{x}{a}\right)^2 + a^2}} \quad (3.2)$$

and should be in the form



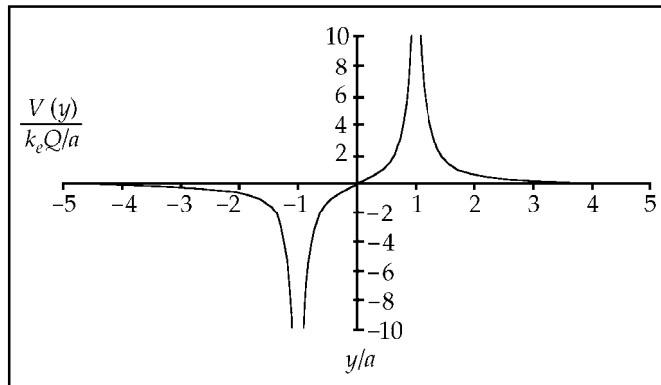
(b) For the points in the y axis and if one of the charge is changed to negative charge, we have

$$\varphi(y) = \frac{k_e Q}{|y-a|} - \frac{k_e Q}{|y+a|} \quad (3.3)$$

which gives

$$\frac{\varphi(y)}{\frac{k_e Q}{a}} = \frac{1}{\left| \frac{y}{a} - 1 \right|} - \frac{1}{\left| \frac{y}{a} + 1 \right|} \quad (3.4)$$

which would gives



Problem 4: 25.31

In Rutherford's famous scattering experiments that led to the planetary model of the atom, alpha particles (charge $+2e$, mass $= 6.64 \times 10^{-27}$ kg) were fired at a gold nucleus (charge $+79e$). An alpha particle, initially very far from the gold nucleus, is fired with a velocity of 2.00×10^7 m/s directly toward the center of the nucleus. How close does the alpha particle get to this center before turning around? Assume the gold nucleus remains stationary.

Solution:

We apply the law of energy conservation again. The alpha particles have kinetic energy at the beginning. So if the direction is head on, that means that the alpha particle would stop at the point where it loses all its kinetic energy in order to gain the potential energy required for it to reach that point. We know that for a point charge, the potential at a point r away from the gold nucleus is

$$V_r = \frac{k_e Q_{gold}}{r} \quad (4.1)$$

Thus, for the alpha particle to reach a point R away from the gold nucleus,

$$PE = \frac{k_e Q_{gold}}{R} q_{alpha} \quad (4.2)$$

Therefore, this would equal to the initial kinetic energy, which is

$$\frac{1}{2}mv^2 = k_e \frac{Q_{gold} q_{alpha}}{R} \quad (4.3)$$

Rearranging, we have

$$R = \frac{2k_e Q_{gold} q_{alpha}}{mv^2} \quad (4.4)$$

which gives

$$\boxed{R_{min} = 2.74 \times 10^{-14} \text{ m}} \quad (4.5)$$

Problem 5: 25.40

When an uncharged conducting sphere of radius a is placed at the origin of an xyz coordinate system that lies in an initially uniform electric field $\mathbf{E} = E_0\mathbf{k}$, the resulting electric potential is

$$V(x, y, z) = V_0 - E_0z + \frac{E_0a^3z}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \quad (5.1)$$

for points outside the sphere, where V_0 is the (constant) electric potential on the conductor. Use this equation to determine the x , y , and z components of the resulting electric field.

Solution:

We know that, since this a conducting sphere, there is no charge inside the sphere, and thus the electric field inside the sphere is zero, according to Gauss's Law.

For $x, y, z > a$, we can obtain the electric field in the three coordinate by taking the partial derivative of the potential respect to the according coordinate, that is

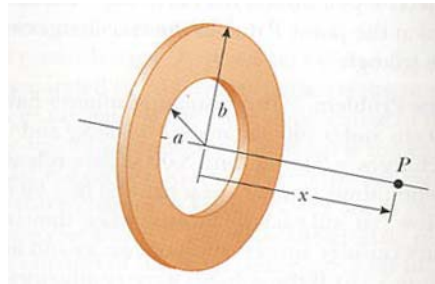
$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z} \quad (5.2)$$

which gives

$$\mathbf{E} = E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k} = \frac{3E_0a^3xz}{(x^2 + y^2 + z^2)^{\frac{5}{2}}}\mathbf{i} + \frac{3E_0a^3yz}{(x^2 + y^2 + z^2)^{\frac{5}{2}}}\mathbf{j} + \left[E_0 + \frac{E_0a^3(2z^2 - x^2 - y^2)}{(x^2 + y^2 + z^2)^{\frac{5}{2}}} \right]\mathbf{k} \quad (5.3)$$

Problem 6: 25.45

Calculate the electric potential at point P on the axis of the annulus shown in Figure P25.45, which has a uniform charge density σ .

**Fig. P25.45****Solution:**

This arrangement of charges would be equal to as if we have a uniformly charged disc with a radius b with charge density σ , superimposed with another uniformly charged disc with radius a with charge density $-\sigma$. So, by what we have done in class, the potential due to a charged disc with radius r at x away is given by

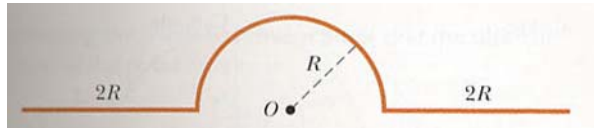
$$V_{disc} = 2\pi\sigma k_e \sqrt{x^2 + r^2} \quad (6.1)$$

Therefore, the potential due to above arrangement is given by

$$\boxed{V = V_{disk a} + V_{disk b} = 2\pi\sigma k_e \left(\sqrt{x^2 + b^2} - \sqrt{x^2 + a^2} \right)} \quad (6.2)$$

Problem 7: 25.46

A wire of finite length that has a uniform linear charge density λ is bent into the shape shown in Figure P25.46. Find the electric potential at point O .

**Fig. P25.46****Solution:**

So we use the formula

$$d\varphi = \frac{dq}{r} \quad (7.1)$$

to calculate the potential at that point, that is we want to know what is the contribution to the potential at point O by each small bit of charge dq along the line of charge. As shown above, the line of charge can be divided into three parts, and each part would have different representation of the formula (7.1). Consider the straight part, which is the part on the left and on the right, we have

$$d\varphi = \frac{dq}{r} = \frac{\lambda dx}{x} \quad (7.2)$$

while for the middle curved part, we have

$$d\varphi = \frac{dq}{r} = \frac{\lambda dl}{R} = \frac{\lambda R d\theta}{R} = \lambda d\theta \quad (7.3)$$

Therefore, we have

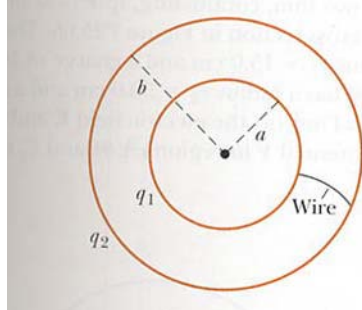
$$\varphi = \int_{-3R}^{-R} \frac{\lambda dx}{x} + \int_{-\pi}^0 \lambda d\theta + \int_R^{3R} \frac{\lambda dx}{x} \quad (7.4)$$

which gives

$$\boxed{\varphi = k_e \lambda (\pi + 2 \ln 3)} \quad (7.5)$$

Problem 8: 25.50

Two concentric spherical conducting shells of radii $a = 0.400$ m and $b = 0.500$ m are connected by a thin wire, as shown in Figure P25.50. If a total charge $Q = 10.0 \mu\text{C}$ is placed on the system, how much charge settles on each sphere?

**Fig. 25.50****Solution:**

Since this is a conductor, and we know that charge only resides on the surface. Since the two shells are connected by a wire, the charge would go as outside as possible, so the charge would just go to the outer sphere. As a result, there would be no charge on the inner sphere, and $10.0 \mu\text{C}$ on the outer sphere.

Problem 9: 25.68

The thin, uniformly charged rod shown in Figure P25.68 has a linear charge density λ . Find an expression for the electric potential at P .

Solution:

Consider a bit of charge dq which sits at x away from the origin, then it would have a distance $\sqrt{x^2 + b^2}$ to the point P . We know that

$$d\varphi = \frac{dq}{r} = \frac{\lambda dx}{\sqrt{x^2 + b^2}} \quad (9.1)$$

and the limit of integration should be from a to $a+l$, thus

$$\varphi = \int_a^{a+l} \frac{\lambda dx}{\sqrt{x^2 + b^2}} \quad (9.2)$$

which gives

$$\boxed{\varphi = k_e \lambda \ln \left(\frac{a+l + \sqrt{(a+l)^2 + b^2}}{a + \sqrt{a^2 + b^2}} \right)} \quad (9.3)$$