Sources of Magnetic Fields: Ampere's Law

Today's Reading Assignment Course Notes: Sections 9.3-9.4, 9.6

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Last Time: Creating Magnetic Fields: Biot-Savart

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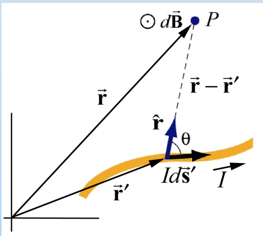
The Biot-Savart Law

Current element of length $d\vec{\mathbf{s}}$ carrying current *I* produces a magnetic field at the point *P*:



$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int_{wire}\frac{Id\vec{\mathbf{s}}'\times(\vec{\mathbf{r}}-\vec{\mathbf{r}}')}{\left|\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right|^3}$$

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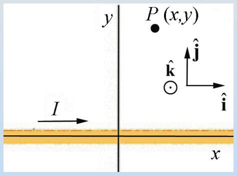
The Biot-Savart Law: Infinite Wire

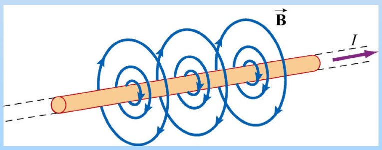


Magnetic Field of an Infinite Wire Carrying Current *I* from Biot-Savart:

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi y}\hat{\mathbf{k}}$$

See W06D3 Problem Solving

http://web.mit.edu/8.02t/www/materials/ProblemSolving/solution05.pdf



More generally:

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\theta}}$$

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3rd Maxwell Equation: Ampere's Law

$$\oint_{closed\ path} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_{open\ surface} \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, da$$

Open surface is bounded by closed path

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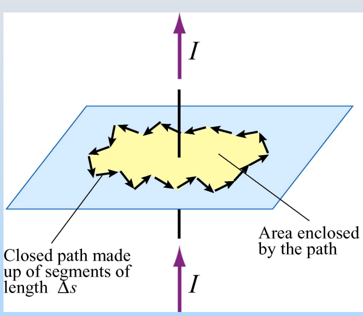
Ampere's Law: The Idea



In order to have a B field around a loop, there must be current punching through the loop

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Concept Question: Line Integral

The integral expression $\oint_{closed\ path} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$

1. is equal to the magnetic work done around a closed path.
2. is an infinite sum of the product of the tangent component of the magnetic field along a small element of the closed path with a small element of the path up to a choice of plus or minus sign.
3. is always zero.
4. is equal to the magnetic potential energy between two points.
5. None of the above.

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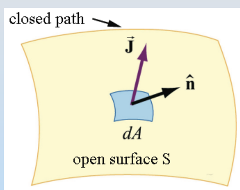
Current Enclosed

$\vec{\mathbf{J}}$ Current density

$$I_{enc} = \iint_{\substack{open \\ surfacce\ S}} \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, dA$$



Current enclosed is the flux of the current density through an open surface *S* bounded by the closed path. Because the unit normal to an open surface is not uniquely defined this expression is unique up to a plus or minus sign.

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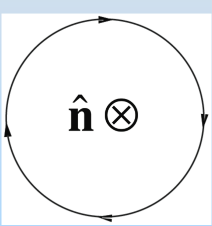
Sign Conventions: Right Hand Rule

$$\oint_{closed\ path} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_{open\ surface} \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, da$$



Integration direction clockwise for line integral requires that unit normal points into page for open surface integral

Current positive into page, negative out of page

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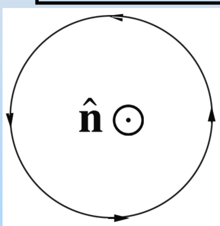
Sign Conventions: Right Hand Rule

$$\oint_{closed\ path} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_{open\ surface} \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, da$$



Integration direction counterclockwise for line integral requires that unit normal points out of page for open surface integral

Current positive out of page, negative into page

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Concept Questions: Ampere's Law

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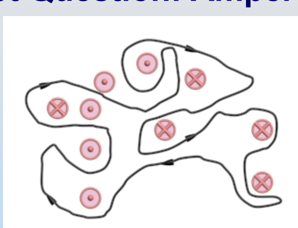
Concept Question: Ampere's Law



Integrating B around the loop shown gives us:

1. a positive number
2. a negative number
3. zero

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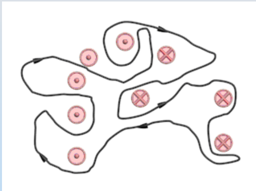
Concept Question: Ampere's Law



Integrating B around the loop in the clockwise direction shown gives us:

1. a positive number
2. a negative number
3. zero

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Applying Ampere's Law

1. Identify regions in which to calculate B field.
2. Choose Amperian closed path such that by symmetry B is 0 or constant magnitude on the closed path!
3. Calculate $\oint_{\text{oriented closed path}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \begin{cases} \text{"}B\text{ times length"} \\ \text{or "zero"} \end{cases}$
4. Calculate current enclosed: $I_{enc} = \mu_0 \iint_{\text{open surface}} \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, da$
5. Apply Ampere's Law to solve for B: check signs

$$\oint_{\text{closed path}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_{\text{open surface}} \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, da$$

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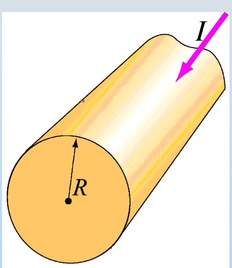
Infinite Wire



A cylindrical conductor has radius R and a uniform current density with total current I. we shall find the direction and magnitude of the magnetic field for the two regions:

(1) outside wire ($r \geq R$)

(2) inside wire ($r < R$)

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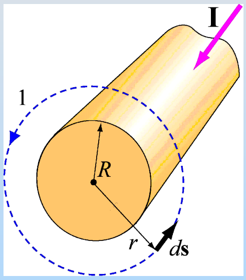
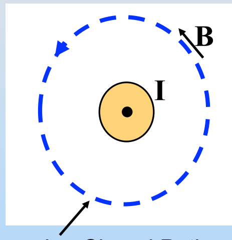
Worked Example: Ampere's Law Infinite Wire





Amperian Closed Path:
B is Constant & Parallel
Current penetrates surface

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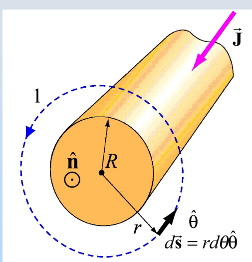
Example: Infinite Wire



Region 1: Outside wire (r ≥ R)

Cylindrical symmetry →
Amperian Circle
B-field counterclockwise

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\oint ds = B(2\pi r)$$

$$= \mu_0 I_{enc} = \mu_0 I$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\theta}}$$

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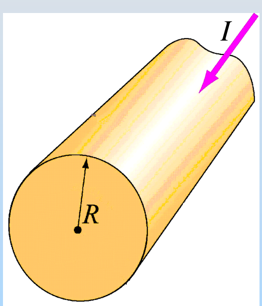
Group Problem: Magnetic Field Inside Wire



We just found B(r>R)

Now you find B(r<R)

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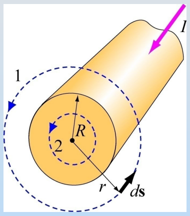
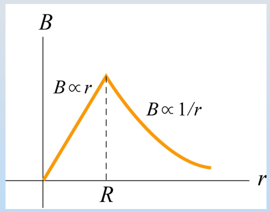
Infinite Wire: Plot of B vs. r





$$B_{in} = \frac{\mu_0 I r}{2\pi R^2} \qquad B_{out} = \frac{\mu_0 I}{2\pi r}$$

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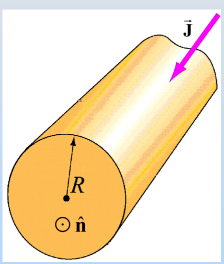
Group Problem: Non-Uniform Cylindrical Wire



A cylindrical conductor has radius R and a non-uniform current density with total current:

$$\vec{\mathbf{J}} = J_0 \frac{R}{r} \hat{\mathbf{n}}$$

Find B everywhere

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Other Geometries

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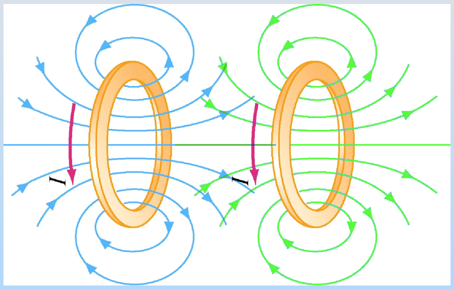
Two Loops



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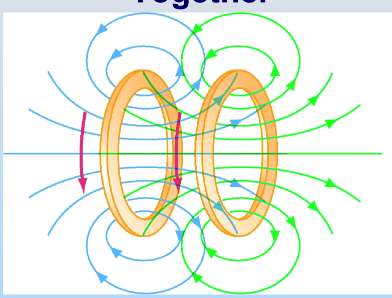
Two Loops Moved Closer Together



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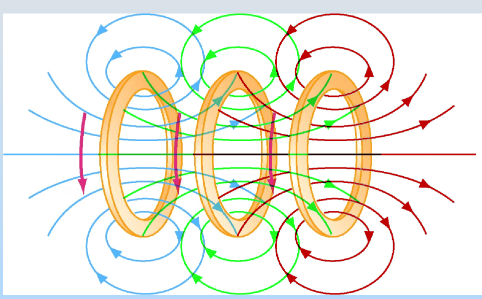
Multiple Wire Loops



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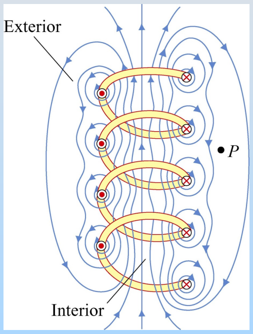
Multiple Wire Loops – Solenoid



http://youtu.be/GI2Prj4CGZI

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Demonstration: Long Solenoid

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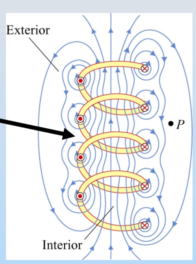
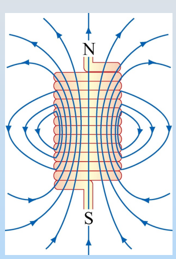
Magnetic Field of Solenoid

Horiz. comp. cancel



loosely wound



tightly wound

For ideal solenoid, B is uniform inside & zero outside

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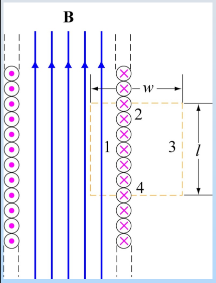
Magnetic Field of Ideal Solenoid



Using Ampere's law: Think!

$$\begin{cases} \vec{\mathbf{B}} \perp d\vec{\mathbf{s}} \text{ along sides 2 and 4} \\ \vec{\mathbf{B}} = 0 \text{ along side 3} \end{cases}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_1 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_2 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_3 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_4 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$$

$$= Bl + 0 + 0 + 0$$

$I_{enc} = nlI$ n: # of turns per unit length

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl = \mu_0 nlI$$

$n = N / L$: # turns/unit length

$$\boxed{B = \frac{\mu_0 nlI}{l} = \mu_0 nI}$$

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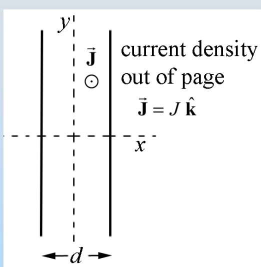
Group Problem: Current Sheet



A sheet of current (infinite in the *y* & *z* directions, of thickness *d* in the *x* direction) carries a uniform current density:

$$\vec{\mathbf{J}} = J\hat{\mathbf{k}}$$

Find the direction and magnitude of *B* as a function of *x*.

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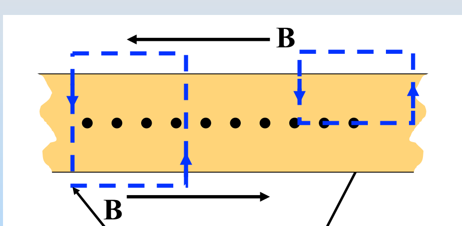
Ampere's Law: Infinite Current Sheet



Amperian Loops:

B is Constant & Parallel OR Perpendicular OR Zero

I Penetrates

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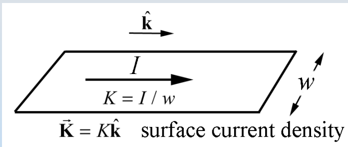
Surface Current Density



A very thin sheet of current of width *w* carrying a current *I* in the positive *z*-direction has a surface current density

$$\vec{\mathbf{K}} = K\hat{\mathbf{k}} \qquad K = I / w$$

For sheet of thickness *d* , width *w*, and current *I*

$$I = Jdw = Kw \Rightarrow J = K / d$$

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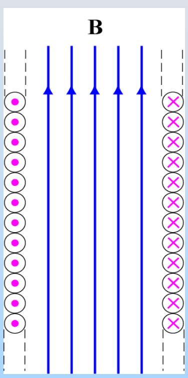
Solenoid is Two Current Sheets



Consider two sheets each of thickness *d* with current density *J*. Then surface current per unit length

$$K = Jd = nI$$

Use either Ampere's Law or superposition principle

$$B = \mu_0 K = \mu_0 Jd = \mu_0 nI$$

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Biot-Savart vs. Ampere

Biot-Savart Law	$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$	general current source ex: finite wire wire loop
Ampere's law	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$	symmetric current source ex: infinite wire infinite current sheet

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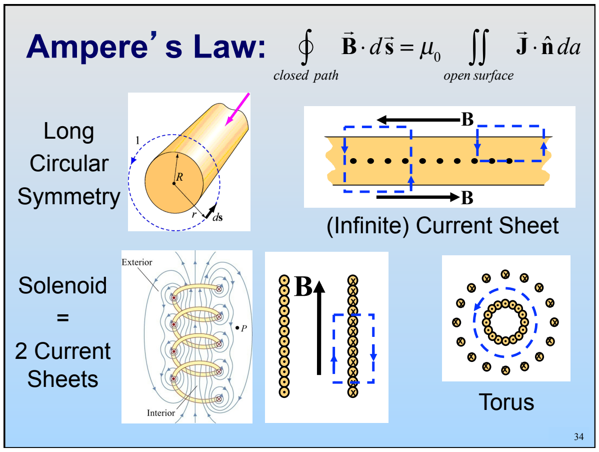
Ampere's Law: $\oint_{closed\ path} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_{open\ surface} \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, da$
Long Circular Symmetry
R
r
ds
B
B
(Infinite) Current Sheet
Solenoid = 2 Current Sheets
Exterior
P
Interior
B
Torus
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