Quantum Physics III (8.06) Spring 2006
Assignment 1

Feb 7, 2006 **Due Feb 14, 2006**

- Please remember to put **your name and section time** at the top of your paper.

Readings

The reading assignment for the first three lectures of 8.06 is:

- Supplementary notes on Natural Units.
- Griffiths, Ch. 5.3
- Cohen-Tannoudji, Ch. XI Complement F.

Useful Facts

- 1 atmosphere = 1.01×10^6 dynes/cm^2 (cgs)
- $M_{\rm sun} = 1.99 \times 10^{33}$ grams
- 1 MeV = 1.602×10^{-6} ergs (cgs)

Problem Set 1

1. **Natural Units (3 points)**

   First, you should verify Equation (12) in the notes on Natural Units, reproduced here:

$$\begin{aligned}
[\text{mass}] &= \text{eV}c^{-2} \\
[\text{time}] &= (\text{eV})^{-1}\hbar \\
[\text{length}] &= (\text{eV})^{-1}\hbar c \\
[\text{momentum}] &= \text{eV}c^{-1} \\
[\text{force}] &= (\text{eV})^2\hbar^{-1}c^{-1} \\
[\text{pressure}] &= (\text{eV})^4\hbar^{-3}c^{-3} \\
[\text{charge}^2] &= \hbar c \\
[\text{magnetic field}] &= (\text{eV})^2\hbar^{-3/2}c^{-3/2} \quad ,
\end{aligned} \tag{1}$$

   Do not turn this in.

   For credit, work out the natural units for conductance, magnetic flux, and magnetic moment.

2. **Magnetic Moments (8 points)**

   (a) A particle's magnetic moment is linearly proportional to its electric charge, e, and to its spin, $\vec{S}$. Write an expression for a particle's magnetic moment in terms of these quantities and its mass m and the fundamental constants $\hbar$ and c. Call the dimensionless constant of proportionality g.

   (b) For an electron, $g \simeq 2$ and its spin is quantized to $\pm\hbar/2$. What is the difference between the energies of electrons with their spins parallel to and antiparallel to a magnetic field of 10^5 gauss? Express your answer in electron volts.

3. **Electron-positron pair production by an electric field (8 points)**

   (a) In natural units [electric field] = $(\text{eV})^2\hbar^{-3/2}c^{-3/2}$. Let us now consider the electric field associated with the rest energy of an electron m_ec^2, i.e. $|\vec{E}| = (m_ec^2)^2(\hbar c)^{-3/2}$. Express this quantity in terms of volts per centimeter.

   (b) The physical meaning of the electric field strength may be understood as follows. In a strong external electric field a quantum fluctuation can lead to electron-positron pair creation. A quantum fluctuation can create an electron positron pair with energy $\delta\epsilon \sim 2m_ec^2$ provided the fluctuation lives less than the time $\delta t \sim \frac{\hbar}{\delta\epsilon}$. In that time, the electron and positron can separate by a distance of order $\delta x \sim c\delta t$. As they separate they gain energy $eE\delta x$, in the electric field with strength E. If they gain enough energy to compensate for their rest mass, they no longer have to annihilate, they can become real. The condition for real e+e- pair creation is therefore that the electric field be greater than a critical value E_c. Determine E_c up to a numerical constant. Compare E_c to the value of electric field in (a).

4. **The Bag Pressure (18 points)**

   The mass, force and energy scales that characterize the physics of quarks are gigantic when expressed in terms of every days units. Here is a problem to show the size of the forces that are at work inside protons and neutrons.

   The proton and neutron ("nucleons" for short) are made of three almost massless quarks. The rest mass of the nucleon is approximately 940 MeV. (Note: $c = 1$.) The quarks are confined to the interior of nucleons because it takes work to "open up" a region of space in which they can be present. (Another way of saying this: the presence of the quarks disturbs the vacuum, and thus costs energy.) The work that must be done in order to make a space in which quarks can live is parametrized by a constant known as the "bag constant", B, which has units of energy per unit volume. That is, the work needed to make a "quark bag" of volume V is BV.

   A simple model of the nucleon (invented here at MIT by Prof. Jaffe and his collaborators in the 1970's) treats it as a spherical bag of radius R, with a rest energy (i.e. rest mass) that includes only two terms: a) the work done to open up a bag, and b) the zero point kinetic energy of the massless quarks confined within the bag. The three massless quarks have kinetic energy $3p$, where p is the mean momentum of a quark within the bag. (Again, note that $c = 1$.) The uncertainty principle tells us that $p \approx 1/R$, (Note: $\hbar = 1$.) So, the rest energy of the nucleon can be written as the sum of the kinetic zero point energy and the bag energy:

$$E(R) = \frac{3}{R} + B\frac{4\pi R^3}{3} \; . \tag{2}$$

In this model, the radius of the nucleon R_0 is the value of R that minimizes $E(R)$.

In the following, you will want to use the conversion factors given in (9) and (13) in the notes on Natural Units.

(a) Restore the factors of $\hbar$ and c to Eq. (2) to make it dimensionally correct. Then set $\hbar = c = 1$ once again for the rest of the problem.

(b) Find R_0. It will be a function of B. Then, find the rest mass of the nucleon as a function of B by substituting R_0 back into Eq. (2).

(c) Given the mass of the nucleon, $M = 940$ MeV, find the numerical value of B. Evaluate B in MeV/fm^3. [Here, 1 MeV= 10^6 eV and 1 fm= 10^{-13} cm.]

(d) Show that energy density and pressure have the same units. (You can use results from Problem 1.)

(e) Evaluate B in atmospheres. This is the pressure that quarks must fight against to open up a bag.

(f) Compute the radius R_0 in fm.

[In reality, the proton does not have a sharp edge, and so R_0 cannot be defined literally as we did in this problem, and the bag model cannot be used to describe all of the properties of the proton. Still, a reasonable characterization of much experimental data is that the radius of the proton is a little less than 1 fm.]

5. **The accelerating universe (5 points)**

(a) One of the biggest recent discoveries in science is that the expansion of the universe is accelerating. This can be described, but not really understood, by reintroducing what Einstein called the cosmological constant Λ into the equations of general relativity[1]. Λ describes a uniform energy density permeated over the whole universe and thus has the dimension of the energy density. What is the most natural scale for Λ in a theory of quantum gravity?

[1] Einstein called this his biggest blunder.

(b) The measured value of Λ — in other words, the Λ that must be introduced in order to parameterize the recently observed accelerating expansion of the universe — is $\Lambda \approx 2000\ \text{eV/cm}^3$. What is the ratio of the observed Λ to the answer you find in (a)? You should find this ratio extremely small. Understanding the smallness of Λ is one of the biggest challenges in theoretical physics.

6. **Fermi energy, velocity and temperature of copper (8 points)**

   Do Griffiths, Problem 5.16.