Quantum Physics III (8.06) Spring 2007
Assignment 6

March 12, 2007 **Due March 20, 2007**

- Please remember to put your name **and section time** at the top of your paper.
- Remember: your **midterm** will be on **Thursday March 22**, in class. (i.e. 12:30pm to 2:00 pm in 4-370.)
- Two **sample midterms** are now available on the course web page.
- There will be no problem set due on Apr 3. Between March 26 and Apr 3, you should focus on the physics of your term paper. Remember that your **paper proposal** is due **in lecture** on **Tuesday Apr 3**. In order to have a good proposal to submit on that date, you want by then to have done much of the work of preparing to write your paper — choosing a topic, finding references, reading, understanding, perhaps doing some calculations, and making a concrete outline of your paper. See the handout describing the term paper project for a description of what your proposal should include.
- Problem Set 7 will be available on the course web page on Apr 3, and will be due on Tuesday April 10.
- When you are studying for your midterm or final, if you want more exercise with perturbation theory Griffiths' problems 6.8, 6.26, 6.30 and 6.39 are good problems. I will not assign them, but I recommend doing them.

Readings

The reading assignment for this problem set is:

- Griffiths all of Chapter 6.
- Cohen-Tannoudji Chapter XI including Complements A-D. Also, Chapter XII. (I could really have put CT Ch. XII on the reading list already last week. It provides more detail (as always with CT!) on a number of classic examples of the application of perturbation theory in atomic physics.)

Problem Set 6

1. **The Feynman-Hellmann Theorem (12 points)**

   Do Griffiths problems 6.32. (That's 6.27 in Griffiths' 1st Ed.)

   [At the end of Problem 6.32, Griffiths asks you to "Compare your answers to Problem 2.12 and the virial theorem predictions." You can skip the comparison to Problem 2.12, but you must do the comparison to what the virial theorem predicts.]

   [Aside: Feynman figured out the Feynman-Hellmann theorem as part of his undergraduate senior thesis, done at MIT in the late 1930's. His senior thesis advisor was Slater, he of the determinant. Note that at MIT we do not *expect* that our undergraduates write senior theses which become part of the standard undergraduate physics curriculum a few decades later. Of course, we don't mind when it happens!

   So, who was Hellmann? You can find a short biography of him at http://www.tc.chemie.uni-siegen.de/hellmann/hellbioe.html. Hans Hellmann was one of the pioneers of quantum chemistry (born in 1903 in Germany). He derived the theorem in 1933. Feynman derived it later (1939), independently of Hellmann. Hellmann's wife was Jewish, and for this reason he was fired as an assistant professor in Hannover in late 1933. He fled to Russia, and in the next several years he wrote pioneering papers on quantum chemistry, and the first text book on the subject. During the "Great Terror" of 1937/38 in the Soviet Union, Hellmann was liquidated at the age of 34 in 1938.]

2. **Energy Shift Due to Finite Nuclear Size (14 points)**

   When you studied the hydrogen atom last semester, you assumed that the Coulomb potential extended all the way to the origin. In reality, the proton's charge is smeared out over a sphere of roughly 10^{-13} cm in radius. This has a small effect on the energy levels of the hydrogen atom. Let's find out how small...

   Model the electric charge distribution of the proton as a uniformly charged sphere of radius R.

   (a) Find the electrostatic potential energy of the electron for all r.
   [Hint: Use Gauss's law $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ to find the electric field everywhere and then integrate $\vec{F} = -e\vec{E}$ to obtain the potential energy.]
   [Answer: $V(r) = -e^2/r$ for $r > R$ and

   $$V(r) = -e^2 \left[\frac{1}{2R^3} \left(R^2 - r^2 \right) + \frac{1}{R} \right]$$

   for $r < R$.]

(b) Use lowest order perturbation theory to calculate the shift in the energy of the ground state of hydrogen due to this modification of the potential. Evaluate your answer numerically, taking $R = 10^{-13}$ cm, and express your answer as a fraction of the binding energy of the ground state (13.6 eV). [Hint: You can simplify the integrals by noticing that the unperturbed wave function varies only slowly over the range $0 < r < R$ and can thus be replaced by the value at $r = 0$.]

(c) Why is this effect most important for states with orbital angular momentum zero? Without doing any calculation, make an estimate of the factor by which this effect is smaller for an $\ell = 1$ state as compared to an $\ell = 0$ state.

3. **Stark Effect for $n = 3$ States of Hydrogen (16 points)**

Consider a hydrogen atom placed in an electric field ε, oriented in the z-direction. The associated perturbation to the Hamiltonian, called the Stark Hamiltonian, is

$$H'_S = -e\varepsilon z = -e\varepsilon r\cos\theta \tag{1}$$

In this problem, you may ignore all fine structure effects, and pretend that the electron has no spin. Ie just think of H'_S as a perturbation to the Bohr Hamiltonian.

We will discuss the first order corrections to the energies of the ground state ($n = 1$) and the first excited states ($n = 2$) in lecture.

In this problem, I ask you to work out the first order corrections to the energies of the $n = 3$ states. In the absence of an electric field, these are 9 degenerate states, so I am asking you to do degenerate perturbation theory with a 9×9 degenerate subspace.

(a) Evaluate the 81 matrix elements of H'_S in the degenerate subspace, but be smart about it.

First of all, use symmetries (and various properties of the spherical harmonics) to argue that many matrix elements are zero. Symmetries also relate some matrix elements to others. If you are sufficiently clever, you can show without calculation that there are only 3 different nonzero matrix elements to calculate.

Once you have boiled the problem down to the evaluation of 3 matrix elements, use dimensional analysis to express each matrix element in terms of a combination of dimensionful constants times a numerical constant. You do not need to evaluate the numerical constant explicitly. Denote them by c_1, c_2, c_3.

(b) Order your basis so that the 9×9 matrix you have constructed in part (a) has a block diagonal form. Find the energies of the 9 states in an electric field, to first order (in terms of c_1, c_2, c_3).

I will not ask you to list the zeroth order good states (ie the eigenstates of the matrix constructed in part (a)). However, show that each of these nine "good" eigenstates is an eigenstate of L_z with eigenvalue m_ℓ, even though many of them are superpositions of states with different values of ℓ and are therefore not eigenstates of L^2.

4. **Van der Waals Interaction between two "toy-model atoms" (18 points)**

Do Griffiths' problem 6.31. (This problem does not occur in Griffiths' 1st Edition.) Note that you should use Gaussian cgs units, as we do in 8.06. That means you should set $4\pi\epsilon_0$ to 1 in Eq. (6.97) and in many subsequent expressions.