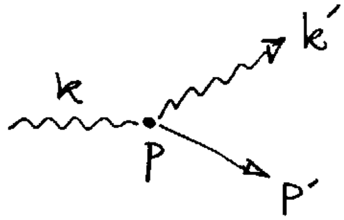
Solutions to Final Exam, 8.20

PROBLEM 2



$k + p = k' + p'$ four vectors

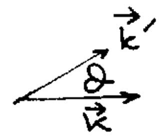
$k - k' + p = p'$

$(p')^2 = m^2c^2 = k^2 + k'^2 + p^2 + 2p \cdot (k - k') - 2k \cdot k' \quad (1)$

$k^2 = 0, \; k'^2 = 0, \; p^2 = m^2c^2$

$2p \cdot (k - k') = 2mc\,(E - E')/c$

$k \cdot k' = \frac{EE'}{c^2}(1 - \cos\theta)$



$(1) \rightarrow \cancel{m^2c^2} = \cancel{m^2c^2} + \cancel{2}m\cancel{c}(E - E')/\cancel{c} - \frac{\cancel{2}EE'}{c^2}(1 - \cos\theta)$

$EE'(1 - \cos\theta) = mc^2(E - E')$

$E = h\nu = hc/\lambda$

$1 - \cos\theta = mc^2\left(\frac{\lambda'}{hc} - \frac{\lambda}{hc}\right) \Rightarrow \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$

PROBLEM 3

a) $\cdot\ p+\bar{p} \to t+\bar{t}$

Minimum energy $\Rightarrow$ t & $\bar{t}$ produced at rest

$\longrightarrow\!\!\bullet\!\!\longrightarrow \quad \longleftarrow\!\!\bullet\!\!\longleftarrow$

$p \qquad \bar{p}$

$$2E = 2m_tc^2 = 300\ (939\ \text{MeV})$$

$$2E = 281.70\ \text{GeV}$$

Each proton starts w/ 939 MeV so

$$2E_{\text{BEAM}} = 281.70 - 2(.939) = 279.82\ \text{GeV}$$

Cost is \$ 2.7982×10^8

b) In rest frame of proton

$\longrightarrow \bullet$

$\bar{p} \qquad p$

$$(p+\bar{p})^2 = 4m_t^2c^2$$

$$2M_p^2c^2 + 2p\cdot\bar{p} = 4m_t^2c^2$$

$$\left.\begin{array}{l} p = (m_pc, \vec{0}) \\ \bar{p} = (\bar{E}/c, 0) \end{array}\right\} \ p\cdot\bar{p} = m_p\bar{E}$$

$$M_p^2c^2 + M_p\bar{E} = 2m_t^2c^2 \quad \to \quad \bar{E} = \frac{2m_t^2 - M_p^2}{M_p}c^2$$

$$E_{\text{BEAM}} = \bar{E} - M_pc^2 = 2\,\frac{m_t^2 - M_p^2}{M_p}c^2 = 2\,[(150)^2 - 1]\,M_pc^2$$

$$= 11249.5\ M_pc^2 = 10\,563.28\ \text{GeV}$$

Cost is \$ 10.563×10^9

Recommendation: build the two rings!

PROBLEM 4

a) $$p_{K\mu} + p_{p\mu} = p_{\pi\mu} + p_{\Lambda\mu}$$

In each case $p_\mu = (E/c, \vec{p})$

b) P and Λ are at rest, isolate p_π:

$$p_K + p_p - p_\Lambda = p_\pi$$

Square $$m_\pi^2 c^2 = (m_p - m_\Lambda)^2 c^2 + m_K^2 c^2 + 2(m_p - m_\Lambda) E_K$$

$$E_K \equiv E^* = \frac{m_K^2 + (m_p - m_\Lambda)^2 - m_\pi^2}{2(m_\Lambda - m_p)} c^2$$

c)

$$E^* = 722.50 \text{ MeV}$$

$$\text{K.E.} = E^* - m_K c^2 = 228.5 \text{ MeV}$$

d) Process no longer works if $E^* < m_K c^2$. Limit occurs at $E^* = m_K c^2$ or

$$2(m_\Lambda - m_p) m_K = m_K^2 + (m_p - m_\Lambda)^2 - m_\pi^2$$

$$m_\pi^2 = (m_K + m_p - m_\Lambda)^2$$

$$m_\pi = m_K + m_p - m_\Lambda$$

$$\text{so } m_\pi c^2 = 317 \text{ MeV}$$

Process occurs provided $m_\pi c^2 < 317$ MeV/c.

PROBLEM 5

a) Neutrinos with $E = 5\,\text{MeV}$ have some speed β. (depends on their mass)

Let t_γ = light travel time
t_ν = neutrino travel time

$$t_\nu - t_\gamma = \frac{d}{\beta c} - \frac{d}{c} \qquad d = \text{distance to LMC}$$

$$\Delta t = \frac{d}{c}\frac{(1-\beta)}{\beta} \approx \frac{d}{c}(1-\beta)$$

$$1-\beta = \frac{c\,\Delta t}{d} = \frac{\Delta t}{(d/c)} \le \frac{1\,\text{hr}}{50000\,\text{yrs}}$$

$$1-\beta < \frac{3600\ \text{seconds}}{5\times10^4 \times \pi \times 10^7\ \text{seconds}} = \frac{3.6\times10^3}{15.7\times10^{11}} = .23\times10^{-8}$$

$$\left.\begin{aligned} \beta &\ge 1 - 2.3\times10^{-9} \\ \beta^2 &\ge 1 - 4.6\times10^{-9} \end{aligned}\right\} \Rightarrow \gamma \ge \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{4.6\times10^{-9}}}$$

$$\gamma \ge \frac{10^5}{6.78} \ge 1.5\times10^4$$

$$m_\nu c^2 \gamma = E_\nu$$

$$m_\nu c^2 < \frac{5\,\text{MeV}}{1.5\times10^4} = 3.33\times10^{-4}\,\text{MeV} = \underline{33.3\ \text{KeV}}$$

b) If neutrinos have a mass m_ν and energy E_ν, then they travel with

$$\gamma = \frac{E_\nu}{m_\nu c^2}$$

The half-life to an earth observer is greater than 50000 yrs.

$$\tau_{1/2}^{earth} \geq 50,000$$

$$\tau_{1/2}^{proper} = \frac{\tau_{1/2}^{earth}}{\gamma} \geq \left(\frac{m_\nu c^2}{E_\nu}\right) 50,000 \text{ yrs.}$$

If we take the lower limit on γ from part a)

$$\tau_{1/2}^{proper} = \frac{50,000}{15,000} = 3\tfrac{1}{3} \text{ yrs.}$$