Massachusetts Institute of Technology
Physics Department

Physics 8.20	IAP 2008
Special Relativity	January 22, 2008
Room 4-370	7:30–9:30 pm

Midterm

Instructions

- **You have two hours for this test. Papers will be picked up at 9:30 pm sharp.**
- **The exam is scored on a basis of 100 points.**
- **Note that this exam has 5 problem, each with multiple parts. This is a long exam, although no individual question requires lengthy computation.**
- **120 minutes / 100 points $\approx$ 1 minute per point. Plan your time accordingly.**
- **Please do ALL your work in the white booklets that have been passed out. There are extra booklets available if you need a second.**
- **Please remember to put your name on the front of your exam.**
- **You may use the single page of notes that you have prepared.**
- **If you have any questions please feel free to ask the proctor(s).**
- **Good luck!**

Information

$c \approx 3 \times 10^8$ m/s

Lorentz transformation (along the x-axis) and its inverse

$$\begin{aligned} x' &= \gamma(x - \beta ct) & x &= \gamma(x' + \beta ct') \\ y' &= y & y &= y' \\ z' &= z & z &= z' \\ ct' &= \gamma(ct - \beta x) & ct &= \gamma(ct' + \beta x') \end{aligned}$$

where $\beta = v/c$, and $\gamma = 1/\sqrt{1-\beta^2}$.

Velocity addition (relative motion along the x-axis):

$$\begin{aligned} u'_x &= \frac{u_x - v}{1 - u_x v/c^2} \\ u'_y &= \frac{u_y}{\gamma(1 - u_x v/c^2)} \\ u'_z &= \frac{u_z}{\gamma(1 - u_x v/c^2)} \end{aligned}$$

Doppler shift

Longitudinal

$$\nu = \sqrt{\frac{1+\beta}{1-\beta}}\ \nu_0$$

Quadratic equation:

$$\begin{aligned} ax^2 + bx + c &= 0 \\ x &= \frac{1}{2a}\left(-b \pm \sqrt{b^2 - 4ac}\right) \end{aligned}$$

Binomial expansion:

$$(1+a)^b = 1 + ba + \frac{b(b-1)}{2}a^2 + \dots$$

Problem 1 [25 points] Short Answer

(a) (4 points) State the postulates on which Einstein's special relativity is based.

(b) (2 points) What is the most important physical constant in this class? What is its numerical value, to three significant digits?

(c) (2 points) Do the Maxwell equations need modification to be correct special relativistically?

(d) (2 points) Does Newton's second law ($F = ma$) need modification to be correct special relativistically?

(e) (6 points) Give three examples of experiments that distinguish between the predictions of special relativity and alternative theories. In each case, state the experiment, the relevant alternative theory, and the outcome.

(f) (2 points) Give one example of an experiment in conflict with special relativity.

(g) (2 points) Define proper length and proper time.

(h) (3 points) What are the allowed values of γ and β? What are their dimensions (units)?

(i) (2 points) Define the invariant interval between two spacetime points.

Problem 2 [16 points] Conference travel

You need to get to a conference on α-Centauri, at a distance of 4 light years. Consider α-Centauri to be roughly at rest with respect to the earth. Your talk is at time t_0, as measured in the frame in which the earth and α-Centauri are at rest.

(a) (8 points) Assuming travel at $\beta = 0.8c$, at what time (relative to time t_0, and measured in the frame in which the earth is at rest) do you need to leave to get there in time for your talk?

(b) (8 points) How much time will you have on the spaceship to prepare your talk? (Equivalently, how much have you aged during the journey?)

Problem 3 [18 points]   Three events

In an inertial frame Σ, there are three spacetime events of interest. Event 1 occurs at $(x, y, z, ct) = (1, 0, 0, 0)$; event 2 occurs at $(x, y, z, ct) = (0, 1, 0, 1)$, and event 3 occurs at $(x, y, z, ct) = (0, 0, 1, 2)$. (Units of meters are implied, so that event 2 occurs at $x = 0$ m, $y = 1$ m, $z = 0$ m, and $ct = 1$ m.)

(a) (4 points) Is there an inertial frame Σ'_{12} in which events 1 and 2 occur at the same time? Justify your answer.

(b) (4 points) Is there an inertial frame Σ'_{23} in which events 2 and 3 occur at the same time? Justify your answer.

(c) (4 points) Is there an inertial frame Σ'_{13} in which events 1 and 3 occur at the same time? Justify your answer.

(d) (6 points) Pick one of the above to which you answered "yes," and find a velocity (magnitude and direction) for Σ' relative to the frame Σ, such that the two events occur at the same time in Σ'. [Hint: Do not get hung up on this. If you see it quickly, great. If not, write down something intelligent and move on.]

Problem 4 [16 points] Relativistic radar

An object A is moving at speed $v = \beta c$ toward an object Y. The object A emits a signal at a frequency ν_A (as measured in object A's rest frame).

(a) (3 points) What is the frequency ν_Y measured by a detector placed on object Y? You can simply state the result, but explain each symbol you use. [Note that you have a formula cheat sheet on page 2 of this exam.]

(b) (2 points) Explain with a couple of sentences where the various factors in your expression in part (a) come from.

(c) (4 points) Check: Is your expression for ν_Y equivalent to the non-relativistic result ($\nu_Y = \frac{c}{c-v}\nu_A$) for $v \ll c$? Hint: Neglect terms of order $(v/c)^2$ in both the relativistic and non-relativistic expressions.

(d) (5 points) Now suppose the object Y emits a signal at the same frequency ν_Y that it measures, and that this signal is recorded by object A, which continues moving toward object Y with speed $v = \beta c$. Show (clearly, and with sufficient explanation) that the frequency ν'_A measured by object A is given by $\nu'_A = \frac{1+\beta}{1-\beta}\nu_A$.

(e) (2 points) What is ν'_A in the limit $v \to c$? Justify this result qualitatively.

This is what happens when a signal (like radar) bounces off an object. The "bouncing off" is equivalent to the object receiving the signal and re-emitting at the same frequency (as measured in the rest frame of the object).

Problem 5 [25 points] Caught speeding

You are speeding down a space highway in a very fast space car. Speed limits are defined with respect to an inertial frame defined by the Intergalactic Transportation Authority (ITA), which roughly corresponds to the inertial frame in which the center of mass of all nearby matter is at rest. In this part of the galaxy, police officers use radar guns that emit at a frequency $\nu = 24$ GHz ($= 24 \times 10^9$/s). Make use of the expression you are asked to prove above in Problem 4(d).

(a) (4 points) Unknown to you, there is a policewoman (Ann) sitting in her space car, which is at rest with respect to the ITA inertial frame. Ann clocks you as you approach her, and measures the frequency of the signal that bounces off of your car and returns to her to be $\nu'_A = 72$ GHz. How fast are you going (with respect to the ITA frame)?

(b) (4 points) Unknown to you, there is another policewoman (Beth) approaching you on the space highway at the speed limit $v_B = c/7$ (measured with respect to the ITA inertial frame). Beth is traveling towards you. What is her speed, as measured in the inertial frame in which you are at rest?

(c) (1 points) What is your speed, as measured in the inertial frame in which Beth is at rest?

(d) (4 points) Beth clocks you with her gun. At what frequency (ν'_B) does Beth detect the signal that bounces off of your car and returns to her?

(e) (4 points) Unknown to you, there is another policewoman (Deb) who has observed you speeding, and is racing after (behind) you at speed $v_D = 2c/3$. As measured in the inertial frame in which you are at rest, what is Deb's speed?

(f) (4 points) Deb clocks you again with her radar gun to check your speed. At what frequency (ν'_D) does she detect the signal that bounces off of your car and returns to her?

(g) (4 points) As measured in the ITA frame, Deb is currently (at time t=0, measured in the ITA frame) 1 light-minute behind you. How long (as measured in the inertial frame in which you are at rest) do you have to think up a good story before Deb catches up to you?