Physics 8.20 Special Relativity IAP 2008

Problem Set #2 Due Friday January 18

Announcements

Topics for this period

- Principles of relativity and the Lorentz transformation
- Consequences: length contraction and time dilation
- Addition of velocities
- Doppler shift

Reading

- Resnick, Chs. 1 – 2 and Supplement A
- French Chs. 1– 5
- Einstein §1 –17 and Appendix 1

Problems

Note: All problems marked **(RH)** *are taken from Resnick and Halliday,* Basic Concepts in Relativity *(MacMillan, New York, 1992).*

1. **A Moving Clock** (7 points)

   A clock moves along the x-axis in your reference frame at a speed of $0.80c$ and reads zero as it passes the origin. What time does it read as it passes the 180 meter mark on this axis? How much time has passed in your reference frame?

2. **How fast a ship?** (7 points)

   A spaceship is moving at such a speed in the laboratory frame that its measured length is one third its proper length. How fast is the spaceship moving relative to the laboratory frame?

3. **String of lights across the desert I** (7 points)

   A series of lights is arrayed in a straight line across the desert. Neighboring lights are separated by a distance d. They are set up to flash in sequence with an interval τ between neighboring lights (as measured in the rest frame of the lights). An observer, O, travels along the same line at a uniform speed v in the same direction of the wave of flashes.

   (a) At what interval do the flashes occur in the rest frame of O?

(b) Suppose O travels in the direction opposite the wave. What is the interval in this case?

(c) For what choices of d, τ, and v do all flashes occur simultaneously in the rest frame of O?

4. **String of lights across the desert II** (7 points)

   In the previous problem you were asked for the times at which the flashed occured in the rest frame of O. Now consider what would be *seen* by the observer O:

   Again let the straight line of lights be separated by d and flash in sequence with interval τ. Now compute the interval between the sequential flashes *as seen by* an observer travelling with uniform speed v along the direction of the flashes. [Hint: in this case you must consider not only the Lorentz transform of each flash event, but you must also consider how long it takes a flash to propagate to the observer O.] For what values of d, τ, and v will the flashes *appear to be* simulataneous to O?

5. **Events in two different frames I** (7 points)

   Two events, A and B, are observed in two different inertial frames, Σ, and Σ'. Frame Σ' moves along the x axis in Σ. Event A occurs at the spacetime origin in both frames,

$$x_A = y_A = z_A = ct_A = 0$$
$$x'_A = y'_A = z'_A = ct'_A = 0$$

   Event B occurs at

$$x_B = 10, \quad y_B = z_B = 0, \quad ct_B = 6$$

   (all distances are in meters).

   The two events occur *simultaneously* in frame Σ'.

   (a) Find the velocity of Σ' with respect to Σ.

   (b) What is the space separation of the two events in the frame Σ'?

   (c) What is the smallest spatial separation between the two events in any inertial frame?

6. **Events in two different frames II** (7 points)

   Two events, A and B, are observed in two different inertial frame, Σ, and Σ'. Event A occurs at the spacetime origin in both frames,

$$x_A = y_A = z_A = ct_A = 0$$
$$x'_A = y'_A = z'_A = ct'_A = 0$$

   Event B occurs at

$$x_B = 2, \quad y_B = z_B = 0, \quad ct_B = 10$$

   (all distances are in meters) as observed in Σ.

   The two events occur *at the same point* in frame Σ'.

(a) Find the velocity of Σ' with respect to Σ.

(b) What is the time separation of the two events in the frame Σ'?

(c) What is the shortest time separation between the two events in any inertial frame?

7. Events in two different frames III (7 points)

Two events, A and B, are observed in two different inertial frame, Σ, and Σ'. Event A occurs at the spacetime origin in both frames,

$$x_A = y_A = z_A = ct_A = 0$$
$$x'_A = y'_A = z'_A = ct'_A = 0$$

Event B occurs at

$$x_B = 2, \quad y_B = z_B = 0, \quad ct_B = 3$$

in Σ, and at

$$x'_B = 3, \quad y'_B = z'_B = 0$$

(all distances are in meters) in Σ'.

(a) What time does event B occur in Σ'?

(b) What is the relative velocity of Σ' relative to Σ?

8. An alternative derivation of the velocity transformation law (9 points)

Spaceship A passes earth at speed v when its clock and the adjacent earth clock read zero (event E_1). When the earth clock reads T, spaceship B passes earth, moving at speed $u > v$ in the same direction as spaceship A (event E_2). Eventually ship B catches up with ship A (event E_3). Let S be the earth frame and S' be the frame of ship A.

(a) Is the interval between E_1 and E_2 a proper time interval in either frame S or frame S'? If so, which one? What about the interval between E_2 and E_3? What about the interval between E_1 and E_3?

(b) Find the time of E_2 according to the S' clocks.

(c) According to S' observers, how far away was the earth at E_2?

(d) Find the time of E_3 according to S' clocks.

(e) Find the time of E_3 according to S clocks.

(f) From these results find the velocity of ship B as measured by observers on ship A (ie, S' observers). This is the relativistic velocity transformation law derived in lecture from the Lorentz transformation.

9. The expanding universe (RH) (7 points)

(a) Galaxy A is reported to be receding from us with a speed of $0.3c$. Galaxy B, located in precisely the opposite direction, is also found to be receding from us at this same speed. What recessional speed would an observer on Galaxy A find i) for our galaxy? ii) for Galaxy B?

(b) It is concluded from measurements of the red shift of the emitted light that quasar Q_1 is moving away from us at a speed of $0.7c$. Quasar Q_2, which lies in the same direction in space, but is closer to us, is moving away from us at speed $0.55c$. What velocity for Q_2 would be measured by an observer on Q_1?

10. Perpendicular velocities (RH) (7 points)

Trains A and B travel on perpendicular tracks. Train A travels north from station S and train B travels east. Both trains leave the station at the same time and travel at a speed $v = 0.75c$ in the rest frame of the station.

(a) Find $\vec{V}_{AB}$, the velocity of train B with respect to train A.

(b) Find $\vec{V}_{BA}$, the velocity of train A with respect to train B.

(c) Comment on the fact that these two relative velocities do not point in opposite directions.

11. Transforming angles I (7 points)

A particle moves with speed u in the $x - y$ plane, making an angle θ with respect to the x-axis in frame Σ. The origin of Σ moves to the right (along the positive x' axis) in the frame Σ' with speed v. What speed u' and angle θ' will the particle appear to have to an observer in Σ'?

12. Transforming angles II (7 points)

A right triangular plate is at rest in the frame Σ. Its legs are placed on the x and y axes and its hypotenuse makes an angle θ with respect to the x-axis. The origin of Σ moves to the right (along the positive x' axis) in the frame Σ' with speed v. What are the angles of the triangle as measured in Σ'?

13. Doppler shift (7 points)

Resnick Chapter 2, Problem 48, P 108.

14. Won't this excuse get you in worse trouble? (7 points)

The wavelength of red light is 650 nm and the wavelength of yellow light is 570 nm. You run a red light, and a policeman pulls you over. You tell him that because of the Doppler shift, you thought the light was still yellow. If the policeman believes you and has taken 8.20, how fast does he conclude you were driving?