The problem is a generalization of Problem 2 of Problem Set 5.

PROBLEM 2: VOLUME OF A CLOSED THREE-DIMENSIONAL SPACE
Problem 3: Tracing Light Rays in a Closed Matter-

Consider a radially light-ray moving through a matter-dominated universe. The following problem was Problem 7 of the Quiz 2 Review Problem.

Dominated Universe (30 Points)

The Schwarzschild metric for a homogeneous, isotropic, closed universe is given by

\[ ds^2 = -\frac{2M}{r} dt^2 + \left(\frac{r}{2M} + 1\right) dr^2 + r^2 d\Omega^2 \]

where \( M \) is the mass enclosed within the light-like ray. The parameter \( a \) is the parameter used to describe the coordinate.

The Schwarzschild metric for a homogeneous, isotropic, closed universe is given by

\[ ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \]

Let the Schwarzschild metric for a homogeneous, isotropic, closed universe be given by

\[ ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \]

The following problem was Problem 7 of the Quiz 2 Review Problem.

Problem 3: Tracing Light Rays in a Closed Matter-

Consider a radially light-ray moving through a matter-dominated universe.
Problem 4: Continuation on next page

- Problem 4

- Problem 4

- Problem 4

- Problem 4

- Problem 4
\[ \varepsilon(\gamma v) = \frac{v^3}{c^3} - \frac{v}{c} = \varepsilon(d) - \frac{v}{c} = d \]

\[ \varepsilon(d) + \varepsilon(e) = \varepsilon(\gamma v) = \varepsilon, \quad \varepsilon(d) = \frac{d^2}{c} = d \]

Energy-Momentum Four-Vector:

Time-Dilation Factor:

Lorentz-Fitzgerald Contraction Factor:

\[ \gamma = \sqrt{1 - \frac{v^2}{c^2}} \]

Special Relativity:

\[ \gamma = \frac{e^{\frac{m}{c}} - 1}{e^{\frac{m}{c}} + 1} \]

Cosmological Redshift:

\[ \frac{n/a}{n/a} - 1 = z \]

\[ (n/a - 1)^{\gamma_{\text{observer}}/\gamma_{\text{source}}} = z \]

Doppler Shift (for motion along a line):

**Quiz 2 Formula Sheet**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>6</td>
<td>1</td>
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</table>

TOTAL: 83
\[
\frac{d^2 \theta}{d \tau^2} + \frac{\dot{\phi} \dot{\theta}}{v^2} \cos \theta + \frac{\dot{\phi} \dot{\theta}}{v^2} = 0
\]

**Schwarzschild Metric:**

The Schwarzschild metric is a solution to the Einstein field equations that describes the spacetime around a non-rotating, spherically symmetric mass. It is often used in general relativity to describe the spacetime around stars and black holes.

**Cosmological Evolution:**

The cosmological evolution of the universe is described by the Friedmann equations, which relate the expansion of the universe to its density and geometry.

**Robertson-Walker Metric:**

The Robertson-Walker metric is a generalization of the Schwarzschild metric to a homogeneous and isotropic universe.

**Cosmological Parameters:**

The cosmological parameters describe the properties of the universe, such as its density, age, and expansion rate. They are determined through observations and theoretical calculations.

**Universe Evolution:**

The evolution of the universe is characterized by its expansion, which is driven by the cosmological constant and dark energy. The current understanding of the universe is that it is expanding, with the expansion rate slowing down due to gravity.

**Kinematics of a Homogeneous Expanding Universe:**

The kinematics of a homogeneous expanding universe describe how objects move within the expanding spacetime. The expansion of the universe is an example of a cosmological phenomenon that affects the motion of all objects within it.
\[ \frac{\xi \eta}{\mu} = \exp \left( \frac{\xi \eta}{\mu} \right) \]