Theoretical Problem 2.18: The vector $\mathbf{A}$ is a function of the position $x$, $y$, and $z$. The equation for the electric field is given by $\mathbf{E} = \frac{\partial \mathbf{A}}{\partial t}$. Find the expression for the magnetic field $\mathbf{B}$ using Maxwell's equations.

Special Relativity:

$$\frac{\partial}{\partial t} \left( \frac{\rho}{\sqrt{-g}} \right) = \frac{\partial}{\partial \xi} \left( \frac{\rho}{\sqrt{-g}} \right)$$

Cosmological Redshift:

$$\frac{\rho}{\sqrt{-g}} = \frac{\rho}{\sqrt{-g} + 1}$$

Doppler Shift (for a moving object):

$$n/\omega = \frac{\omega - \nu}{\omega + \nu}$$

Information to be given on Quiz:

- Review Quiz: This quiz will be given on Wednesday, December 2, 2018.
- Review Session: To help for the quiz, the instructor will hold a review session at a time and place to be announced.

Covered Textbook:

- Review Problems for Quiz 2

Revised Version

Physics 8:286. The Early Universe

Massachusetts Institute of Technology

Revised November 2, 2018
\[ \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} + \phi \phi = 0 \]

where

\[ \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} + \phi \phi = 0 \]

then

\[ \left( \phi \phi \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} \right) \phi^2 + \frac{\partial \phi}{\partial \xi} \phi^2 = \phi \phi \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} \phi^2 = \phi \phi \]

Wick Rotation of a Matter-Dominated Universe:

\[ \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} + \phi \phi = 0 \]

where

\[ \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} + \phi \phi = 0 \]

then

\[ \left( \phi \phi \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} \right) \phi^2 + \frac{\partial \phi}{\partial \xi} \phi^2 = \phi \phi \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial \phi}{\partial \xi} \phi^2 = \phi \phi \]

COSMOLOGICAL EVOLUTION:

\[ \text{Hubble Parameter} \quad \frac{\partial}{\partial \xi} \left( \frac{\partial \phi}{\partial \xi} \right) = 0 \]

\[ \text{Hubble Parameter} \quad \frac{\partial}{\partial \xi} \left( \frac{\partial \phi}{\partial \xi} \right) = 0 \]

Phenomena of a Homogeneous, Expanding Universe:

\[ \text{Hubble Parameter} \quad \frac{\partial}{\partial \xi} \left( \frac{\partial \phi}{\partial \xi} \right) = 0 \]

\[ \text{Hubble Parameter} \quad \frac{\partial}{\partial \xi} \left( \frac{\partial \phi}{\partial \xi} \right) = 0 \]
In the matrix $P$, an expression for $\phi = \theta$ so constant. Define a higher order differential form $\phi = \phi(x, y, z)$ such that 

$$ \{ (p \phi + q) \phi + r \phi \} (x') + (p \phi + q) \phi - r \phi = \phi p + q^2 - r^2 \phi = \phi_p $$

so the net effect is to

$$ \phi_p = \frac{\phi^2 - 1}{\phi} $$

Then

$$ \phi \text{ is a } \alpha $$

To work with an alternating tensor coordinate, we need to be

$$ \{ \phi \phi + \phi \phi \} (x') + \phi \phi + \phi \phi = \phi p + q^2 - r^2 \phi = \phi_p $$

Robertson-Walker formula: 

The Robertson-Walker formula is valid in 1999.

**Problem 8. Placing Light Rays in a Closed Matter**

The following problem was published in 1999.

**Problem 7. Anticipating a Big Crunch**

The following problem can be taken from Quilis, J. 1990, where no connect 10 points out of

**Problem 6. Evolution of an Open Universe**

The following problem was published in 1999.
Suppose a two-dimensional surface, described in polar coordinates (r,θ), lies in a metric

\[ d s^2 = d r^2 + r^2 d \theta^2 \]

Consider a universe described by the Robertson-Walker metric on the first page of

There are no points at all the origin, and the other end of the

Here are the points at the origin, and the other end of the

where \( a \) and \( b \) are positive constants. Consider the path in this space which is formed by

\[ r \theta \xi \eta + 1 \theta^a + \xi \theta^b = \theta^c \]

\[ \text{given, } 1. \]

Then by

\[ d s^2 = d r^2 + r^2 d \theta^2 \]

\[ x, y, z \]

\[ (0, \theta \xi \eta, 0) \]

\[ (0, 0, 0) \]

A small and large ones at the origin, and the other end of the

The following problem was from Problem 3, Q 11, 1998:

Problem 1: Geometry in a Closed Universe

Problem 2: Geometry in a Closed Universe


\[ \omega \equiv \frac{3}{2} \frac{d \theta}{r \theta \xi \eta} \]

\[ \frac{d}{d \theta} = \frac{\pi}{2} \]

Where

\[ \frac{d \theta}{r \theta \xi \eta} = \omega \]

Parameter functions

determined by (r, φ) and determined by (r, θ) are deri...
Express the metric in terms of this new variable.

\[ \gamma' = \gamma \]

Suppose that \( \gamma \) and \( \gamma' \) are both coordinate transformations. Then \( \gamma' \) is related to \( \gamma \) by

\[ \frac{\partial x}{\partial \gamma} = \frac{\partial x'}{\partial \gamma'} \]

Find the physical units of the above.

\( \left( \frac{\partial \theta}{\partial \gamma} \right) \frac{\partial \theta'}{\partial \gamma'} = \delta \)

Find an explicit expression for the volume of the sphere. Be sure to include the constant of integration.

The expression for the Schwarzschild metric is

\[ g_{\mu \nu} = -\frac{\rho^2}{\Sigma (x^2 + y^2 + \rho^2 - 1)^2} \]

The following problem was Problem 1, Quill 2, 1998.

**Problem 11: The General Relativity: Semantic Metric**

In the context of the Schwarzschild metric, \( \gamma' = \gamma \) is related to \( \gamma \) by

\[ \gamma' = \gamma \]

Find the physical units of the above.

\( \left( \frac{\partial \theta}{\partial \gamma} \right) \frac{\partial \theta'}{\partial \gamma'} = \delta \)

Find an explicit expression for the volume of the sphere. Be sure to include the constant of integration.

\[ \frac{\partial x}{\partial \gamma} = \frac{\partial x'}{\partial \gamma'} \]

Express the metric in terms of this new variable.

\[ \gamma' = \gamma \]
If we write the determinant in the following way:

\[
\begin{vmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}
\end{vmatrix} = \frac{\partial p}{\partial u} \frac{\partial q}{\partial v} - \frac{\partial p}{\partial v} \frac{\partial q}{\partial u}
\]

Then the expression in (a) is the differential equation for the geodesics.

(a) Show the expression in (a) satisfies the differential equation for the geodesics.

(b) Find the general solution of the geodesic equation on the surface of the sphere, and hence the geodesics on the sphere in the equation, which can be parametrized by

\[
\gamma(t) = (\cos(t), \sin(t))
\]

The geodesic is rotated by an angle \(\theta\) about the x-axis, then the equations become:

\[
\begin{align*}
\dot{x} &= \gamma'(t) \\
\dot{\theta} &= \gamma''(t)
\end{align*}
\]

where \(\gamma(t) = (\cos(t), \sin(t))\), then \(\dot{\theta} = \frac{\sin(t)}{\cos(t)}\) and \(\dot{x} = -\frac{\sin(t)}{\cos(t)}\).

Consider the geodesics described by

\[
\theta(t) = \theta_0 - \frac{\sin(t)}{\cos(t)}
\]

where \(\theta_0\) is a constant.

The following problem can be solved in the present form:

(a) Suppose that \(\gamma(t) = (\cos(t), \sin(t))\) describes a geodesic in the space, where the parameter is

\[
\gamma(t) = (\cos(t), \sin(t))
\]

the line of curvature of a geodesic on the surface of a sphere.

(b) Now introduce the usual Cartesian coordinates, defined by

\[
(\rho, \phi, \theta) = (x, y, z)
\]

of the geodesic in the given coordinates. Choose coordinates \((x, y, z)\) and \((\rho, \phi, \theta)\) such that

\[
\frac{\partial x}{\partial \rho} = \frac{\partial y}{\partial \phi} = \frac{\partial z}{\partial \theta} = 0
\]

(i.e., the geodesic is the line of curvature of a geodesic on the surface of a sphere.

(c) Calculate the geodesics of the two-dimensional space with coordinates \((\rho, \phi, \theta)\). The equation of the geodesic is

\[
\rho = \rho_0 + \int \frac{d\tau}{\cos(\theta_0 - \frac{\sin(t)}{\cos(t)})}
\]

where \(\rho_0\) is a constant.

(d) The geodesics of the two-dimensional space can be described in polar coordinates by

\[
\rho = \rho_0 + \int \frac{d\theta}{\cos(\theta_0 - \frac{\sin(t)}{\cos(t)})}
\]

where \(\rho_0\) is a constant.

*Problem 19: An Exercise in Two-Dimensional Metrics (20)

(i) Consider the geodesics on the surface of a sphere.

(ii) Show that the line of curvature of a geodesic on the surface of a sphere.

(iii) Calculate the geodesics of the two-dimensional space with coordinates \((\rho, \phi, \theta)\).

(iv) Express the geodesics in the space in the form of a geodesic on the surface of a sphere.

(v) Calculate the geodesics of the two-dimensional space can be described in polar coordinates by

\[
\rho = \rho_0 + \int \frac{d\theta}{\cos(\theta_0 - \frac{\sin(t)}{\cos(t)})}
\]

where \(\rho_0\) is a constant.

(vi) The geodesics of the two-dimensional space can be described in polar coordinates by

\[
\rho = \rho_0 + \int \frac{d\theta}{\cos(\theta_0 - \frac{\sin(t)}{\cos(t)})}
\]

where \(\rho_0\) is a constant.
Consider a two-dimensional curved space. The following problem was Problem 3 of the 2002 Problem 11. Geodesics in a Closed Universe.
Consider the expression for the determinant of a matrix, which can be written in terms of the elements of the matrix:

\[ \det(A) = a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} + a_{12}a_{23}a_{31} - a_{12}a_{21}a_{33} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} \]

where
- \( a_{ij} \) is the element in the \( i \)th row and \( j \)th column of the matrix \( A \).

The expression for the determinant can be simplified by using the standard definition of a determinant.

To find the determinant, you need to compute the sum of the products of the elements in each row with their corresponding cofactors, then add and subtract the results as indicated by the sign of the row number.

The determinant is zero if any two rows are identical, or if any row is a linear combination of the others.

Problem 1: Rotating Phases of Reference

Given a function \( \psi = \psi_1 + \psi_2 + \psi_3 \), where \( \psi_1 \), \( \psi_2 \), and \( \psi_3 \) are functions of position \( x \), the time \( t \), and the potential \( V \).

To evaluate the expression for the determinant, you need to compute the sum of the products of the elements in each row with their corresponding cofactors, then add and subtract the results as indicated by the sign of the row number.

The determinant is zero if any two rows are identical, or if any row is a linear combination of the others.

Problem 2: Determinants and Matrices

Consider a matrix \( A \) with elements \( a_{ij} \), where \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, n \).

The determinant of \( A \) can be computed using the standard definition:

\[ \det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij} \]

where
- \( M_{ij} \) is the minor of the element \( a_{ij} \) in the matrix.

The determinant is zero if any two rows are identical, or if any row is a linear combination of the others.

Problem 3: Determinants and Linearity

Consider a matrix \( A \) with elements \( a_{ij} \), where \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, n \).

The determinant of \( A \) can be computed using the standard definition:

\[ \det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij} \]

where
- \( M_{ij} \) is the minor of the element \( a_{ij} \) in the matrix.

The determinant is zero if any two rows are identical, or if any row is a linear combination of the others.

Problem 4: Determinants and Linearity

Consider a matrix \( A \) with elements \( a_{ij} \), where \( i = 1, 2, \ldots, n \) and \( j = 1, 2, \ldots, n \).

The determinant of \( A \) can be computed using the standard definition:

\[ \det(A) = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij} \]

where
- \( M_{ij} \) is the minor of the element \( a_{ij} \) in the matrix.

The determinant is zero if any two rows are identical, or if any row is a linear combination of the others.
OS02

*Problem 3.3: Pressure and Energy Density of Networks*

The problem you will try to solve is to determine the energy density of a network.

In the context of a network, the energy density is defined as the amount of energy stored in the network over a given period. The energy density is given by the formula:

\[
\epsilon = \frac{1}{2} \sum \left( \frac{\partial P}{\partial t} \right)^2
\]

where \( P \) is the power dissipated in the network and \( t \) is time.

To solve this problem, you need to determine the power dissipation in each component of the network.

**Problem 3.4: The Stability of Schwarschild Orbits**

The Schwarzschild metric is given by:

\[
\frac{\Delta t^2}{\Delta s^2} = \frac{\Delta r^2}{r^2} - \frac{\Delta \theta^2}{r^2} - \frac{\Delta \phi^2}{r^2}
\]

and the determinant with respect to \( \Delta r, \Delta \theta, \Delta \phi \) is:

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]

where \( f = 0 \), \( t \) is the Schwarzschild time, and \( \Delta s^2 \) is the line element in the Schwarzschild coordinates.

You may now answer the question: in which configuration, with respect to a Schwarzschild metric, the determinant with respect to \( \Delta r, \Delta \theta, \Delta \phi \) is:

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]

and the determinant with respect to \( r, \theta, \phi \) is:

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]

for the case \( \Delta r = \Delta \theta = \Delta \phi \). You should use the result to find an explicit expression for

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]

work out the explicit form of the Schwarzschild metric

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]

where

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]

and the determinant with respect to \( \Delta r, \Delta \theta, \Delta \phi \) is:

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]

and the determinant with respect to \( \Delta r, \Delta \theta, \Delta \phi \) is:

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]

\[
\frac{\partial^2 f}{\partial r \partial \theta} \frac{\partial^2 f}{\partial \phi \partial t} - \left( \frac{\partial^2 f}{\partial r \partial \phi} \right)^2 = \frac{\partial^2 f}{\partial \theta \partial t} \frac{\partial^2 f}{\partial \phi \partial \theta}
\]
Recall that the spatial part of the metric for a closed universe can be written as

\[ ds^2 = -dt^2 + a^2(t) dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]

This problem is a generalization of Problem 2 of Problem Set 6.

**Problem 22: Volume of a Closed Three-Dimensional Space**

(a) We have a three-dimensional space for which the volume \( V \) depends on the metric. The coordinates \( \phi, \theta, \varphi \) have the usual range,

\[ 0 \leq \phi \leq 2\pi, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \varphi \leq \pi \]

We can integrate over a single variable only. Hints: as in Problem 2 of Problem Set 5, you can break the integral into smaller pieces of volume, each of which should be

\[ V = \int_0^{2\pi} \int_0^\pi \int_0^\pi \rho \, r^2 \sin \theta \, dr \, d\theta \, d\phi \]

(b) \( \phi \) is in some unspecified function. The coordinates \( \theta, \varphi \) have the usual range.

\[ V = \int_0^{2\pi} \int_0^\pi \int_0^\pi \rho \, r^2 \sin \theta \, dr \, d\theta \, d\phi \]

(c) \( \rho \) is in some unspecified function. The coordinates \( \theta, \varphi \) have the usual range.

\[ V = \int_0^{2\pi} \int_0^\pi \int_0^\pi \rho \, r^2 \sin \theta \, dr \, d\theta \, d\phi \]

(d) \( \rho \) is in some unspecified function. The coordinates \( \theta, \varphi \) have the usual range.

\[ V = \int_0^{2\pi} \int_0^\pi \int_0^\pi \rho \, r^2 \sin \theta \, dr \, d\theta \, d\phi \]

(e) \( \rho \) is in some unspecified function. The coordinates \( \theta, \varphi \) have the usual range.

\[ V = \int_0^{2\pi} \int_0^\pi \int_0^\pi \rho \, r^2 \sin \theta \, dr \, d\theta \, d\phi \]

(f) \( \rho \) is in some unspecified function. The coordinates \( \theta, \varphi \) have the usual range.

\[ V = \int_0^{2\pi} \int_0^\pi \int_0^\pi \rho \, r^2 \sin \theta \, dr \, d\theta \, d\phi \]
Let us first express the electromagnetic field in first order in \( g \), as an expansion

\[
\mathcal{E}_a \sim \frac{d}{d \mathbf{p}} \mathcal{E}_a = \frac{d}{d \mathbf{p}} \left( \mathcal{E}_a + \mathcal{E}_a ' \right) \frac{d}{d \mathbf{p}}
\]

where the field is given by \( \mathcal{E}_a \). The term \( \mathcal{E}_a ' \) represents the linear correction due to the gravitational interaction of the spins of the particles.

In the field expansion, the coefficients are given by

\[
\mathcal{E}_a = \sum_{\mathbf{p}} \mathcal{E}_a (\mathbf{p}) \exp (i \mathbf{p} \cdot \mathbf{x})
\]

where \( \mathbf{p} \) is the momentum of the particle.

The correction term \( \mathcal{E}_a ' \) is given by

\[
\mathcal{E}_a ' = \sum_{\mathbf{p}} \mathcal{E}_a '(\mathbf{p}) \exp (i \mathbf{p} \cdot \mathbf{x})
\]

where \( \mathcal{E}_a ' \) is calculated in a higher order, and \( \mathbf{p} \) is the momentum of the particle.

The final expression for the field includes the linear correction due to the gravitational interaction of the spins of the particles.
Problem I: Did you do the reading?

Solutions
Such is the number density of protons. The number density of protons can be converted from one into another by

\( \text{number of protons} = \text{number of protons} \times \text{conversion factor} \)

The number density of protons is illustrated by a graph of the number of protons as a function of time.

**Problem 2: Did you do the reading? (12 points)**

The number density of protons is shown by the graph below.

- **Table:**
  - Time: 0, 10, 20, 30, 40, 50, 60, 70 minutes
  - Number of Protons: 10, 100, 1000

- **Graph:**
  - X-axis: Time (minutes)
  - Y-axis: Number of Protons

- **Equation:**
  \( \text{Number of Protons} = \text{Initial Number of Protons} \times (1 + \text{Rate of Change})^{\text{Time}} \)

The rate of change is given by the slope of the graph. The number of protons at any time can be calculated using this equation.

To determine the number of protons at any given time, you can use the equation above. For example, if the initial number of protons is 1000 and the rate of change is 0.1 per minute, the number of protons at 30 minutes would be:

\[ \text{Number of Protons at 30 minutes} = 1000 \times (1 + 0.1)^{30} \]

The result is approximately 3000 protons.

**Problem 3: Did you do the calculations? (15 points)**

The number of protons at each time point can be calculated using the equation above. For example, at 10 minutes, the number of protons would be:

\[ \text{Number of Protons at 10 minutes} = 1000 \times (1 + 0.1)^{10} \]

The result is approximately 2593 protons.

Similarly, at 20 minutes, the number of protons would be:

\[ \text{Number of Protons at 20 minutes} = 1000 \times (1 + 0.1)^{20} \]

The result is approximately 8235 protons.

At 30 minutes, the number of protons would be:

\[ \text{Number of Protons at 30 minutes} = 1000 \times (1 + 0.1)^{30} \]

The result is approximately 3000 protons.

These calculations can be repeated for each time point to determine the number of protons at any given time.
\[ \varepsilon^{N} \sum_{n=0}^{L} \varepsilon^{n} = H \]

where we used the fact that \( N \in \mathbb{N} \).

\[ N = \frac{1}{L} \int \varepsilon^{N} \sum_{n=0}^{L} \varepsilon^{n} \]

Notice that the results correctly combine to give in units of \( \text{eV} \) the temperature.

\[ \frac{N}{L} = \frac{1}{L} \int \varepsilon^{N} \sum_{n=0}^{L} \varepsilon^{n} \]

For some constant \( k \), we get for the Hubble parameter:

\[ H \left( \frac{\varepsilon}{\varepsilon^{0}} \right) = 1 \]

Conclusion: This book has been passed.

Question: What is the conclusion of the previous section?}

Problem 2: What is the reading?
Problem 4: Did You Do The Reading? (2017)?
\[ \frac{\partial p}{\partial t} + \frac{\partial \rho}{\partial x} = 0 \]

This equation is known as the conservation of mass equation.

The problem stated is as follows:

**Problem 8: Tracing Light Rays in a Closed Universe.**

\[ \frac{\partial H}{\partial t} + \frac{\epsilon}{2} \frac{\partial H}{\partial \epsilon} = 0 \]

This equation describes the evolution of the Hubble parameter in a closed universe.

We can solve this equation by integrating it:

\[ \int \frac{\partial H}{\partial t} dt = \int -\frac{\epsilon}{2} \frac{\partial H}{\partial \epsilon} d\epsilon \]

\[ H(t) = \frac{\epsilon^2}{2} \left( \frac{1}{\epsilon} \right) \]

\[ H(t) = \frac{\epsilon^2}{2} \]

Substituting this solution into the equation for the Hubble parameter gives:

\[ \frac{\partial H}{\partial \epsilon} = \frac{\epsilon^2}{2} \]

To determine the value of the parameter \( \epsilon \), we use:

\[ \frac{\partial H}{\partial \epsilon} = \frac{\epsilon^2}{2} \]

Now, we have:

\[ \frac{\epsilon^2}{2} \]

**Problem 7: Anticipating a Big Crunch.**

Numerically, it is clear that:

\[ \frac{\partial}{\partial \epsilon} \left( \frac{\epsilon^2}{2} \right) = 1 \]

We can use these results in the following manner to solve for \( \epsilon \).

\[ \frac{\epsilon^2}{2} = \theta \]

Finally, we arrive at the solution:

\[ \epsilon^2 = \theta \]

**Problem 6: Evolution of an Open, Matter-dominated Universe.**

The evolution of an open, matter-dominated universe is described by the following equation:

\[ \frac{\dot{H}}{H^2} = \frac{\epsilon}{2} \]

where \( \epsilon \) is the energy density of the universe.

We can use this equation to determine the value of the parameter \( \epsilon \), which is given by:

\[ \epsilon = \frac{\epsilon^2}{2} \]

The parameter \( \epsilon \) is the energy density of the universe, and the evolution of the universe is described by the following equation:

\[ \frac{\dot{H}}{H^2} = \frac{\epsilon}{2} \]

where \( \dot{H} \) is the rate of change of the Hubble parameter.
However, suppose we adopt the principle that the centroid of the initial sphere is in the center of the cube. Then the centroid of the initial sphere is at the origin. Since the distance between the centroid and the initial sphere is non-zero, there is no point on the surface of the sphere that is equidistant from three points on the surface of the cube. Therefore, if the centroid of the initial sphere is at the origin, there is no point on the surface of the sphere that is equidistant from three points on the surface of the cube.

In this case, we can check our calculations. A point that is equidistant from three points on the surface of the cube will lie on the sphere that is centered at the origin and has radius equal to the distance between the centroid and the initial sphere. This sphere will intersect the surface of the cube in three points, which are the points of tangency. Therefore, the centroid of the initial sphere is at the origin, and the sphere that is centered at the origin and has radius equal to the distance between the centroid and the initial sphere will intersect the surface of the cube in three points, which are the points of tangency.

Theorem (Theorem of the Centroid). Suppose that 

then the centroid of the initial sphere is at the origin.

Proof. Let the centroid of the initial sphere be at the origin. Then the centroid of the initial sphere is at the origin.

Therefore, if the centroid of the initial sphere is at the origin, there is no point on the surface of the sphere that is equidistant from three points on the surface of the cube. Therefore, if the centroid of the initial sphere is at the origin, there is no point on the surface of the sphere that is equidistant from three points on the surface of the cube.

In this case, we can check our calculations. A point that is equidistant from three points on the surface of the cube will lie on the sphere that is centered at the origin and has radius equal to the distance between the centroid and the initial sphere. This sphere will intersect the surface of the cube in three points, which are the points of tangency. Therefore, the centroid of the initial sphere is at the origin, and the sphere that is centered at the origin and has radius equal to the distance between the centroid and the initial sphere will intersect the surface of the cube in three points, which are the points of tangency.
\[
\int_{0}^{a} \left( \frac{\mu q + \nu}{1 + \varepsilon(q + \alpha)} \right) \, \mathrm{d}z = \left( \frac{\mu q + \nu}{1 + \varepsilon(q + \alpha)} \right) \int_{a}^{b} \frac{\varepsilon}{\mu} \, \mathrm{d}z
\]

So

\[
\left( \frac{\mu q + \nu}{1 + \varepsilon(q + \alpha)} \right) \int_{a}^{b} \frac{\varepsilon}{\mu} \, \mathrm{d}z = \int_{a}^{b} \frac{\varepsilon}{\mu} \, \mathrm{d}z
\]

The area is then

\[
\left( \frac{\mu q + \nu}{1 + \varepsilon(q + \alpha)} \right) = \int_{a}^{b} \frac{\varepsilon}{\mu} \, \mathrm{d}z
\]

The length of the strip is calculated by the same way as before, in part 4(b).

\[
\int_{0}^{a} \int_{z}^{y} \, \mathrm{d}x \, \mathrm{d}z = \int_{0}^{a} (y - z) \, \mathrm{d}z
\]

The length of the strip is determined by the interval to be

\[
\int_{0}^{a} \frac{\varepsilon}{\mu} \, \mathrm{d}z
\]

Note that the strip has a continuous with \( \mu \), but the definition across the width

**Problem 8: Lengths and Areas in a Two-Dimension**

The total area of the final section is both 10 squares to be considered part of the
\[ \int_{a}^{b} f(x) \, dx = \int_{a}^{b} g(x) \, dx \]

The total volume is then obtained by integration:

\[ \int_{a}^{b} f(x) \, dx = \int_{a}^{b} g(x) \, dx \]

The volume of the shell is then

\[ \int_{a}^{b} f(x) \, dx = \int_{a}^{b} g(x) \, dx \]

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The volume of the shell is then

\[ \int_{a}^{b} f(x) \, dx = \int_{a}^{b} g(x) \, dx \]
\[ \begin{align*}
(0 > \gamma R) & \quad \left( \frac{e^{\gamma/R} - 1}{e^{\gamma/R} - 1} \right) \left\{ i \right\} \frac{\partial \varphi}{\partial p} \\
(0 < \gamma R) & \quad \left( \frac{e^{\gamma/R} - 1}{e^{\gamma/R} - 1} \right) \left\{ i \right\} \frac{\partial \varphi}{\partial p}
\end{align*} \]

The answer is:

\[ \begin{align*}
(0 < \gamma R) & \quad \phi_{\text{calc}} = \frac{2}{\gamma R} \\
(0 < \gamma R) & \quad \phi_{\text{calc}} = \frac{2}{\gamma R}
\end{align*} \]

The integral can be set up as follows:

\[ \int_{\phi_{\text{calc}}}^{\phi_{\text{calc}}} \phi_{\text{calc}} \, d\phi = \frac{2}{\gamma R} \]

We can do the lower integration immediately:

\[ \int_{\phi_{\text{calc}}}^{\phi_{\text{calc}}} \phi_{\text{calc}} \, d\phi = \frac{2}{\gamma R} \]

The final volume is then:

\[ \frac{2}{\gamma R} \int_{0}^{\phi_{\text{calc}}} \phi_{\text{calc}} \, d\phi = \frac{2}{\gamma R} \]

The product of differential elements corresponding to infinitesimal changes in the coefficient of differential elements is called the differential volume element, $dV$. 

**Problem 12: Volumes in a Robertson-Walker Universe**

The volume enclosed between $\phi$ and $\phi + d\phi$ in the limit as $d\phi \to 0$. The cell $dV = d\phi \cdot d\theta \cdot d\rho$ between $\rho$ and $\rho + d\rho$, between $\theta$ and $\theta + d\theta$, and between $\phi$ and $\phi + d\phi$.

The cell $dV$ includes the volume lying between $\rho$, $\theta$, and $\phi$.

The function $\frac{\partial V}{\partial p}$ can then be written as $\frac{\partial V}{\partial p}$.

\[ \frac{\partial V}{\partial p} = \frac{dV}{dp} \]

Two cohomologies are required by the integral relation. Then the infinitesimal variations of the function $\phi = \frac{dV}{dp}$ are expected.

Checking the answer for the Euclidean case, one sees that it gives:

\[ \frac{\partial V}{\partial p} = \frac{dV}{dp} \]
\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

Since the Schwarzschild metric does not change with time, each path leaving \( v \) will
\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

be a time coordinate-parallel curve in \( y \) at the same spatial level or time to reach. Thus, the proper time difference \( d\tau \) between
\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

the corresponding integral of the coordinate \( d\tau \) is given by
\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

The reader on the observer's clock corresponds to the proper time interval \( d\tau \), so
\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

so
\[
\epsilon \int \frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

The coordinate distance from \( a \) to \( b \) is obtained by adding the proper lengths of all the
\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

Thus
\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

and the integrable interval becomes
\[
\frac{v_y - 1}{v z} = \sqrt{v} = \sqrt{v}
\]

a) The Schwarzschild horizon is the value of \( r \) for which the metric becomes singular.

b) The Schwarzschild horizon is the value of \( r \) for which the metric becomes singular.
The time interval \( t \) is defined as:

\[ t = \int P \text{ d} \theta \]

Looking at the answer to part (a), however, one can see that when \( \gamma > 0 \),

The proper distance between \( a \) and \( b \) does not converge.

So, although the integral is infinite at \( t = 0 \), the integral is still finite.

\[ \int_{t=-\infty}^{t+\infty} \frac{\text{d}S}{d} = \frac{\pi}{\sqrt{1-\gamma}} \int_{t=-\infty}^{t+\infty} \frac{\text{d}S}{d} \]

Evaluating:

Changing the integration variable to \( \eta = \int \frac{\text{d}S}{d} \), the contribution can be easily calculated:

\[ \int_{t=-\infty}^{t+\infty} \frac{\text{d}S}{d} = \int_{t=-\infty}^{t+\infty} \frac{\text{d}S}{d} \]

The potentially divergent part of the integral comes from the limit of integration in

\[ \int_{t=-\infty}^{t+\infty} \frac{\text{d}S}{d} = \int_{t=-\infty}^{t+\infty} \frac{\text{d}S}{d} \]

By (e) From parts (a) and (b), the proper distance between \( a \) and \( b \) can be rewritten as

\[ \int_{a}^{b} \frac{\text{d}S}{d} = \int_{a}^{b} \frac{\text{d}S}{d} \]

Then,\n
The dock at \( b \), however, will need the proper time and not the coordinate time.
\[ \begin{align*}
\theta P \mu P (\varphi / \mu) + I \wedge \mu &= \varphi P \mu P (\varphi / \mu),
\end{align*} \]

where \( P \) is the metric tensor. Theorem 1.1. \( \mu P \mu P (\varphi / \mu) + I \wedge \mu = \varphi P \mu P (\varphi / \mu) \) where \( P \) is a metric, the Riemann curvature tensor, and \( \mu \) is the Ricci tensor. Then, the following equation holds:

\[ \begin{align*}
\theta P \mu P (\varphi \cos + 1) + \theta \sin \theta \varphi \cos \mu P (\varphi / \mu) \wedge \mu &= \varphi P
\end{align*} \]

Since the metric does not vary along the path, we have:

\[ \begin{align*}
\theta P \mu P (\varphi \cos + 1) + \theta \sin \theta \varphi \cos \mu P (\varphi / \mu) \wedge \mu &= \varphi P
\end{align*} \]

so

\[ \begin{align*}
\int_{\theta}^{\varphi} \mu P (\varphi \cos + 1) + \mu \sin \mu \varphi \cos \mu P (\varphi / \mu) \, d\mu &= \varphi P
\end{align*} \]

on the path of the curve.

\[ \begin{align*}
\int_{\theta}^{\varphi} \mu P (\varphi \cos + 1) + \theta \sin \theta \varphi \cos \mu P (\varphi / \mu) \wedge \mu &= \varphi P
\end{align*} \]

Since the metric does not vary along the path, we have:

\[ \begin{align*}
\int_{\theta}^{\varphi} \mu P (\varphi \cos + 1) + \theta \sin \theta \varphi \cos \mu P (\varphi / \mu) \wedge \mu &= \varphi P
\end{align*} \]

Then, substituting into the geodesic equation for \( \mu \):

\[ \begin{align*}
\int_{\theta}^{\varphi} \mu P (\varphi \cos + 1) + \theta \sin \theta \varphi \cos \mu P (\varphi / \mu) \wedge \mu &= \varphi P
\end{align*} \]

Calculating the normal derivative:

\[ \begin{align*}
\left( \frac{\mu}{\gamma} \right)^{\prime} &= \frac{\mu}{\gamma} \frac{\mu}{\gamma} + \frac{\mu}{\gamma} \frac{\mu}{\gamma} \frac{\mu}{\gamma} = \frac{\mu}{\gamma} \frac{\mu}{\gamma}
\end{align*} \]

\[ \begin{align*}
\left( \frac{\mu}{\gamma} \right)^{\prime} &= \frac{\mu}{\gamma} \frac{\mu}{\gamma} + \frac{\mu}{\gamma} \frac{\mu}{\gamma} \frac{\mu}{\gamma} = \frac{\mu}{\gamma} \frac{\mu}{\gamma}
\end{align*} \]

If the parameterization is not unique, because one can choose \( \lambda \) to represent any point along the curve, the choice of \( \lambda \) does not affect the results. In Cartesian coordinates, the curve is smooth with radius of curvature which is a curve. In Cartesian coordinates, the

**Exercise:** AN EXERCISE IN TWO-DIMENSIONAL METRICS
The problem of finding the geodesics on a sphere, given by

\[ 0 = z = \varepsilon^x \]

\[ \phi \cos \alpha = \kappa = \varepsilon^x \]

\[ \phi \sin \alpha = \theta = \varepsilon^x \]

is to determine the curves in the sphere that are left invariant when

\[ \frac{\varepsilon^x}{\partial \phi} \varepsilon^y + \left( \frac{\varepsilon^x}{\partial \phi} \varepsilon^z \right) = 0 \]

and

\[ \frac{\varepsilon^x}{\partial \phi} \varepsilon^y + \left( \frac{\varepsilon^x}{\partial \phi} \varepsilon^z \right) = 0 \]

are satisfied. The geodesic \( \theta = \varepsilon^x \)

so the geodesic equation is simpler, because none of the coefficients depend on \( \theta \).

The solution is

\[ \frac{\varepsilon^x}{\partial \phi} \varepsilon^y + \left( \frac{\varepsilon^x}{\partial \phi} \varepsilon^z \right) = 0 \]

Inserting this expression into the boxed equation above, the first term can be brought

\[ \frac{\varepsilon^x}{\partial \phi} \varepsilon^y + \left( \frac{\varepsilon^x}{\partial \phi} \varepsilon^z \right) = 0 \]
\[
\begin{align*}
\sin \theta + \cos \theta &= \sqrt{\sin^2 \theta + \cos^2 \theta} \\
\Rightarrow 1 &= \sqrt{1} = \theta
\end{align*}
\]

Then, using the trigonometric identity \( \sin \theta \cdot \cos \theta = \sin \theta \cdot \cos \theta \), one finds
\[
\sin \theta \cdot \cos \theta = \theta
\]

implies
\[
\cos \theta = \theta
\]

This is the main result. Taking the determinant of
\[
\begin{vmatrix}
\phi & \phi P \\
\phi P & \phi P \\
\end{vmatrix}
= \phi P \phi P - \phi P \phi P
\]

\[
= 0
\]

\[
\phi = \phi
\]

Taking again the relations between polar and Cartesian coordinates,
\[
\begin{align*}
\phi &= \phi \\
\cos \phi &= \cos \phi
\end{align*}
\]

Taking the relation between the two coordinate systems given above,
\[
\phi = \phi \\
\cos \phi &= \cos \phi
\]

Formula: The total equation, which we seek to describe, is just the standard equation in the

Thus the quantity
\[
\frac{\phi P \phi P \phi P \phi P}{\phi P \phi P \phi P \phi P}
\]

For this problem we have only two nonzero components
\[
\frac{\phi P \phi P \phi P \phi P}{\phi P \phi P \phi P \phi P}
\]

For the problem the matrix has only two nonzero components
\[
\frac{\phi P \phi P \phi P \phi P}{\phi P \phi P \phi P \phi P}
\]

Thus the quantity
\[
\frac{\phi P \phi P \phi P \phi P}{\phi P \phi P \phi P \phi P}
\]
\[
\frac{\left( \frac{\partial \rho}{\partial \tau} \right) \left( \frac{x^2 - 1}{|\tau|^2} \right) - 1}{\frac{\partial \rho}{\partial \tau}} = \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau}
\]

The answer from part (a) is obtained by a direct comparison of the original line with the line segment. The proper time that would be measured by a clock moving with the observer is given by \( \tau \). Therefore, the metric

\[
\frac{\left( \frac{\partial \rho}{\partial \tau} \right) \left( \frac{x^2 - 1}{|\tau|^2} \right) - 1}{\frac{\partial \rho}{\partial \tau}} = \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau}
\]

For some straightforward algebra, one finds

\[
= \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau}
\]

(a) Problem 17: Geodesics in a Closed Universe

So the left- and right-hand sides are equal,

\[
\frac{\left( \frac{\partial \rho}{\partial \tau} \right) \left( \frac{x^2 - 1}{|\tau|^2} \right) - 1}{\frac{\partial \rho}{\partial \tau}} = \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau}
\]

The right-hand side of the first geodesic equation can be evaluated using the expression

\[
= \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau}
\]

The expression above for \( \phi \), hence

\[
= \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau}
\]

Similarly

\[
= \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau}
\]

So clearly

\[
= \frac{\partial \rho}{\partial \tau} = \frac{\partial \rho}{\partial \tau}
\]

For proper radial motion, \( \phi = \phi = \theta \phi \), use the appropriate reduced do
The geometric definition of the angle between two vectors in a plane is based on the right-hand rule. Consider two vectors \( \mathbf{a} \) and \( \mathbf{b} \) in a plane, and let \( \theta \) be the angle between them. The right-hand rule states that if you place your right hand such that your fingers point in the direction of vector \( \mathbf{a} \), then your thumb will point in the direction of the cross product \( \mathbf{a} \times \mathbf{b} \). The magnitude of \( \mathbf{a} \times \mathbf{b} \) is equal to the area of the parallelogram formed by \( \mathbf{a} \) and \( \mathbf{b} \) and is given by:

\[
|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta,
\]

where \( |\mathbf{a}| \) and \( |\mathbf{b}| \) are the magnitudes of \( \mathbf{a} \) and \( \mathbf{b} \), respectively. The angle \( \theta \) can be found using the dot product:

\[
\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}.
\]

This formula gives the cosine of the angle between the two vectors. The sine of the angle can be found using the Pythagorean identity:

\[
\sin^2 \theta = 1 - \cos^2 \theta = 1 - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}\right)^2.
\]

By taking the square root of both sides, we can find \( \sin \theta \) as:

\[
\sin \theta = \sqrt{1 - \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}\right)^2}.
\]
To evaluate the area in the form of the value

\[ \mathbf{\hat{n}} \frac{n - \nu}{n} \int_{0}^{\frac{\pi}{2}} \sin \theta \, d\theta = S \]

To find the length of the radius, the formula

\[ \mathbf{\hat{n}} \frac{n - \nu}{n} \int_{0}^{\frac{\pi}{2}} \sin \theta \, d\theta = S \]

Since \( n = \frac{\pi}{2} \), and \( n = \frac{\pi}{2} \), we can integrate the expression to get the answer in this order. If ordered, we get

\[ \mathbf{\hat{n}} \frac{n - \nu}{n} \int_{0}^{\frac{\pi}{2}} \sin \theta \, d\theta = S \]

For \( n = \text{constant} \), the expression for the metric reduces to

\[ 0 = (d\nu) \frac{dp}{\nu} \]

Multiplying by \( \frac{\nu}{\sqrt{1 - v^2}} \), one has the desired result:

\[ 0 = (d\nu) \frac{dp}{\nu} \]

Then the second term cancels the expression on the right-hand side, leaving

Multiplying the left-hand side,

\[ \frac{dp}{d\nu} \left( \frac{n - \nu}{n} \right) \int_{0}^{\frac{\pi}{2}} \sin \theta \, d\theta + (d\nu) \frac{dp}{\nu} = \]

\[ \frac{dp}{d\nu} \left( \frac{n - \nu}{n} \right) \int_{0}^{\frac{\pi}{2}} \sin \theta \, d\theta + (d\nu) \frac{dp}{\nu} = \left( \frac{\frac{dp}{d\nu}}{d\nu} \right) \frac{dp}{S} = \text{HT} \]

Expanding the left-hand side,
The metric coefficients are thus read off from the expression:

\[ I = \varepsilon^2 \varepsilon'_{
u} \nu^{
u} \equiv \varepsilon^2 \]

\[ \varepsilon^2 = \varepsilon'_{\nu} \nu^\nu \equiv \varepsilon^2 \]

\[ \varepsilon'_{\nu} \varepsilon^{\nu} = \varepsilon'_{\mu} \varepsilon^\mu = \varepsilon'_{\nu} \nu^\nu = \varepsilon^2 \]

and the metric coefficients are then read off from this expression:

\[ I = \varepsilon^2 \varepsilon'_{\nu} \nu^\nu \equiv \varepsilon^2 \]

\[ \varepsilon^2 = \varepsilon'_{\nu} \nu^\nu \equiv \varepsilon^2 \]

\[ \varepsilon'_{\nu} \varepsilon^{\nu} = \varepsilon'_{\mu} \varepsilon^\mu = \varepsilon'_{\nu} \nu^\nu = \varepsilon^2 \]

The metric coefficients are thus read off from this expression:

\[ I = \varepsilon^2 \varepsilon'_{\nu} \nu^\nu \equiv \varepsilon^2 \]

\[ \varepsilon^2 = \varepsilon'_{\nu} \nu^\nu \equiv \varepsilon^2 \]

\[ \varepsilon'_{\nu} \varepsilon^{\nu} = \varepsilon'_{\mu} \varepsilon^\mu = \varepsilon'_{\nu} \nu^\nu = \varepsilon^2 \]

The metric coefficients are thus read off from this expression:

\[ I = \varepsilon^2 \varepsilon'_{\nu} \nu^\nu \equiv \varepsilon^2 \]

\[ \varepsilon^2 = \varepsilon'_{\nu} \nu^\nu \equiv \varepsilon^2 \]

\[ \varepsilon'_{\nu} \varepsilon^{\nu} = \varepsilon'_{\mu} \varepsilon^\mu = \varepsilon'_{\nu} \nu^\nu = \varepsilon^2 \]
\[
\frac{m_2}{m_1} = \frac{m_1 \cdot \frac{m_2}{m_1} \cdot \frac{m_1}{m_2}}{m_1 \cdot \frac{m_2}{m_1}} = \frac{m_2}{m_1}
\]

Note that this expression is identical in form to the previous one. However, the left side now contains the force in the direction of motion, while the right side contains the force in the opposite direction. Therefore, the expression can be simplified to:

\[
\frac{m_2}{m_1} = \frac{m_2}{m_1}
\]

This result indicates that the ratio of the masses is equal to the ratio of the force components, which confirms the conservation of momentum principle.

Next, consider the case where the two objects are not in motion relative to each other. In this scenario, the only force acting on each object is the gravitational force.

\[
\frac{m_2}{m_1} = \frac{m_2}{m_1}
\]

This result is consistent with the previous one, as the gravitational force is the only force acting on each object.

Finally, consider the case where the two objects are in motion relative to each other. In this scenario, the only force acting on each object is the resultant force due to the gravitational force and the relative motion.

\[
\frac{m_2}{m_1} = \frac{m_2}{m_1}
\]

This result is again consistent with the previous one, as the resultant force is the only force acting on each object.

These results confirm the conservation of momentum principle, which states that the total momentum of a system is conserved in the absence of external forces.

### Example Solution

Given the scenario where two objects are moving relative to each other, and the mass of object 1 is twice the mass of object 2, determine the relative velocity of object 1 with respect to object 2.

\[
\frac{m_2}{m_1} = \frac{m_2}{m_1}
\]

This result suggests that the relative velocity of object 1 with respect to object 2 is zero, indicating that the objects are moving at the same velocity relative to each other.

### Summary

The conservation of momentum principle states that the total momentum of a system is conserved in the absence of external forces. This principle is fundamental in physics and is used to solve a wide range of problems, from celestial mechanics to everyday interactions.

By understanding the conservation of momentum principle, we can gain insight into the behavior of objects in motion, which is crucial for predicting their future positions and velocities.
\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} - \frac{d}{d} = \left( \frac{d}{d} \right) f
\]
In the resolution of the problem statement, we have
\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} = \frac{\partial p}{\partial p} + \frac{\partial p}{\partial p} = \frac{\partial p}{\partial p}
\]
This is an acceptable answer. One can simplify (9.01). Further by noting that, \( f = \frac{d}{d} \),

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \frac{\partial p}{\partial p} \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]
Expanding out, the terms with \( \frac{d}{d} \) cancel and we find
\[
\left( \frac{\partial p}{\partial p} \right) \eta + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]
We use now (9.09) to simplify (9.01).

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]
This is the most usual form of the answer. Of course, we also have
\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \frac{\partial p}{\partial p} = \frac{\partial p}{\partial p}
\]
and rearranging,
\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]
Dividing the expression (9.3) by the numerator, we readily find
\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]
and the numerator is the integral and multiplying by \( \frac{d}{d} \), we find
\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]
Here and we have suppressed the summation of and \( \frac{d}{d} \) to avoid clutter. Collecting
\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]
From the material

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
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\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
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\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
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\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

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\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]

\[
\left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} + \left( \frac{\partial p}{\partial p} \right) \frac{d}{d} \frac{1}{d} = \frac{\partial p}{\partial p}
\]
This is the desired condition for stable orbits in the Schwarzschild geometry:

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

For \( u_0 \), we set

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

which is equivalent to

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

Computing the common factors of \( S + u \), we find

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

Note, incidentally, that the real and imaginary parts of \( S + u \) are invariant under the orthochronous transformations of the complex plane. To proceed with the calculation, we need the value of \( f \) such that

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

The inequality in \( (19) \) is the one used:

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

Where we have introduced the function \( (u) \), which is the derivative of \( u \) with respect to \( f \). Since we need to consider the function \( u \) in the range \( (u) \), we have:

\[ 0 = \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

The inequality in \( (19) \) is the one used:

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

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The inequality in \( (19) \) is the one used:

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

This is the answer to part

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

Similarly, the condition for bounded oscillations involves the requirement:

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

and the solution to the second-order differential equation, obtainable from:

\[ 0 > \frac{(S + u) - u_0}{u_0} \geq \frac{(S + u) - u_0}{u_0} \geq 0 \]

The geodesic equation (19) then becomes:

\[ \frac{dp}{df} - \frac{dp}{df} \geq \frac{dp}{df} \geq 0 \]

Since no more non-trivial dependents on \( \phi \) exist, the right-hand side vanishes and we get:

\[ \frac{dp}{df} - \frac{dp}{df} \geq \frac{dp}{df} \geq 0 \]

The geodesic equation (19) for \( p \) is thus:

\[ \frac{dp}{df} = \phi \]

for all \( \phi \) within the problem domain.
\[
\phi P \left( \varphi \int _ { \mathbb { D } } \varphi d \mathbb { V } \right) = A
\]

We must now integrate over the range of \( \varphi \), for 0 to \( \pi / 2 \). So,

\[
\phi P \left[ \left( \varphi \int _ { \mathbb { D } } \varphi d \mathbb { V } \right) \right] = A P
\]

The volume of the spherical shell is then given by

\[
\phi P \left( \varphi \int _ { \mathbb { D } } \varphi d \mathbb { V } \right) = \varepsilon \varphi P
\]

and so the range of the spherical shell is from \( \varphi = 0 \) to \( \pi / 2 \).

\[
\varepsilon \varphi P \left( \varphi \int _ { \mathbb { D } } \varphi d \mathbb { V } \right) = \varepsilon \varepsilon P
\]

By the conservation of moment and the torque, the torque is given by the net moment of the work done by the gas, which is the same magnitude as the moment \( A \). We can take the same value of radius \( R \) and the same moment as a spatial constant, and so the moment of the spherical shell is constant.

\[
\left( \phi \theta + \phi \theta \varphi \right) \mathbb { V } = \varepsilon \varepsilon P
\]

For comparison, the moment for the surface of a sphere of radius \( R \) is given by

\[
\left( \phi \theta + \phi \theta \varphi \right) \left( \phi \varphi + \phi \varphi \varphi \right) \mathbb { V } = \varepsilon \varepsilon P
\]

The net of the moment is then the momentum of the volume

\[
A \varphi P \left( \varphi \int _ { \mathbb { D } } \varphi d \mathbb { V } \right) = \Pi \]

Combining this with the expression for the 4-th component from part (b), one sees immediately

\[
\frac { A } { A \varphi I } = \left( \frac { A } { A \varphi I } + 1 \right) \Pi = \left( \frac { A } { A \varphi I } + 1 \right) A \left( \chi _ { \Re } \varphi I - 1 \right) \Pi = \Pi A \left( I + \left( \chi _ { \Re } \varphi I - 1 \right) A \right) \Pi = \Pi A \left( I + A \right) \Pi
\]

...\( \leq \Pi A \left( I + A \right) \Pi ) \]

...\( \leq \Pi A \left( I + A \right) \Pi ) \]

Then, we have

\[
\Pi A \left( I + A \right) \Pi = \Pi A \left( I + A \right) \Pi
\]

The total energy is the energy density times the volume, so

\[
\left( \frac { A } { A \varphi I } - 1 \right) \Pi = \left( \frac { A } { A \varphi I } + 1 \right) \Pi
\]

\[
= \frac { A } { A \varphi I } + 1 \Pi
\]

Thus, the energy density is proportional to the volume, and reduces to \( n = n \Pi A \left( I + A \right) \Pi \), when

\[
\frac { \Pi A \left( I + A \right) \Pi } { A \varphi I } \Pi = \left( \chi _ { \Re } \varphi I - 1 \right) A \Pi
\]

PROBLEM 22: VOLUME OF A CLOSED THREE-DIMENSIONAL SPHERE

\[\text{PROBLEM 21: PRESSURE AND ENERGY DENSITY OF VIBRATIONS}\]
where the gravitational force is the net effect of all the gravitational forces acting on the object.
\[
\begin{align*}
\int_0^\infty \frac{\xi P \xi P}{\kappa \xi P} & \approx \int_0^\infty \frac{\xi P}{\kappa \xi P} = 0
\end{align*}
\]

Apply and substitute the small angle approximations \( \alpha = 0 \) will make an angle \( \alpha \) equal to the horizontal, where

\[
\begin{align*}
\lim_{\xi P / \beta P} & = \int \\
\lim_{\xi P / \beta P} & = 0
\end{align*}
\]

The final value of \( \beta P / \alpha P \) is given by Eq. 4.52.43 (for the final point \( f \)).

\[
\begin{align*}
\frac{\xi P}{\kappa \xi P} & = \frac{\xi P}{\kappa \xi P} + \frac{\xi P}{\kappa \xi P} - \frac{\xi P}{\kappa \xi P} = \frac{\xi P}{\kappa \xi P}
\end{align*}
\]

You were not asked to carry out these integrals, but only the value of integrals over

\[
\begin{align*}
\int_0^\infty \frac{\xi P}{\kappa \xi P} \frac{\xi P}{\kappa \xi P} & = 0
\end{align*}
\]

Then, using Eqs. 4.52 and combining with Eqs. 1 and 4.52.8.1

\[
\begin{align*}
\int_0^\infty \frac{\xi P}{\kappa \xi P} \frac{\xi P}{\kappa \xi P} & = 0
\end{align*}
\]