Problem 1

(a) Using the expression

\[ \text{Re } Z(\zeta) = -2S(\zeta) = -2e^{-\zeta^2} \int_0^\zeta e^{t^2} dt \]

show by successive integration by parts that for \( \zeta \ll 1 \),

\[ S(\zeta) \simeq \zeta - \frac{2}{3} \zeta^3 + \ldots \]

(b) Show that for \( \zeta \gg 1 \), an asymptotic expansion of \( S(\zeta) \) is given by

\[ S(\zeta) = \frac{1}{2\sqrt{\pi}} P \int_{-\infty}^{\infty} \frac{e^{-t^2}}{t + \zeta} dt \simeq \frac{1}{2\zeta} \left[ 1 + \frac{1}{2\zeta^2} + \frac{3}{4\zeta^4} + \ldots \right] \]
Problem 2

The collisionless components of the stress tensor elements (in addition to pressure, \( \nabla p_{i\perp} \)) are given by

\[
\begin{align*}
\pi_{xx} &= -\pi_{yy} = -\frac{p_{\perp}}{2\Omega} \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) \\
\pi_{xy} &= \pi_{yx} = -\frac{p_{\perp}}{2\Omega} \left( \frac{\partial v_y}{\partial y} - \frac{\partial v_x}{\partial x} \right)
\end{align*}
\]

where \( p_{\perp} = nT_{\perp} \) (see W. B. Thomson, Introduction to Plasma Physics). Now consider the electrostatic ion cyclotron wave, \( k_z^2 \ll k_{\perp}^2 \). By including finite ion temperature in the fluid equations, and keeping the finite ion Larmor radius terms \( b_i = k_{\perp}^2 r_{ci}^2 \ll 1 \), as well as \( \nabla p_{i\perp} \), show that to \( 0(b_i) \) the electrostatic ion cyclotron wave dispersion relationship is given by

\[
\omega^2 = \omega_{ci}^2 \left[ 1 + \frac{k_{\perp}^2 T_{e\parallel}}{\omega_{ci}^2 m_i} \right]
\]

**even when** \( T_{e\parallel} \approx T_{i\perp} \). We also assumed \( 1 \gg k_{\perp}^2 \lambda_D^2 \to 0 \).

Hint: Take \( k_{\perp} \approx k_y, k_x = 0 \), and use the two-fluid equations. Assume \( \omega \gtrsim \omega_{ci} \ll \omega_{pi}, k_zv_{ti} \ll \omega \ll k_zv_{te} \), and consider strongly magnetized electrons. The two fluid equations for each species are:

\[
\begin{align*}
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{v}_i) &= 0 \\
m_i n_i \left( \frac{\partial \vec{v}_i}{\partial t} + \vec{v}_i \cdot \nabla \vec{v}_i \right) &= q_i n_i \left( \vec{E} + \frac{\vec{v}_i \times \vec{B}}{c} \right) - \nabla p - \nabla \cdot \vec{\pi}
\end{align*}
\]

Solve these equations by the usual linearization procedure, assuming that \( E_0 = 0 \).
Problem 3

Do (a) or (b)

Using the hot plasma electrostatic dispersion relationship, show that in the small, but finite Larmor radius limit (i.e. $b \ll 1$ for both ions and electrons):

(a) the upper hybrid (Bernstein) wave for $b_e \ll 1$, $k_{\parallel}^2/k_{\perp}^2 \ll 1$, and $\omega_{pe}^2 \ll \omega_{ce}^2 \approx \omega^2$ is

$$\omega^2 \approx \omega_{ce}^2 + \omega_{pe}^2 \left(1 - b_e - \frac{k_{\parallel}^2}{k_{\perp}^2}\right); \text{ Here } b_e = k_{\perp}^2 r_{ce} = k_{\perp}^2 T_e/m_e \omega_{ce}.$$  

(b) The lower hybrid dispersion relationship is given by

$$\omega^2 = \omega_{LH}^2 \left[1 + \frac{k_{\parallel}^2}{k_{\perp}^2} \frac{m_i}{m_e} + \frac{3}{2} \frac{k_{\perp}^2 v_{ti}^2}{\omega^2} \left(1 + \frac{1}{4} y^4 \frac{T_e}{T_i}\right)\right]$$

where we assumed $\omega_{ce}^2 \gg \omega^2 \gg \omega_{ci}^2$; $\omega_{pe}^2 \gg \omega^2$; $y^2 = \omega^2/\omega_{ce} \omega_{ci}$; $v_{ti}^2 = 2T_i/m_i$. 