Neural circuits for cognition

Nonnormal amplification

MIT Course 9.49/9.490

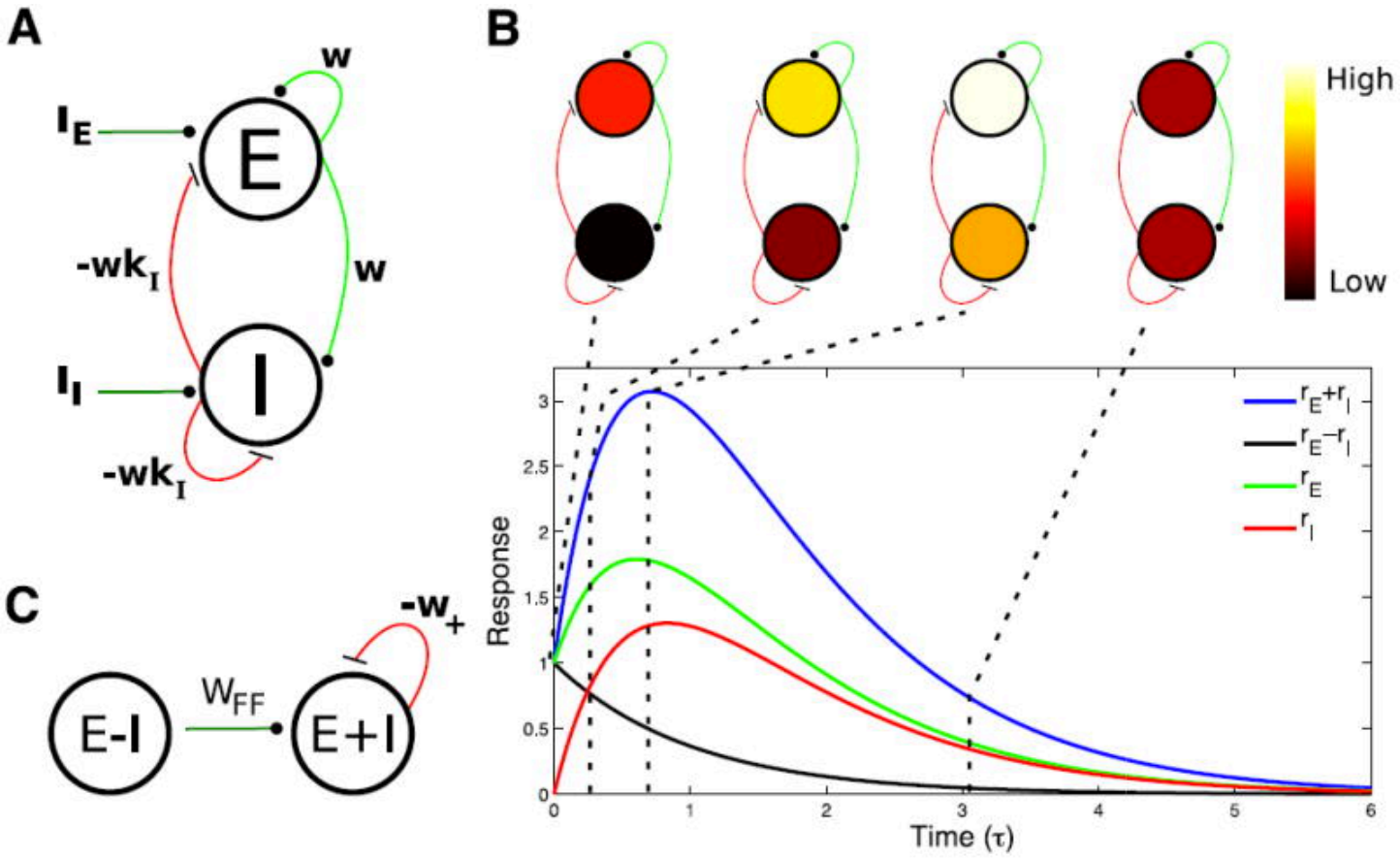
Instructor: Professor Ila Fiete

Complete and recap: Fast amplification in non-normal networks

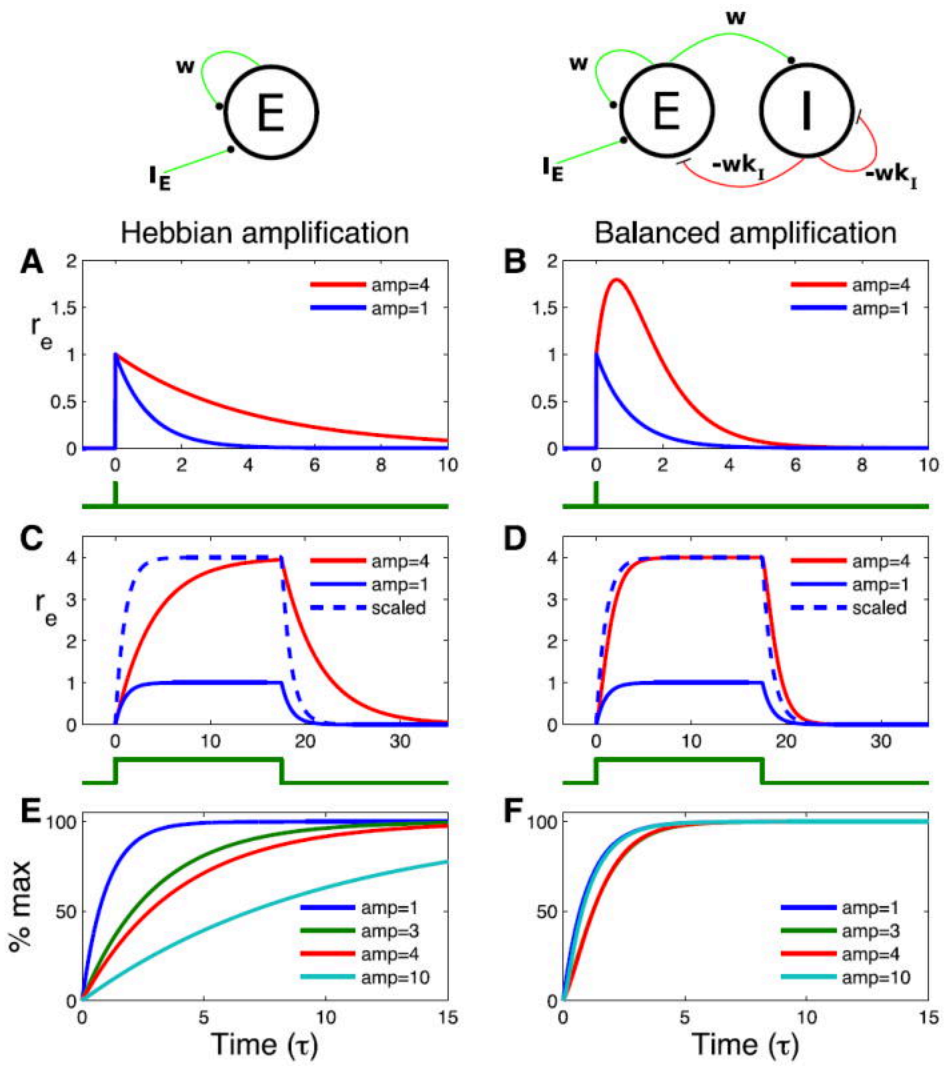
Murphy & Miller, 2009

Continue derivation/interpretation of nonnormal dynamics on board…

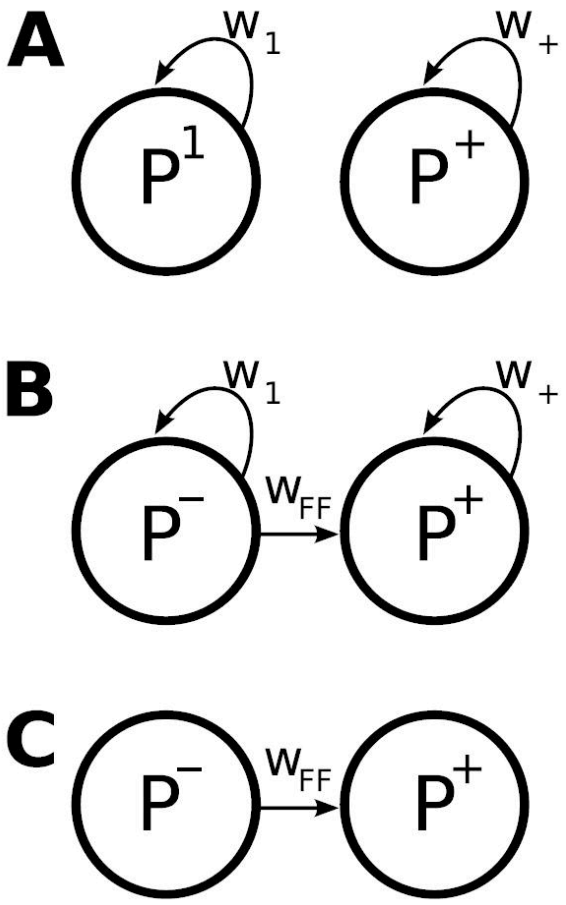
Transient amplifying modes in a non-normal network



Comparison: normal vs non-normal amplification



Decomposition of network dynamics



Normal network, eigenvector basis

Non-normal network, Schur basis (diagonal entries = self-interaction; upper-triangular entries = ff connections)

Balanced non-normal network: models do not excite themselves, looks like just ff chain

Schur decomposition

$$A \in \mathbf{C}^{n \times n}$$

$$A = QUQ^{-1}$$

where Q is unitary ($Q^{-1} = Q^H$) and U is upper-triangular; diagonal of U consists of the eigenvalues of A.

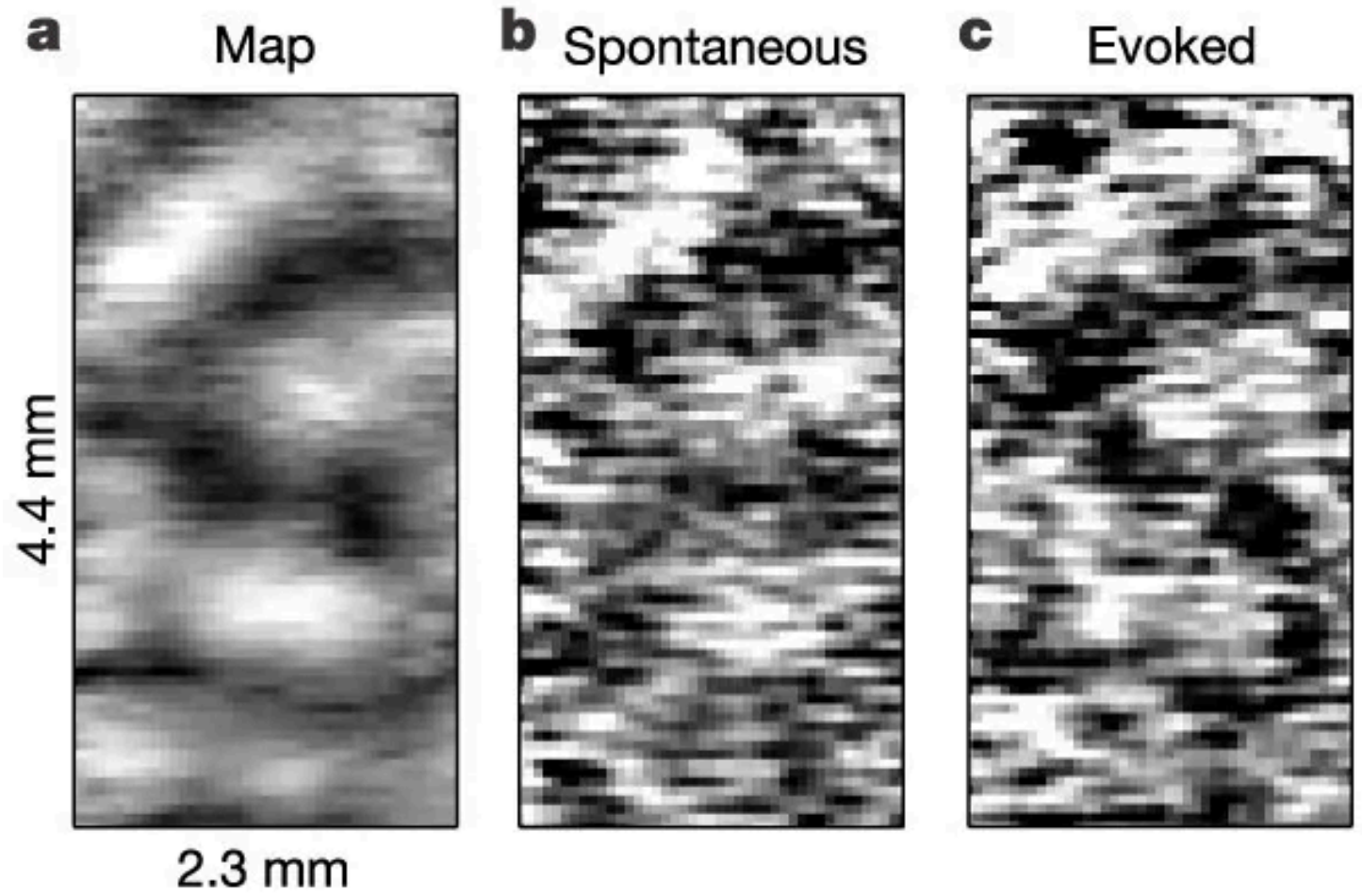
Implications of highly non-normal dynamics

- Large excursions of state away from a stable fixed point.
- In linear systems with a single fixed point at 0, these excursions are transient and the global fixed point is restored after the transient has decayed.
- In nonlinear systems with a stable fixed point at 0 in the linearized dynamics, even small perturbations can lead to large excursions can push the system out of the neighborhood of 0 so it becomes trapped in the nonlinear dynamics away from the linearized stable fixed point at 0.
- Famous example: Onset of turbulence in fluid flows.
- Might the non-existence of a fixed point at 0 in the sharply nonlinear EI networks be related to this?

Why relevant for neuroscience?

- Non-normal matrices are more generic than normal ones.
- E, I populations make neural systems predisposed towards non-normality.
- Sensory systems that depend on amplification and speed simultaneously might exploit nonnormal amplification.
- Some signatures that could be consistent with nonnormal dynamics in neural data (next slides).

Spontaneous activity patterns in cortex (V1)

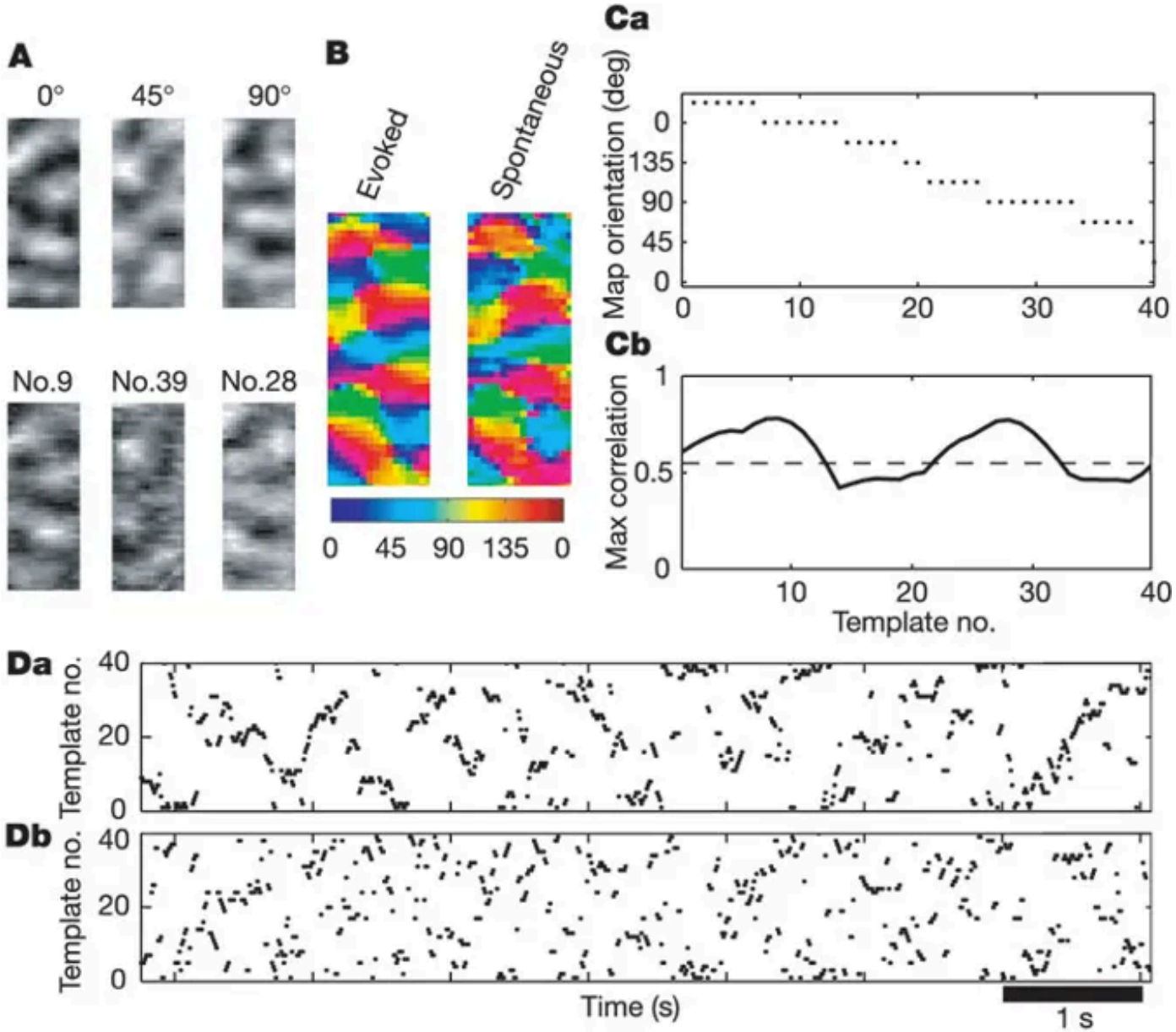


Voltage sensitive dye imaging

Spontaneously emerging cortical representations of visual attributes. Kenet, Bibitchkov, Tsodyks, Grinvald, Arieli

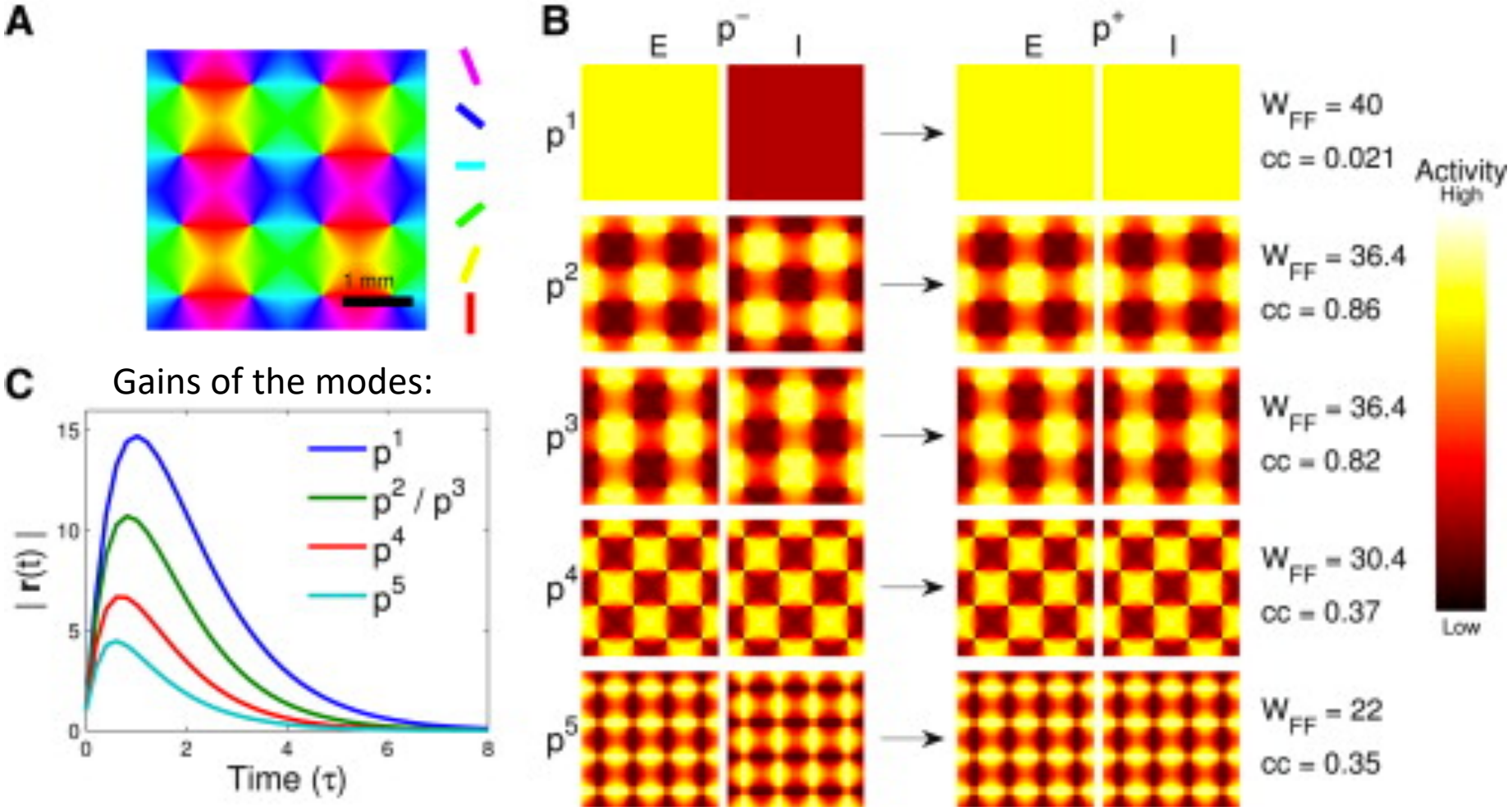
evoked

spontaneous



Spatially extended balanced amplification model

Orientation map (wts setup to produce this)



p^- difference modes p^+ sum modes

Interpretations of spontaneous activity

- States are stable fixed points, and the system moves or switches between them during spontaneous activity through some other temporal process.

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- States are the sub-critical (non-stable) eigenvectors of the system, with transient amplification of the dynamics leading to amplification of the state before decay.