

PARI-GP Reference Card

(PARI-GP version 2.1.0)

Note: optional arguments are surrounded by braces {}.

Starting & Stopping GP

to enter GP, just type its name: `gp`
to exit GP, type `\q` or `quit`

Help

describe function `?function`
extended description `??keyword`
list of relevant help topics `???pattern`

Input/Output & Defaults

output previous line, the lines before `%, %', %'', etc.`
output from line `%n`
separate multiple statements on line `;`
extend statement on additional lines `\`
extend statements on several lines `{seq1; seq2;}`
comment `/* ... */`
one-line comment, rest of line ignored `\\ ...`
set default `d` to `val` `default({d}, {val}, {fl})`
mimic behaviour of GP 1.39 `default(compatible,3)`

Metacommands

toggle timer on/off `#`
print time for last result `##`
print `%n` in raw format `\a n`
print `%n` in pretty format `\b n`
print defaults `\d`
set debug level to `n` `\g n`
set memory debug level to `n` `\gm n`
enable/disable logfile `\l {filename}`
print `%n` in pretty matrix format `\m`
set output mode (raw, default, prettyprint) `\o n`
set `n` significant digits `\p n`
set `n` terms in series `\ps n`
quit GP `\q`
print the list of PARI types `\t`
print the list of user-defined functions `\u`
read file into GP `\r filename`
write `%n` to file `\w n filename`

GP Within Emacs

to enter GP from within Emacs: `M-x gp, C-u M-x gp`
word completion `<TAB>`
help menu window `M-\c`
describe function `M-?`
display \TeX 'd PARI manual `M-x gpman`
set prompt string `M-\p`
break line at column 100, insert `M-\l`
PARI metacommand `\letter` `M-\letter`

Reserved Variable Names

$\pi = 3.14159\dots$ `Pi`
Euler's constant `= .57721\dots` `Euler`
square root of -1 `I`
big-oh notation `O`

PARI Types & Input Formats

`t_INT`. Integers $\pm n$
`t_REAL`. Real Numbers $\pm n.ddd$
`t_INTMOD`. Integers modulo m `Mod(n, m)`
`t_FRAC`. Rational Numbers n/m
`t_COMPLEX`. Complex Numbers $x + I * y$
`t_PADIC`. p -adic Numbers $x + O(p^k)$
`t_QUAD`. Quadratic Numbers $x + y * \text{quadgen}(D)$
`t_POLMOD`. Polynomials modulo g `Mod(f, g)`
`t_POL`. Polynomials $a * x^n + \dots + b$
`t_SER`. Power Series $f + O(x^k)$
`t_QFI/t_QFR`. Imag/Real bin. quad. forms `Qfb(a, b, c, {d})`
`t_RFRAC`. Rational Functions f/g
`t_VEC/t_COL`. Row/Column Vectors $[x, y, z]$, $[x, y, z]$ -
`t_MAT`. Matrices $[x, y; z, t; u, v]$
`t_LIST`. Lists `List([x, y, z])`
`t_STR`. Strings `"aaa"`

Standard Operators

basic operations `+, -, *, /, ^`
`i=i+1, i=i-1, i=i*j, ...` `+++, i--, i*=j, ...`
euclidean quotient, remainder `x\y, x\y, x%y, divrem(x, y)`
shift `x` left or right `n` bits `x<<n, x>>n` or `shift(x, n)`
comparison operators `<=, <, >=, >, ==, !=`
boolean operators (or, and, not) `||, &&, !`
sign of `x = -1, 0, 1` `sign(x)`
maximum/minimum of `x` and `y` `max, min(x, y)`
integer or real factorial of `x` `x!` or `fact(x)`

Conversions

Change Objects
make `x` a vector, matrix, set, list, string `Vec, Mat, Set, List, Str`
create PARI object (`x mod y`) `Mod(x, y)`
make `x` a polynomial of `v` `Pol(x, {v})`
as above, starting with constant term `Polrev(x, {v})`
make `x` a power series of `v` `Ser(x, {v})`
PARI type of object `x` `type(x, {t})`
object `x` with precision `n` `prec(x, {n})`
evaluate `f` replacing vars by their value `eval(f)`

Select Pieces of an Object

length of `x` `length(x)`
`n`-th component of `x` `component(x, n)`
`n`-th component of vector/list `x` `x[n]`
(m, n)-th component of matrix `x` `x[m, n]`
row `m` or column `n` of matrix `x` `x[m,], x[, n]`
numerator of `x` `numerator(x)`
lowest denominator of `x` `denominator(x)`
Conjugates and Lifts
conjugate of a number `x` `conj(x)`
conjugate vector of algebraic number `x` `conjvec(x)`
norm of `x`, product with conjugate `norm(x)`
square of L^2 norm of vector `x` `norml2(x)`
lift of `x` from Mods `lift, centerlift(x)`

Random Numbers

random integer between 0 and $N - 1$ `random({N})`
get random seed `getrand()`
set random seed to `s` `setrand(s)`

Lists, Sets & Sorting

sort `x` by k th component `vecsort(x, {k}, {{fl} = 0})`
Sets (= row vector of strings with strictly increasing entries)
intersection of sets `x` and `y` `setintersect(x, y)`
set of elements in `x` not belonging to `y` `setminus(x, y)`
union of sets `x` and `y` `setunion(x, y)`
look if `y` belongs to the set `x` `setsearch(x, y, {fl})`

Lists

create empty list of maximal length `n` `listcreate(n)`
delete all components of list `l` `listkill(l)`
append `x` to list `l` `listput(l, x, {i})`
insert `x` in list `l` at position `i` `listinsert(l, x, i)`
sort the list `l` `listsort(l, {fl})`

Programming & User Functions

Control Statements (`X`: formal parameter in expression `seq`)
eval. `seq` for $a \leq X \leq b$ `for(X = a, b, seq)`
eval. `seq` for `X` dividing `n` `fordiv(n, X, seq)`
eval. `seq` for primes $a \leq X \leq b$ `forprime(X = a, b, seq)`
eval. `seq` for $a \leq X \leq b$ stepping `s` `forstep(X = a, b, s, seq)`
multivariable `for` `forvec(X = v, seq)`
if $a \neq 0$, evaluate `seq1`, else `seq2` `if(a, {seq1}, {seq2})`
evaluate `seq` until $a \neq 0$ `until(a, seq)`
while $a \neq 0$, evaluate `seq` `while(a, seq)`
exit `n` innermost enclosing loops `break({n})`
start new iteration of n th enclosing loop `next({n})`
return `x` from current subroutine `return(x)`
error recovery (try `seq1`) `trap({err}, {seq2}, {seq1})`

Input/Output

prettyprint args with/without newline `printp(), printp1()`
print args with/without newline `print(), print1()`
read a string from keyboard `input()`
reorder priority of variables $[x, y, z]$ `reorder({{x, y, z}})`
output `args` in \TeX format `printtex(args)`
write `args` to file `write, write1, writetex(file, args)`
read file into GP `read({file})`

Interface with User and System

allocates a new stack of `s` bytes `allocatemem({s})`
execute system command `a` `system(a)`
as above, feed result to GP `extern(a)`
install function from library `install(f, code, {gpf}, {lib})`
alias `old` to `new` `alias(new, old)`
new name of function `f` in GP 2.0 `whatnow(f)`

User Defined Functions

`name(formal vars) = local(local vars); seq`
`struct.member = seq`
kill value of variable or function `x` `kill(x)`
declare global variables `global(x, ...)`

Iterations, Sums & Products

numerical integration `intnum(X = a, b, expr, {fl})`
sum `expr` over divisors of `n` `sumdiv(n, X, expr)`
sum $X = a$ to $X = b$, initialized at `x` `sum(X = a, b, expr, {x})`
sum of series `expr` `suminf(X = a, expr)`
sum of alternating/positive series `sumalt, sumpos`
product $a \leq X \leq b$, initialized at `x` `prod(X = a, b, expr, {x})`
product over primes $a \leq X \leq b$ `prodeuler(X = a, b, expr)`
infinite product $a \leq X \leq \infty$ `prodinf(X = a, expr)`
real root of `expr` between `a` and `b` `solve(X = a, b, expr)`

Vectors & Matrices

dimensions of matrix x	<code>matsize(x)</code>
concatenation of x and y	<code>concat(x, {y})</code>
extract components of x	<code>vecextract(x, y, {z})</code>
transpose of vector or matrix x	<code>mattranspose(x)</code> or x -
adjoint of the matrix x	<code>matadj(x)</code>
eigenvectors of matrix x	<code>mateigen(x)</code>
characteristic polynomial of x	<code>charpoly(x, {v}, {fl})</code>
trace of matrix x	<code>trace(x)</code>

Constructors & Special Matrices

row vec. of $expr$ eval'ed at $1 \leq X \leq n$	<code>vector(n, {X}, {expr})</code>
col. vec. of $expr$ eval'ed at $1 \leq X \leq n$	<code>vectorv(n, {X}, {expr})</code>
matrix $1 \leq X \leq m, 1 \leq Y \leq n$	<code>matrix(m, n, {X}, {Y}, {expr})</code>
diagonal matrix whose diag. is x	<code>matdiagonal(x)</code>
$n \times n$ identity matrix	<code>matid(n)</code>
Hessenberg form of square matrix x	<code>mathess(x)</code>
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	<code>mathilbert(n)</code>
$n \times n$ Pascal triangle $P_{ij} = \binom{i}{j}$	<code>matpascal(n - 1)</code>
companion matrix to polynomial x	<code>matcompanion(x)</code>

Gaussian elimination

determinant of matrix x	<code>matdet(x, {fl})</code>
kernel of matrix x	<code>matker(x, {fl})</code>
intersection of column spaces of x and y	<code>matintersect(x, y)</code>
solve $M * X = B$ (M invertible)	<code>matsolve(M, B)</code>
as solve, modulo D (col. vector)	<code>matsolvemod(M, D, B)</code>
one sol of $M * X = B$	<code>matinverseimage(M, B)</code>
basis for image of matrix x	<code>matimage(x)</code>
supplement columns of x to get basis	<code>mat supplement(x)</code>
rows, cols to extract invertible matrix	<code>matindexrank(x)</code>
rank of the matrix x	<code>matrank(x)</code>

Lattices & Quadratic Forms

upper triangular Hermite Normal Form	<code>mathnf(x)</code>
HNF of x where d is a multiple of $\det(x)$	<code>mathnfmod(x, d)</code>
vector of elementary divisors of x	<code>matsnf(x)</code>
LLL-algorithm applied to columns of x	<code>qflll(x, {fl})</code>
like <code>qflll</code> , x is Gram matrix of lattice	<code>qflllgram(x, {fl})</code>
LLL-reduced basis for kernel of x	<code>matkerint(x)</code>
\mathbf{Z} -lattice \longleftrightarrow \mathbf{Q} -vector space	<code>matrixqz(x, p)</code>

Quadratic Forms

signature of quad form ${}^t y * x * y$	<code>qfsgn(x)</code>
decomp into squares of ${}^t y * x * y$	<code>qfgaussred(x)</code>
find up to m sols of ${}^t y * x * y \leq b$	<code>qfminim(x, b, m)</code>
eigenvals/eigenvecs for real symmetric x	<code>qfjacobi(x)</code>

Formal & p-adic Series

truncate power series or p -adic number	<code>truncate(x)</code>
valuation of x at p	<code>valuation(x, p)</code>
Dirichlet and Power Series	
Taylor expansion around 0 of f w.r.t. x	<code>taylor(f, x)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(x, y)</code>
$f = \sum a_k * t^k$ from $\sum (a_k/k!) * t^k$	<code>serlaplace(f)</code>
reverse power series F so $F(f(x)) = x$	<code>serreverse(f)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x, y)</code>
Dirichlet Euler product (b terms)	<code>direuler(p = a, b, expr)</code>
p-adic Functions	
square of x , good for 2-adics	<code>sqr(x)</code>
Teichmuller character of x	<code>teichmuller(x)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>

PARI-GP Reference Card

(PARI-GP version 2.1.0)

Polynomials & Rational Functions

degree of f	<code>poldegree(f)</code>
coefficient of degree n of f	<code>polcoeff(f, n)</code>
round coeffs of f to nearest integer	<code>round(f, {\&e})</code>
gcd of coefficients of f	<code>content(f)</code>
replace x by y in f	<code>subst(f, x, y)</code>
discriminant of polynomial f	<code>poldisc(f)</code>
resultant of f and g	<code>polresultant(f, g, {fl})</code>
as above, give $[u, v, d]$, $xu + yv = d$	<code>bezoutres(x, y)</code>
derivative of f w.r.t. x	<code>deriv(f, x)</code>
formal integral of f w.r.t. x	<code>intformal(f, x)</code>
reciprocal poly $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
interpolating poly evaluated at a	<code>polinterpolate(X, {Y}, {a}, {\&e})</code>
initialize t for Thue equation solver	<code>thueinit(f)</code>
solve Thue equation $f(x, y) = a$	<code>thue(t, a, {sol})</code>

Roots and Factorization

number of real roots of f , $a < x \leq b$	<code>polsturm(f, {a}, {b})</code>
complex roots of f	<code>polroots(f)</code>
symmetric powers of roots of f up to n	<code>polysym(f, n)</code>
roots of f mod p	<code>polrootsmod(f, p, {fl})</code>
factor f	<code>factor(f, {lim})</code>
factorization of f mod p	<code>factormod(f, p, {fl})</code>
factorization of f over \mathbf{F}_{p^a}	<code>factorff(f, p, a)</code>
p -adic fact. of f to prec. r	<code>factorpadic(f, p, r, {fl})</code>
p -adic roots of f to prec. r	<code>polrootspadic(f, p, r)</code>
p -adic root of f cong. to a mod p	<code>padicappr(f, a)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>

Special Polynomials

n th cyclotomic polynomial in var. v	<code>polcyclo(n, {v})</code>
d -th degree subfield of $\mathbf{Q}(\zeta_n)$	<code>polsubcyclo(n, d, {v})</code>
n -th Legendre polynomial	<code>pollegendre(n)</code>
n -th Tchebicheff polynomial	<code>pol tchebi(n)</code>
Zagier's polynomial of index n, m	<code>polzagier(n, m)</code>

Transcendental Functions

real, imaginary part of x	<code>real(x), imag(x)</code>
absolute value, argument of x	<code>abs(x), arg(x)</code>
square/ n th root of x	<code>sqr(x), sqrtn(x, n, &z)</code>
trig functions	<code>sin, cos, tan, cotan</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
exponential of x	<code>exp(x)</code>
natural log of x	<code>ln(x) or log(x)</code>
gamma function $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$	<code>gamma(x)</code>
logarithm of gamma function	<code>lngamma(x)</code>
$\psi(x) = \Gamma'(x)/\Gamma(x)$	<code>psi(x)</code>
incomplete gamma function ($y = \Gamma(s)$)	<code>incgam(s, x, {y})</code>
exponential integral $\int_x^\infty e^{-t}/t dt$	<code>eint1(x)</code>
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(x)</code>
dilogarithm of x	<code>dilog(x)</code>
m th polylogarithm of x	<code>polylog(m, x, {fl})</code>
U -confluent hypergeometric function	<code>hyperu(a, b, u)</code>
J -Bessel function $J_{n+1/2}(x)$	<code>besseljh(n, x)</code>
K -Bessel function of index nu	<code>besselk(nu, x)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
give bit number n of integer x	<code>bittest(x, n)</code>
ceiling of x	<code>ceil(x)</code>
floor of x	<code>floor(x)</code>
fractional part of x	<code>frac(x)</code>
round x to nearest integer	<code>round(x, {\&e})</code>
truncate x	<code>truncate(x, {\&e})</code>
gcd of x and y	<code>gcd(x, y)</code>
LCM of x and y	<code>lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

add primes in v to the prime table	<code>addprimes(v)</code>
the n th prime	<code>prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>preprime(x)</code>
factorization of x	<code>factor(x, {lim})</code>
reconstruct x from its factorization	<code>factorback(fa, {nf})</code>

Divisors

number of distinct prime divisors	<code>omega(x)</code>
number of prime divisors with mult	<code>bigomega(x)</code>
number of divisors of x	<code>numdiv(x)</code>
row vector of divisors of x	<code>divisors(x)</code>
sum of (k -th powers of) divisors of x	<code>sigma(x, {k})</code>

Special Functions and Numbers

binomial coefficient $\binom{x}{y}$	<code>binomial(x, y)</code>
Bernoulli number B_n as real	<code>bernreal(n)</code>
Bernoulli vector B_0, B_2, \dots, B_{2n}	<code>bernvec(n)</code>
n th Fibonacci number	<code>fibonacci(n)</code>
Euler ϕ -function	<code>eulerphi(x)</code>
Möbius μ -function	<code>moebius(x)</code>
Hilbert symbol of x and y (at p)	<code>hilbert(x, y, {p})</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>

Miscellaneous

integer or real factorial of x	<code>x!</code> or <code>fact(x)</code>
integer square root of x	<code>sqrint(x)</code>
solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $xu + yv = \gcd(x, y)$	<code>bezout(x, y)</code>
multiplicative order of x (intmod)	<code>znorder(x)</code>
primitive root mod prime power q	<code>znprimroot(q)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
continued fraction of x	<code>contfrac(x, {b}, {lmax})</code>
last convergent of continued fraction x	<code>contfracpnqn(x)</code>
best rational approximation to x	<code>bestappr(x, k)</code>

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>issquare(x, {\&n})</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Based on an earlier version by Joseph H. Silverman
November 2000 v2.11. Copyright © 2000 K. Belabas
GP copyright by The PARI Group

Permission is granted to make and distribute copies of this card provided the copyright and this permission notice are preserved on all copies.
Send comments and corrections to <Karim.BELABAS@math.u-psud.fr>

PARI-GP Reference Card (2)

(PARI-GP version 2.1.0)

Elliptic Curves

Elliptic curve initially given by 5-tuple $E = [a_1, a_2, a_3, a_4, a_6]$.

Points are $[x, y]$, the origin is $[0]$.

Initialize elliptic struct. ell , i.e create `ellinit($E, \{fl\}$)`

$a_1, a_2, a_3, a_4, a_6, b_2, b_4, b_6, b_8, c_4, c_6, disc, j$. This data can be recovered by typing $ell.a_1, \dots, ell.j$. If fl omitted, also

E defined over \mathbf{R}

x -coords. of points of order 2	<code>ell.roots</code>
real and complex periods	<code>ell.omega</code>
associated quasi-periods	<code>ell.eta</code>
volume of complex lattice	<code>ell.area</code>

E defined over $\mathbf{Q}_p, |j|_p > 1$

x -coord. of unit 2 torsion point	<code>ell.roots</code>
Tate's $[u^2, u, q]$	<code>ell.tate</code>
Mestre's w	<code>ell.w</code>

change curve E using $v = [u, r, s, t]$ `ellchangecurve(ell, v)`

change point z using $v = [u, r, s, t]$ `ellchangept(z, v)`

cond, min mod, Tamgawa nmb $[N, v, c]$ `ellglobalred(ell)`

Kodaira type of p fiber of E `elllocalred(ell, p)`

add points $z_1 + z_2$ `elladd(ell, z_1, z_2)`

subtract points $z_1 - z_2$ `ellsub(ell, z_1, z_2)`

compute $n \cdot z$ `ellpow(ell, z, n)`

check if z is on E `ellisoncurve(ell, z)`

order of torsion point z `ellorder(ell, z)`

torsion subgroup with generators `elltors(ell)`

y -coordinates of point(s) for x `ellordinate(ell, x)`

canonical bilinear form taken at z_1, z_2 `ellbil(ell, z_1, z_2)`

canonical height of z `ellheight($ell, z, \{fl\}$)`

height regulator matrix for pts in x `ellheightmatrix(ell, x)`

p th coeff a_p of L -function, p prime `ellap(ell, p)`

k th coeff a_k of L -function `ellak(ell, k)`

vector of first n a_k 's in L -function `ellan(ell, n)`

$L(E, s)$, set $A \approx 1$ `elllseries($ell, s, \{A\}$)`

root number for $L(E, \cdot)$ at p `ellrootno($ell, \{p\}$)`

modular parametrization of E `elltaniyama(ell)`

point $[\wp(z), \wp'(z)]$ corresp. to z `ellztopoint(ell, z)`

complex z such that $p = [\wp(z), \wp'(z)]$ `ellpointtoz(ell, p)`

Elliptic & Modular Functions

arithmetic-geometric mean `agm(x, y)`

elliptic j -function $1/q + 744 + \dots$ `ellj(x)`

Weierstrass σ function `ellsigma($ell, z, \{fl\}$)`

Weierstrass \wp function `ellwp($ell, \{z\}, \{fl\}$)`

Weierstrass ζ function `ellzeta(ell, z)`

modified Dedekind η func. $\prod(1 - q^n)$ `eta($x, \{fl\}$)`

Jacobi sine theta function `theta(q, z)`

k -th derivative at $z=0$ of $\theta(q, z)$ `thetanullk(q, k)`

Weber's f functions `weber($x, \{fl\}$)`

Riemann's zeta $\zeta(s) = \sum n^{-s}$ `zeta(s)`

Graphic Functions

crude graph of $expr$ between a and b `plot($X = a, b, expr$)`

High-resolution plot (immediate plot)

plot $expr$ between a and b `ploth($X = a, b, expr, \{fl\}, \{n\}$)`

plot points given by lists lx, ly `plothraw($lx, ly, \{fl\}$)`

terminal dimensions `plotsizes()`

Rectwindow functions

init window w , with size x, y `plotinit(w, x, y)`

erase window w `plotkill(w)`

copy w to w_2 with offset (dx, dy) `plotcopy(w, w_2, dx, dy)`

scale coordinates in w `plotscale(w, x_1, x_2, y_1, y_2)`

ploth in w `plotrecth($w, X = a, b, expr, \{fl\}, \{n\}$)`

plothraw in w `plotrecthraw($w, data, \{fl\}$)`

draw window w_1 at $(x_1, y_1), \dots$ `plotdraw($[[w_1, x_1, y_1], \dots]$)`

Low-level Rectwindow Functions

set current drawing color in w to c `plotcolor(w, c)`

current position of cursor in w `plotcursor(w)`

write s at cursor's position `plotstring(w, s)`

move cursor to (x, y) `plotmove(w, x, y)`

move cursor to $(x + dx, y + dy)$ `plotrmove(w, dx, dy)`

draw a box to (x_2, y_2) `plotbox(w, x_2, y_2)`

draw a box to $(x + dx, y + dy)$ `plotrbox(w, dx, dy)`

draw polygon `plotlines($w, lx, ly, \{fl\}$)`

draw points `plotpoints(w, lx, ly)`

draw line to $(x + dx, y + dy)$ `plotrline(w, dx, dy)`

draw point $(x + dx, y + dy)$ `plotrpoint(w, dx, dy)`

Postscript Functions

as ploth `psploth($X = a, b, expr, \{fl\}, \{n\}$)`

as plothraw `psplothraw($lx, ly, \{fl\}$)`

as plotdraw `psdraw($[[w_1, x_1, y_1], \dots]$)`

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) `Qfb($a, b, c, \{d\}$)`

reduce x ($s = \sqrt{D}, l = \lfloor s \rfloor$) `qfbred($x, \{fl\}, \{D\}, \{l\}, \{s\}$)`

composition of forms $x * y$ or `qfbnucomp(x, y, l)`

n -th power of form x^n or `qfbnupow(x, n)`

composition without reduction `qfbcompraw(x, y)`

n -th power without reduction `qfbpowraw(x, n)`

prime form of disc. x above prime p `qfbprimeform(x, p)`

class number of disc. x `qfbclassno(x)`

Hurwitz class number of disc. x `qfbhclassno(x)`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ `quadgen(x)`

minimal polynomial of ω `quadpoly(x)`

discriminant of $\mathbf{Q}(\sqrt{D})$ `quaddisc(x)`

regulator of real quadratic field `quadregulator(x)`

fundamental unit in real $\mathbf{Q}(x)$ `quadunit(x)`

class group of $\mathbf{Q}(\sqrt{D})$ `quadclassint($D, \{fl\}, \{t\}$)`

Hilbert class field of $\mathbf{Q}(\sqrt{D})$ `quadhilbert($D, \{fl\}$)`

ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ `quadray($D, f, \{fl\}$)`

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure nf `nfinit($f, \{fl\}$)`

nf members:

polynomial defining $nf, f(\theta) = 0$	<code>nf.pol</code>
number of [real,complex] places	<code>nf.sign</code>
discriminant of nf	<code>nf.disc</code>
T_2 matrix	<code>nf.t2</code>
vector of roots of f	<code>nf.roots</code>
integral basis of \mathbf{Z}_K as powers of θ	<code>nf.zk</code>
different	<code>nf.diff</code>
codifferent	<code>nf.codiff</code>

recompute nf using current precision `nfnewprec(nf)`

init relative rnf given by $g = 0$ over K `rnfinit(nf, g)`

init big number field structure bnf `bnfinit($f, \{fl\}$)`

bnf members: same as nf , plus

underlying nf	<code>bnf.nf</code>
classgroup	<code>bnf.clgp</code>
regulator	<code>bnf.reg</code>
fundamental units	<code>bnf.fu</code>
torsion units	<code>bnf.tu</code>
$[tu, fu], [fu, tu]$	<code>bnf.tufu/futu</code>

compute a bnf from small bnf `bnfmake($sbnf$)`

add S -class group and units, yield $bnfs$ `bnfsunit(nf, S)`

init class field structure bnr `bnrinit($bnf, m, \{fl\}$)`

bnr members: same as bnf , plus

underlying bnf	<code>bnr.bnf</code>
structure of $(\mathbf{Z}_K/m)^*$	<code>bnr.zkst</code>

Simple Arithmetic Invariants (nf)

Elements are rational numbers, polynomials, polmods, or column vectors (on integral basis $nf.zk$).

integral basis of field def. by $f = 0$ **nfbasis**(f)
 field discriminant of field $f = 0$ **nfdisc**(f)
 reverse polmod $a = A(X) \bmod T(X)$ **modreverse**(a)
 Galois group of field $f = 0$, $\deg f \leq 11$ **polgalois**(f)
 smallest poly defining $f = 0$ **polredabs**($f, \{fl\}$)
 small polys defining subfields of $f = 0$ **polred**($f, \{fl\}, \{p\}$)
 small polys defining suborders of $f = 0$ **polredord**(f)
 poly of degree $\leq k$ with root $x \in \mathbf{C}$ **algdep**(x, k)
 small linear rel. on coords of vector x **lindep**(x)
 are fields $f = 0$ and $g = 0$ isomorphic? **nfisom**(f, g)
 is field $f = 0$ a subfield of $g = 0$? **nfisincl**(f, g)
 compositum of $f = 0, g = 0$ **polcompositum**($f, g, \{fl\}$)
 basic element operations (prefix **nfelt**):

 (**nfelt**)**mul, pow, div, diveuc, mod, divrem, val**
 express x on integer basis **nfaltobasis**(nf, x)
 express element x as a polmod **nfbasistoalg**(nf, x)
 quadratic Hilbert symbol (at p) **nfhilbert**($nf, a, b, \{p\}$)
 roots of g belonging to **nf** **nfroots**(nf, g)
 factor g in **nf** **nffactor**(nf, g)
 factor g mod prime pr in **nf** **nffactormod**(nf, g, pr)
 number of roots of 1 in **nf** **nfrootof1**(nf)
 conjugates of a root θ of **nf** **nfgaloisconj**($nf, \{fl\}$)
 apply Galois automorphism s to x **nfgaloisapply**(nf, s, x)
 subfields (of degree d) of **nf** **nfsubfields**($nf, \{d\}$)

Dedekind Zeta Function ζ_K

ζ_K as Dirichlet series, $N(I) < b$ **dirzetak**(nf, b)
 init nfz for field $f = 0$ **zetakinit**(f)
 compute $\zeta_K(s)$ **zetak**($nfz, s, \{fl\}$)
 Artin root number of K **bnrrootnumber**($bnr, chi, \{fl\}$)

Class Groups & Units (bnf, bnr)

$a1, \{a2\}, \{a3\}$ usually $bnr, subgp$ or $bnf, module, \{subgp\}$
 remove GRH assumption from bnf **bnfcertify**(bnf)
 expo. of ideal x on class gp **bnfisprincipal**($bnf, x, \{fl\}$)
 expo. of ideal x on ray class gp **bnrisprincipal**($bnr, x, \{fl\}$)
 expo. of x on fund. units **bnfisunit**(bnf, x)
 as above for S -units **bnfissunit**($bnfs, x$)
 fundamental units of bnf **bnfunit**(bnf)
 signs of real embeddings of $bnf.fu$ **bnfsignunit**(bnf)

Class Field Theory

ray class group structure for mod. m **bnrclass**($bnf, m, \{fl\}$)
 ray class number for mod. m **bnrclassno**(bnf, m)
 discriminant of class field ext **bnrdisc**($a1, \{a2\}, \{a3\}$)
 ray class numbers, l list of mods **bnrclassnolist**(bnf, l)
 discriminants of class fields **bnrdisc**($bnf, l, \{arch\}, \{fl\}$)
 decode output from **bnrdisc**list **bnfdecodemodule**(nf, fa)
 is modulus the conductor? **bnrisconductor**($a1, \{a2\}, \{a3\}$)
 conductor of character chi **bnrconductorofchar**(bnr, chi)
 conductor of extension **bnrconductor**($a1, \{a2\}, \{a3\}, \{fl\}$)
 conductor of extension def. by g **rnfconductor**(bnf, g)
 Artin group of ext. def'd by g **rnfnormgroup**(bnr, g)
 subgroups of **bnr**, index $\leq b$ **subgroup**($bnr, b, \{fl\}$)
 rel. eq. for class field def'd by sub **rnfkummer**($bnr, sub, \{d\}$)
 same, using Stark units (real field) **bnrstark**($bnr, sub, \{fl\}$)

PARI-GP Reference Card (2)

(PARI-GP version 2.1.0)

Ideals

Ideals are elements, primes, or matrix of generators in HNF.
 is id an ideal in **nf**? **nfisideal**(nf, id)
 is x principal in **bnf**? **bnfisprincipal**(bnf, x)
 principal ideal generated by x **idealprincipal**(nf, x)
 principal ideale generated by x **ideleprincipal**(nf, x)
 give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$ **idealtwoelt**($nf, x, \{a\}$)
 put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form **idealhnf**($nf, a, \{b\}$)
 norm of ideal x **idealnrm**(nf, x)
 minimum of ideal x (direction v) **idealmin**(nf, x, v)
 LLL-reduce the ideal x (direction v) **idealred**($nf, x, \{v\}$)

Ideal Operations

add ideals x and y **idealadd**(nf, x, y)
 multiply ideals x and y **idealmul**($nf, x, y, \{fl\}$)
 intersection of ideals x and y **idealintersect**($nf, x, y, \{fl\}$)
 n -th power of ideal x **idealpow**($nf, x, n, \{fl\}$)
 inverse of ideal x **idealinv**(nf, x)
 divide ideal x by y **idealdiv**($nf, x, y, \{fl\}$)
 Find $[a, b] \in x \times y, a + b = 1$ **idealaddtoone**($nf, x, \{y\}$)

Primes and Multiplicative Structure

factor ideal x in **nf** **idealfactor**(nf, x)
 recover x from its factorization in **nf** **factorback**(x, nf)
 decomposition of prime p in **nf** **idealprimedec**(nf, p)
 valuation of x at prime ideal pr **idealval**(nf, x, pr)
 weak approximation theorem in **nf** **idealchinese**(nf, x, y)
 give $bid = \text{structure of } (\mathbf{Z}_K/id)^*$ **idealstar**($nf, id, \{fl\}$)
 discrete log of x in $(\mathbf{Z}_K/bid)^*$ **ideallog**(nf, x, bid)
idealstar of all ideals of norm $\leq b$ **ideallist**($nf, b, \{fl\}$)
 add archimedean places **ideallistarch**($nf, b, \{ar\}, \{fl\}$)
 init **prmod** structure **nfmodprinit**(nf, pr)
 kernel of matrix M in $(\mathbf{Z}_K/pr)^*$ **nfkermodpr**($nf, M, prmod$)
 solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$ **nfsolvemodpr**($nf, M, B, prmod$)

Relative Number Fields (rnf)

Extension L/K is defined by $g \in K[x]$. We have $order \subset L$.
 absolute equation of L **rnfequation**($nf, g, \{fl\}$)

Lifts and Push-downs

absolute \rightarrow relative repres. for x **rnfeltabstorel**(rnf, x)
 relative \rightarrow absolute repres. for x **rnfeltreltoabs**(rnf, x)
 lift x to the relative field **rnfeltup**(rnf, x)
 push x down to the base field **rnfeltdown**(rnf, x)
 idem for x ideal: (**rnfideal**)**reltoabs, abstorel, up, down**
 relative **nfaltobasis** **rnfaltobasis**(rnf, x)
 relative **nfbasistoalg** **rnfbasistoalg**(rnf, x)
 relative **idealhnf** **rnfidealhnf**(rnf, x)
 relative **idealmul** **rnfidealmul**(rnf, x, y)
 relative **idealtwoelt** **rnfidealtwoelt**(rnf, x)

Projective \mathbf{Z}_K -modules, maximal order

relative **polred** **rnfpolred**(nf, g)
 relative **polredabs** **rnfpolredabs**(nf, g)
 characteristic poly. of $a \bmod g$ **rnfcharpoly**($nf, g, a, \{v\}$)
 relative Dedekind criterion, prime pr **rnfdedekind**(nf, g, pr)
 discriminant of relative extension **rnfdisc**(nf, g)
 pseudo-basis of \mathbf{Z}_L **rnfipseudobasis**(nf, g)
 relative HNF basis of $order$ **rnfhnfbasis**($bnf, order$)
 reduced basis for $order$ **rnfllgram**($nf, g, order$)
 determinant of pseudo-matrix A **rnfdet**(nf, A)
 Steinitz class of $order$ **rnfsteinitz**($nf, order$)
 is $order$ a free \mathbf{Z}_K -module? **rnfisfree**($bnf, order$)
 true basis of $order$, if it is free **rnfbasis**($bnf, order$)

Norms

absolute norm of ideal x **rnfidealnrmabs**(rnf, x)
 relative norm of ideal x **rnfidealnrmrel**(rnf, x)
 solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$ **bnfisintnorm**(bnf, x)
 is $x \in \mathbf{Q}$ a norm from K ? **bnfisnorm**($bnf, x, \{fl\}$)
 is $x \in K$ a norm from L ? **rnfisnorm**($bnf, ext, x, \{fl\}$)

Based on an earlier version by Joseph H. Silverman
 November 2000 v2.11. Copyright © 2000 K. Belabas
 GP copyright by The PARI Group

Permission is granted to make and distribute copies of this card provided
 the copyright and this permission notice are preserved on all copies.

Send comments and corrections to <Karim.BELABAS@math.u-psud.fr>