

Measuring the Balmer Lines and the Proton-Deuterium Nuclear Mass Ratio via Optical Spectroscopy

Matthew Houston

Junior Lab

Theory: A Model of the Atom

Solving the 3D Schrödinger equation for a Coulomb potential yields energy eigenvalues:

$$E_n = -R_\infty \frac{ch}{n^2} \qquad R_\infty = \frac{m_e e^4}{8h^3 \epsilon_0^2 c}$$

- Assuming transitions between energy states occur due to emission of photons:

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \quad m \leq n \quad n, m = 1, 2, 3, \dots$$

- Transitions with $m=2$ are known as the Balmer Lines

Theory: The Effects of Finite Mass

The assumption of an infinitely massive nucleus is reasonable:

$$\frac{m_e}{M_H} \approx \frac{1}{1800} \ll 1$$

- Borrowing a more exact result from astrophysics:

$$\mu = \frac{m_e M}{m_e + M} \qquad R_\mu = \frac{\mu}{m_e} R_\infty$$

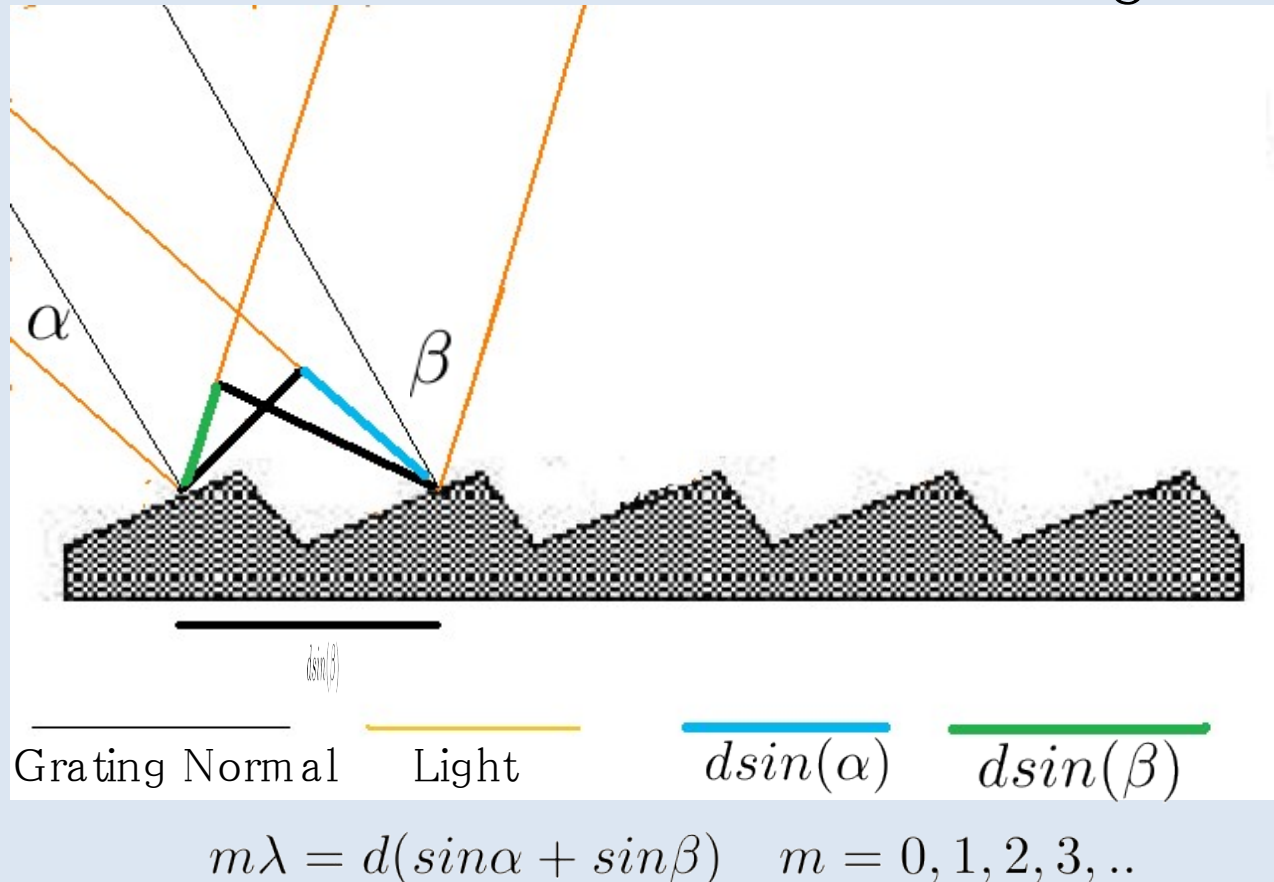
$$\frac{1}{\lambda} = R_\mu \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \quad m \leq n \quad n, m = 1, 2, 3, \dots$$

- Which when applied hydrogen and deuterium:

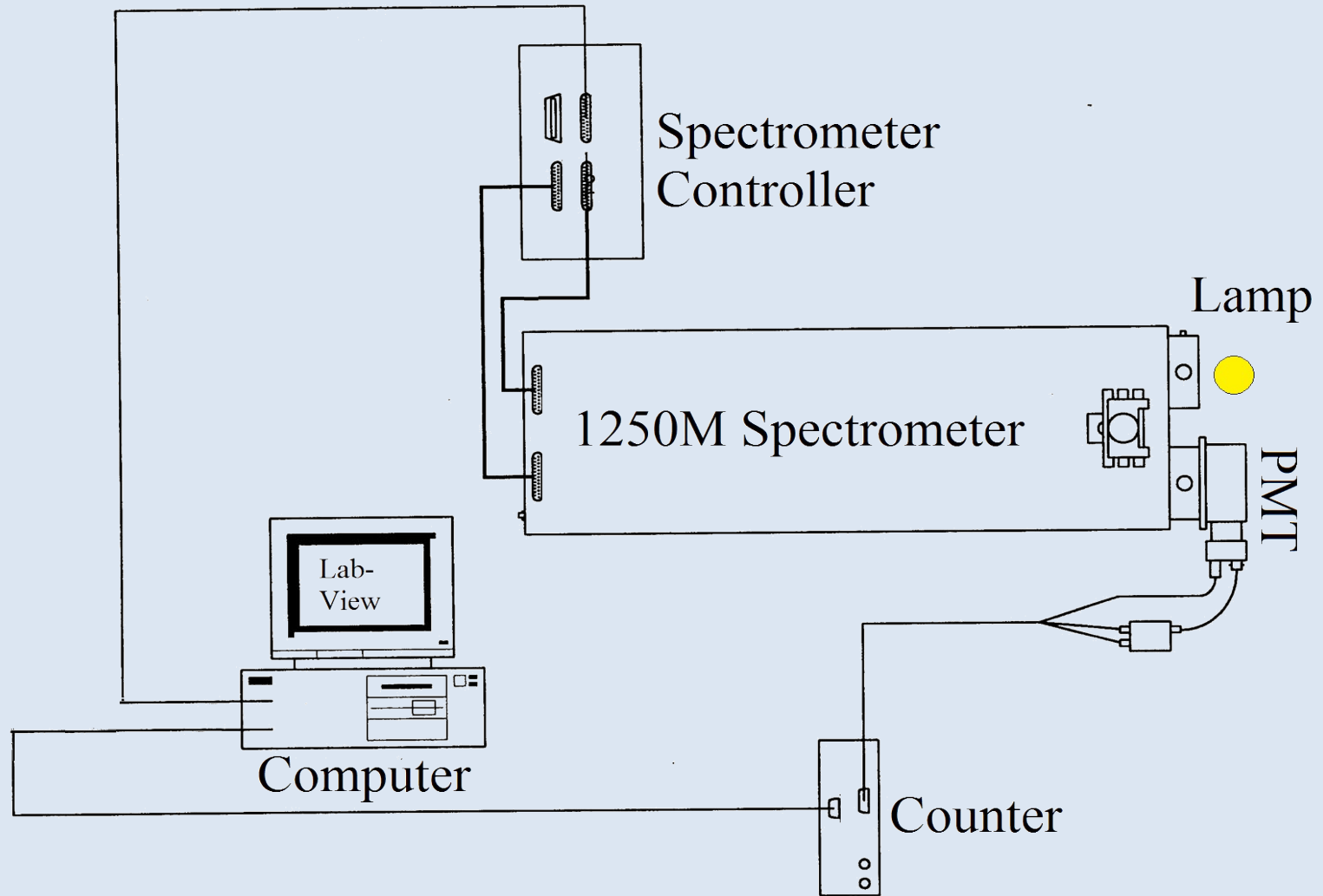
$$\frac{M_H}{M_D} = 1 - \left(1 + \frac{M_H}{m_e} \right) \frac{\Delta\lambda}{\lambda_H}$$

Theory: Spectrometry

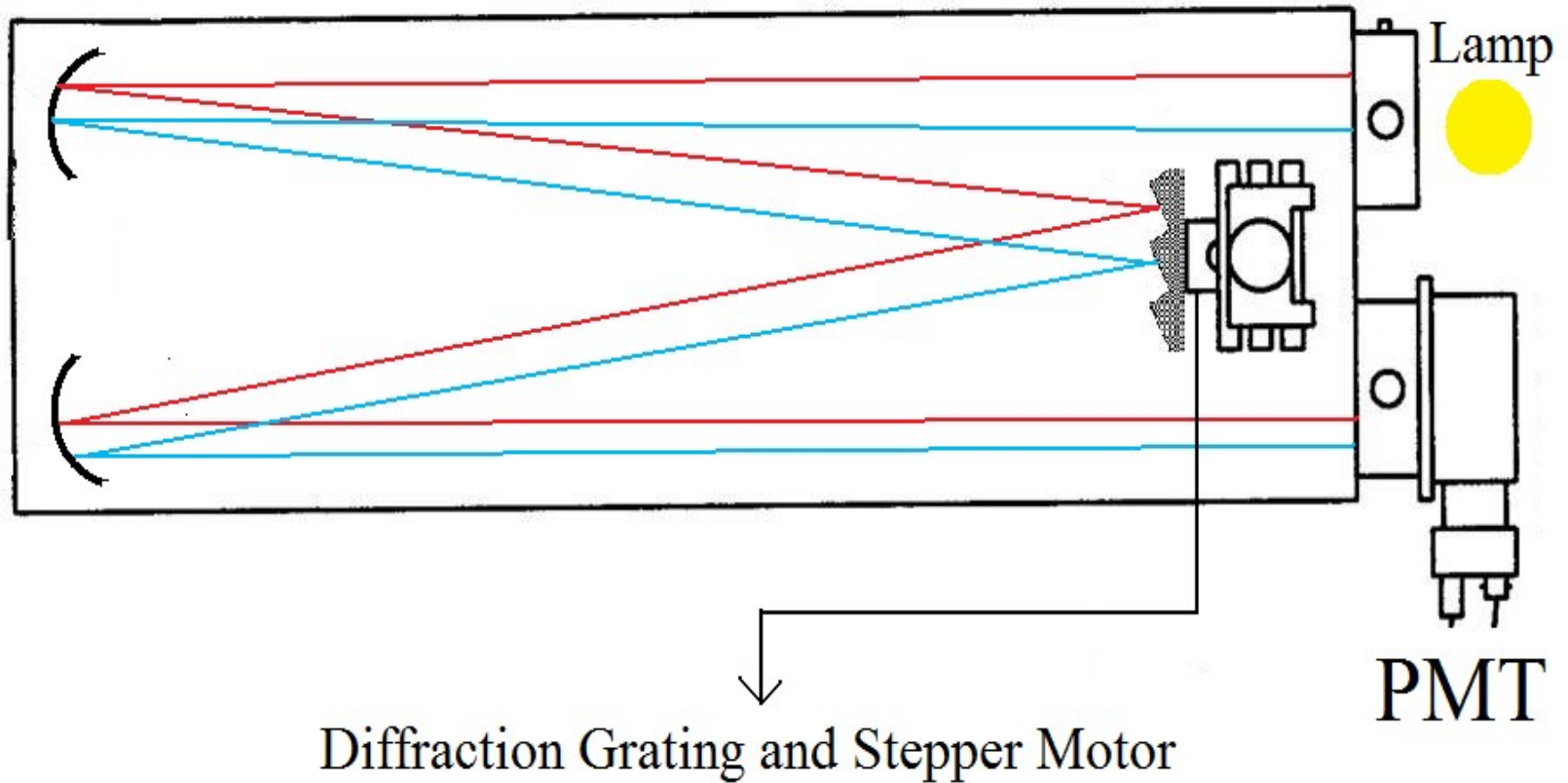
By creating differences in path length, it is possible to isolate various wavelengths



Experiment Setup: Overview



Experimental Set Up:



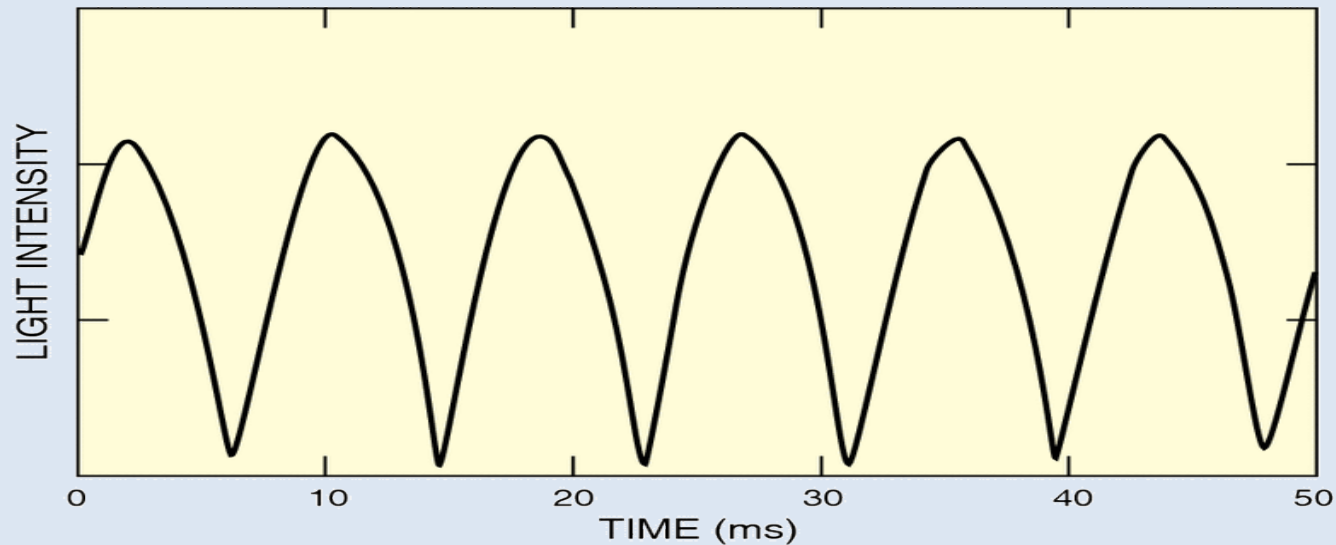
- Not to Scale
- Line colors do **NOT** indicate wavelength

Analysis: Sources of Error

Experience from previous experiments and dark count data → Poisson counting

Time-dependent output from lamps is another source of error

Short term temporal behavior of 65010 Cesium Lamp*



*<http://www.newport.com/High-Power-Spectral-Calibration-Lamps/378229/1033/catalog.aspx>

Analysis: Overview

Spectral lines appear as peaks

These peaks can be fit to Gaussians giving a wavelength

Wavelengths from the spectrometer are in error, necessitating calibration

Analysis: Fitting to the Peaks

Hg, H, and D peaks are fit to a sum of

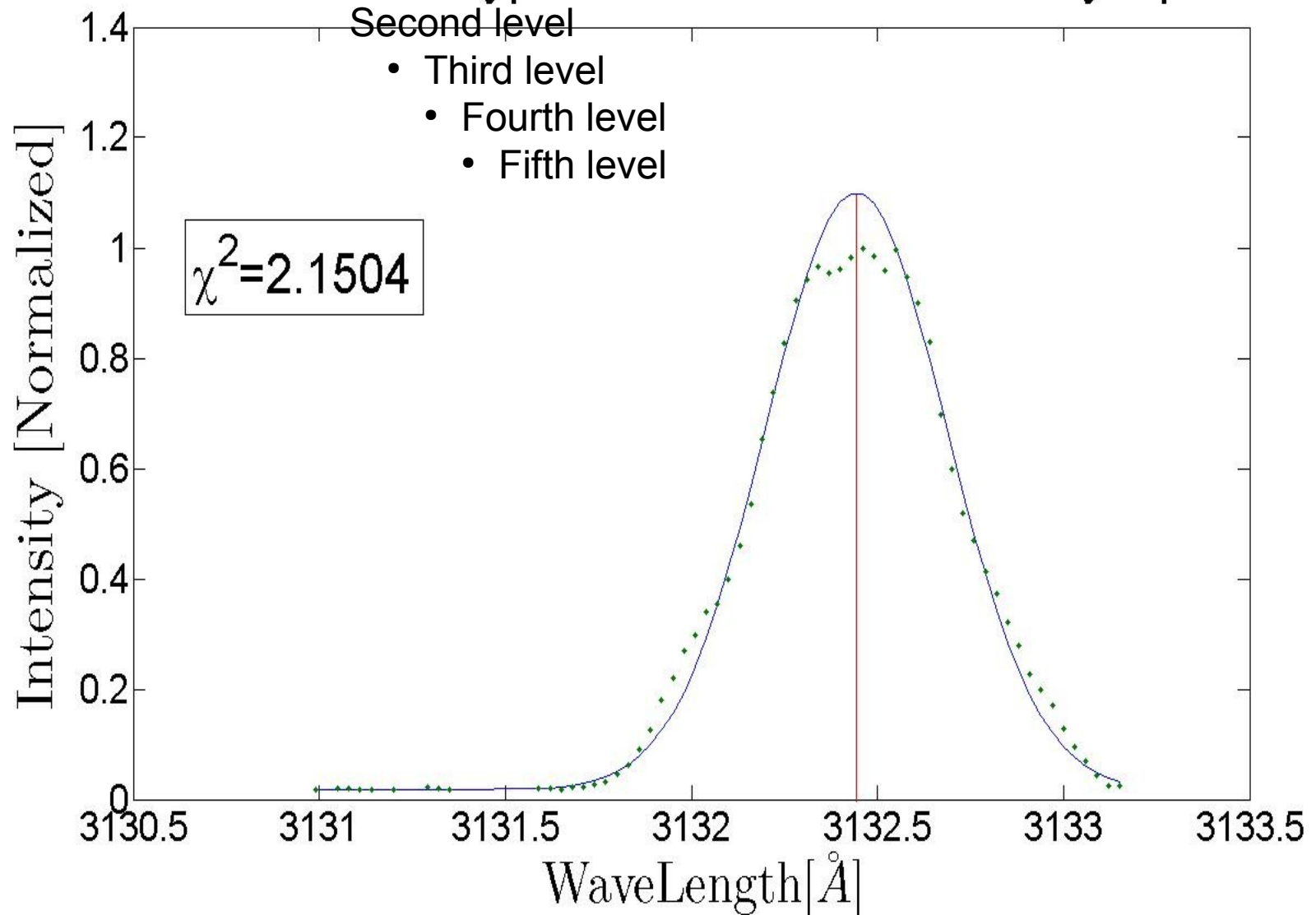
Gaussians:

$$I(\lambda) = a_1 \exp\left(-\left(\frac{\lambda - a_2}{a_3}\right)^2\right) + a_4 \exp\left(-\left(\frac{\lambda - a_5}{a_6}\right)^2\right) + a_7$$

- Chi-squared values range from 0.5–3
- In the mercury spectrum, the second Gaussian accounts for doublet splitting.
- In deuterium, the first Gaussian fits the hydrogen peak and the second fits the deuterium peak.

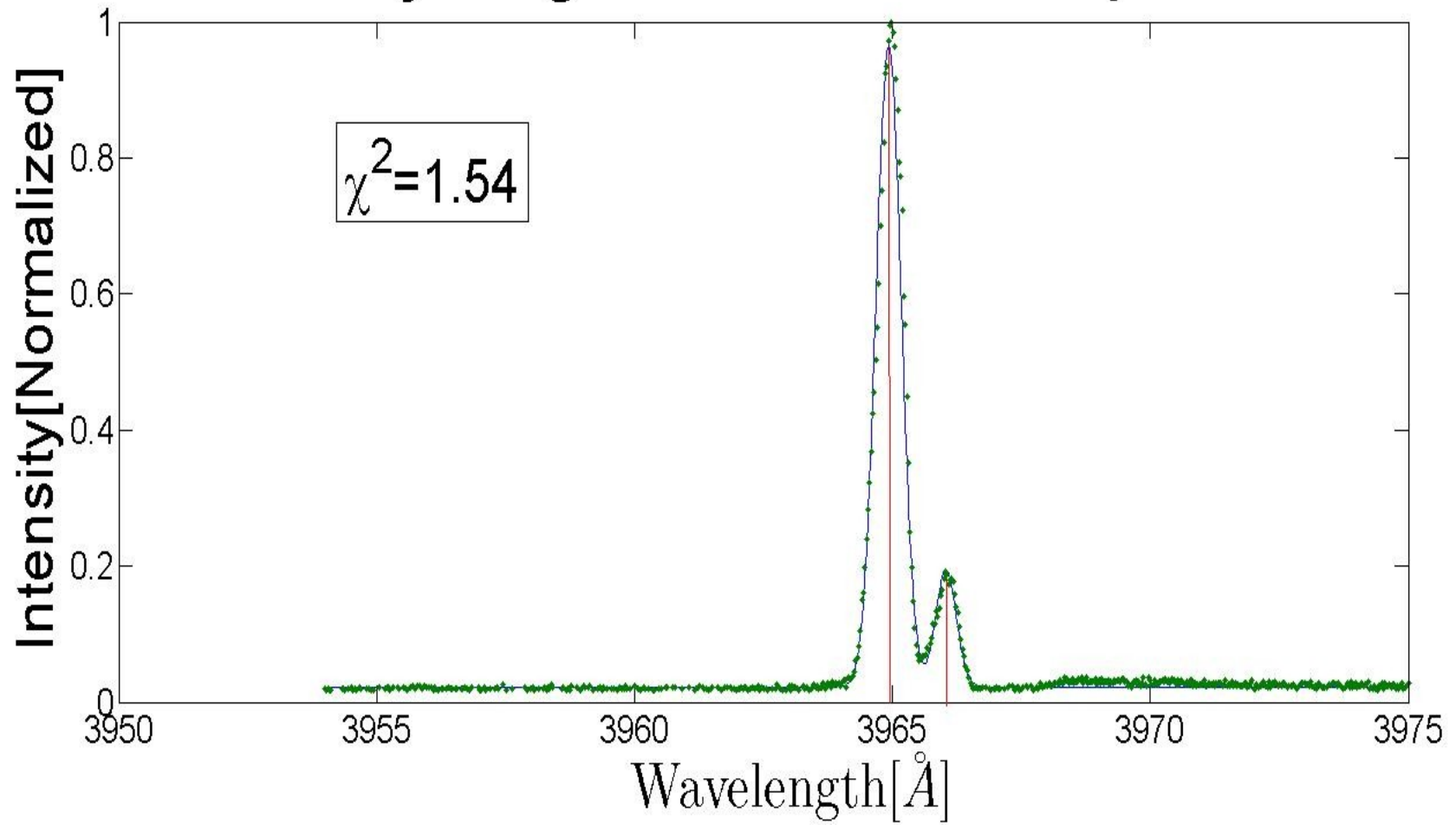
Analysis: Fitting to the Peaks

Gaussian Fit to a Typical Peak in the Mercury Spectrum



Analysis: Fitting to the Peaks

Sum of Gaussians Fit to a Typical Peak in the Hydrogen-Deuterium Spectrum



Analysis: The Calibration Curve

Comparing known mercury lines to observed values gives a calibration function:

$$\lambda_{obs} = -1.071 \cdot 10^{-6} \lambda_{true}^2 + 1.002 \lambda_{true} + 5.94$$

- This function has residuals ≈ 0.2 Angstroms
- Quadratic term indicates a linear approximation in the spectrometer

Analysis: Final Steps

Once peaks are identified and fit to the calibration function gives true wavelengths

Deuterium peaks are plugged into:

$$\frac{M_H}{M_D} = 1 - \left(1 + \frac{M_H}{m_e}\right) \frac{\Delta\lambda}{\lambda_H}$$

- First order error propagation.

Conclusion: The Balmer Lines

Actual [Å]	Measured [Å]	Error/ σ	Rydberg's Result [Å]
6562.85	6562.66 $\pm$ 0.18	1.02	6564.67
4861.33	4861.26 $\pm$ 0.19	0.36	4862.72
4340.47	4340.48 $\pm$ 0.19	0.04	4341.71
4101.74	4101.91 $\pm$ 0.17	1.04	4102.92
3970.07	3970.29 $\pm$ 0.21	1.09	3971.22
3889.05	3888.73 $\pm$ 0.23	1.39	3890.17

Strongly supports the Quantum model of the atom
Lends credibility to the Rydberg formula

Conclusion: The Hydrogen Deuterium Mass Ratio

The Hydrogen Deuterium Mass Ratio was measured to be:

$$\frac{M_H}{M_D} = 0.478 \pm 0.1002$$

- The actual value is 0.49996 which differs from our value by 0.207 standard deviations.
- Assuming knowledge of protons and neutrons, our results suggest a 1-proton 1-neutron nucleus.

Conclusion: Wave Particle Duality

The spectrometer operates via destructive interference suggesting that light is a wave.

The photomultiplier tube observes distinct counts suggesting light is a particle.

Questions?